

White Paper

The CBOE Volatility Index - VIX®

➤ The powerful and flexible trading and risk management tool from the Chicago Board Options Exchange

In 1993, the Chicago Board Options Exchange® (CBOE®) introduced the CBOE Volatility Index® (VIX® Index), which was originally designed to measure the market's expectation of 30-day volatility implied by at-the-money S&P 100® Index (OEX® Index) option prices. The VIX Index soon became the premier benchmark for U.S. stock market volatility. It is regularly featured in the Wall Street Journal, Barron's and other leading financial publications, as well as business news shows on CNBC, BloombergTV and CNN/Money, where VIX is often referred to as the "fear index."

Ten years later in 2003, CBOE together with Goldman Sachs, updated the VIX to reflect a new way to measure expected volatility, one that continues to be widely used by financial theorists, risk managers and volatility traders alike. The new VIX is based on the S&P 500® Index (SPXSM), the core index for U.S. equities, and estimates expected volatility by averaging the weighted prices of SPX puts and calls over a wide range of strike prices. By supplying a script for replicating volatility exposure with a portfolio of SPX options, this new methodology transformed VIX from an abstract concept into a practical standard for trading and hedging volatility.

In 2014, CBOE enhanced the VIX Index to include series of SPX WeeklysSM. First introduced by CBOE in 2005, weekly options are now available on hundreds of indexes, equities, ETFs and ETNs and have become a very popular and actively-traded risk management tool. Today, SPX Weeklys account for one-third of all SPX options traded, and average over a quarter of a million contracts traded per day¹.

The inclusion of SPX Weeklys allows the VIX Index to be calculated with S&P 500 Index option series that most precisely match the 30-day target timeframe for expected volatility that the VIX Index is intended to represent. Using SPX options with more than 23 days and less than 37 days to expiration ensures that the VIX Index will always reflect an interpolation of two points along the S&P 500 volatility term structure.

Volatility as a tradable asset: VIX Futures & Options

On March 24, 2004, CBOE introduced the first exchange-traded VIX futures contract on its new, all-electronic CBOE Futures ExchangeSM (CFE®). Two years later in February 2006, CBOE launched VIX options, the most successful new product in CBOE history. In just ten years since the launch, combined trading activity in VIX options and futures has grown to over 800,000 contracts per day.

The negative correlation of volatility to stock market returns is well documented and suggests a diversification benefit to including volatility in an investment portfolio. VIX futures and options are designed to deliver pure volatility exposure in a single, efficient package. CBOE/CFE provides a continuous, liquid and transparent market for VIX products that are available to all investors from the smallest retail trader to the largest institutional money managers and hedge funds.

Beyond VIX

In addition to the VIX Index, CBOE calculates several other volatility indexes including the CBOE Short-Term Volatility Index (VXSTSM) - which reflects 9-day expected volatility of the S&P 500 Index, as well as the CBOE Nasdaq-100® Volatility Index (VXNSM), CBOE DJIA® Volatility Index (VXDSM), CBOE Russell 2000® Volatility Index (RVXSM) and CBOE S&P 500® 3-Month Volatility Index (VXVSM) and the CBOE S&P 500® 6-Month Volatility Index (VXMTSM). Currently, VXST, VXN and RVX futures are listed on CFE; VXST and RVX options trade on CBOE.

In 2008, CBOE pioneered the use of the VIX methodology to estimate expected volatility of certain commodities and foreign currencies. The CBOE Crude Oil ETF Volatility Index (OVXSM), CBOE Gold ETF Volatility Index (GVZSM) and CBOE EuroCurrency ETF Volatility Index (EVZSM) use exchange-traded fund options based on the United States Oil Fund, LP (USO), SPDR Gold Shares (GLD) and CurrencyShares Euro Trust (FXE), respectively.

¹ YTD through August 2014.

CBOE has since introduced several new volatility indexes, including volatility indexes based on individual stocks:

CBOE U.S. Energy Sector ETF Volatility Index (VXXLESM)

CBOE Emerging Markets ETF Volatility Index (VXEEMSM)

CBOE EFA ETF Volatility Index (VXEFASM)

CBOE Gold Miners ETF Volatility Index (VXGDXSM)

CBOE Silver ETF Volatility Index (VXSLVSM)

CBOE Brazil ETF Volatility Index (VXEZWSM)

CBOE China ETF Volatility Index (VXFXISM)

CBOE Equity VIX® on Apple (VXAPLSM)

CBOE Equity VIX® on Amazon (VXAZNSM)

CBOE Equity VIX® on Goldman Sachs (VXGSSM)

CBOE Equity VIX® on Google (VXGOGSM)

CBOE Equity VIX® on IBM (VXIBMSM)

As of August 2014, security futures on OVX, GVZ, VXEEM and VXEZW are listed at CFE. CBOE lists options on OVX, GVZ, VXEEM and VXEZW as well.

Historical Prices: VIX and Other Volatility Indexes

Perhaps one of the most valuable features of the VIX Index is the existence of more than 20 years of historical prices. This extensive data set provides investors with a useful perspective of how option prices have behaved in response to a variety of market conditions. Price history for the original CBOE Volatility Index (VXO) based on OEX options is available from 1986 to the present. CBOE has created a similar historical record for the new VIX Index dating back to 1990 so that investors can compare the new VIX Index with VXO, which reflects information about the volatility “skew” or “smile.” Historical prices for VIX, VXO and CBOE’s other volatility indexes may be found on the CBOE website at <http://www.cboe.com/volatility> under *CBOE Volatility Indexes*.

The VIX Calculation: Step-by-Step

Stock indexes, such as the S&P 500, are calculated using the prices of their component stocks. Each index employs rules that govern the selection of component securities and a formula to calculate index values.

The VIX Index is a volatility index comprised of options rather than stocks, with the price of each option reflecting the market's expectation of future volatility. Like conventional indexes, the VIX calculation employs rules for selecting component options and a formula to calculate index values.

The generalized formula used in the VIX calculation⁵ is:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \quad (1)$$

WHERE...

σ is	$VIX/100 \Rightarrow VIX = \sigma \times 100$
T	Time to expiration
F	Forward index level desired from index option prices
K_0	First strike below the forward index level, F
K_i	Strike price of the ith out-of-the-money option; a call if $K_i > K_0$; and a put if $K_i < K_0$; both put and call if $K_i = K_0$.
ΔK_i	Interval between strike prices - half the difference between the strike on either side of K_i :

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

(Note: ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

R	Risk-free interest rate to expiration
$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike K_i .

⁵ Please see "More than you ever wanted to know about volatility swaps" by Kresimir Demeterfi, Emanuel Derman, Michael Kamal and Joseph Zou, Goldman Sachs Quantitative Strategies Research Notes, March 1999.

GETTING STARTED

The VIX calculation measures 30-day expected volatility of the S&P 500 Index. The components of the VIX calculation are near- and next-term put and call options with more than 23 days and less than 37 days to expiration. These include SPX options with “standard” 3rd Friday expiration dates and “weekly” SPX options that expire every Friday, except the 3rd Friday of each month. Once each week, the SPX options used to calculate VIX “roll” to new contract maturities. For example, on the second Tuesday in October, the VIX index would be calculated using SPX options expiring 24 days later (i.e., “near-term”) and 31 days later (i.e., “next-term”). On the following day, the SPX options that expire in 30 calendar days would become the “near-term” options and SPX options that expire in 37 calendar days would be the “next-term” options.

In this hypothetical example, the near-term options are “standard” SPX options with 25 days to expiration, the next-term options are P.M.-settled SPX Weeklys with 32 days to expiration; and the calculation reflects prices observed at 9:46 a.m. Chicago time. For the purpose of calculating time to expiration, “standard” SPX options are deemed to expire at the open of trading on SPX settlement day - the third Friday of the month², and “weekly” SPX options are deemed to expire at the close of trading (i.e., 3:00 p.m. CT).

The VIX calculation measures time to expiration, T , in calendar days and divides each day into minutes in order to replicate the precision that is commonly used by professional option and volatility traders. The time to expiration is given by the following expression:

$$T = \{M_{\text{Current day}} + M_{\text{Settlement day}} + M_{\text{Other days}}\} / \text{Minutes in a year}$$

WHERE...

$M_{\text{Current Day}}$ = minutes remaining until midnight of the current day

$M_{\text{Settlement day}}$ = minutes from midnight until 8:30 a.m. for “standard” SPX expirations; or minutes from midnight until 3:00 p.m. for “weekly” SPX expirations

$M_{\text{Other days}}$ = total minutes in the days between current day and expiration day

Using 9:46 a.m. as the time of the calculation, T for the near-term and next-term options, T_1 and T_2 , respectively, is:

$$T_1 = \{854 + 510 + 34,560\} / 525,600 = 0.0683486$$

$$T_2 = \{854 + 900 + 44,640\} / 525,600 = 0.0882686$$

The risk-free interest rates, R_1 and R_2 , are the bond-equivalent yields of the U.S. T-bill maturing closest to the expiration dates of relevant SPX options. As such, the VIX calculation may use different risk-free interest rates for near- and next-term options. In this example, assume that $R_1 = 0.0305\%$ for the near term options and that $R_2 = 0.0286\%$ for the next term options. Note in this example, T_2 uses a value of 900 for $M_{\text{Settlement day}}$, which reflects the 3:00 p.m. expiration time of the next-term SPX Weeklys options.

Since many of the interim calculations are repetitive, only representative samples appear below. The complete set of SPX option data and calculations may be found in *Appendix 1*.

²Technically, the expiration date for “standard” SPX options is the “Saturday following the 3rd Friday of the expiration month.” In this example, however, expiration is deemed to take place at the determination of the exercise settlement value of the SPX, which is based on the opening prices of SPX component securities.

STEP 1: Select the options to be used in the VIX calculation

The selected options are out-of-the-money SPX calls and out-of-the-money SPX puts centered around an at-the-money strike price, K_0 . Only SPX options quoted with non-zero bid prices are used in the VIX calculation.

One important note: as volatility rises and falls, the strike price range of options with non-zero bids tends to expand and contract. As a result, the number of options used in the VIX calculation may vary from month-to-month, day-to-day and possibly, even minute-to-minute.

For **each** contract month:

- Determine the forward SPX level, F , by identifying the strike price at which the absolute difference between the call and put prices is smallest. The call and put prices in the following table reflect the average of each option's bid / ask quotation. As shown below, the difference between the call and put prices is smallest at the 1965 strike for the near- and the 1960 strike for the next-term options.

Near Term Options				Next Term Options			
Strike Price	Call	Put	Difference	Strike Price	Call	Put	Difference
.
1940	38.45	15.25	23.20	1940	41.05	18.80	22.25
1945	34.70	16.55	18.15	1945	37.45	20.20	17.25
1950	31.10	18.25	12.85	1950	34.05	21.60	12.45
1955	27.60	19.75	7.85	1955	30.60	23.20	7.40
1960	24.25	21.30	2.95	1960	27.30	24.90	2.40
1965	21.05	23.15	2.10	1965	24.15	26.90	2.75
1970	18.10	25.05	6.95	1970	21.10	28.95	7.85
1975	15.25	27.30	12.05	1975	18.30	31.05	12.75
1980	12.75	29.75	17.00	1980	15.70	33.50	17.80

Using the **1965** call and put in the near-term, and the **1960** call and put in the next-term contract applied to the formula:

$$F = \text{Strike Price} + e^{RT} \times (\text{Call Price} - \text{Put Price})$$

the forward index prices, F_1 and F_2 , for the near- and next-term options, respectively, are:

$$F_1 = 1965 + e^{(0.000305 \times 0.0683486)} \times (21.05 - 23.15) = 1962.89996$$

$$F_2 = 1960 + e^{(0.000286 \times 0.0882686)} \times (27.30 - 24.90) = 1962.40006$$

- Next, determine K_0 - the strike price immediately below the forward index level, F - for the near- and next-term options. In this example, $K_{0,1} = 1960$ and $K_{0,2} = 1960$.
- Select out-of-the-money put options with strike prices $< K_0$. Start with the put strike immediately lower than K_0 and move to successively lower strike prices. Exclude any put option that has a bid price equal to zero (i.e., no bid). As shown below, once two puts with consecutive strike prices are found to have zero bid prices, no puts with lower strikes are considered for inclusion. (Note that the 1350 and 1355 put options are not included despite having non-zero bid prices)

Put Strike	Bid	Ask	Include?
1345	0	0.15	<i>Not considered following two zero bids</i>
1350	0.05	0.15	
1355	0.05	0.35	
1360	0	0.35	No
1365	0	0.35	No
1370	0.05	0.35	Yes
1375	0.1	0.15	Yes
1380	0.1	0.2	Yes
.	.	.	.

- Next, select out-of-the-money call options with strike prices $> K_0$. Start with the call strike immediately higher than K_0 and move to successively higher strike prices, excluding call options that have a bid price of zero. As with the puts, once two consecutive call options are found to have zero bid prices, no calls with higher strikes are considered. (Note that the 2225 call option is not included despite having a non-zero bid price.)

Call Strike	Bid	Ask	Include?
.	.	.	.
2095	0.05	0.35	Yes
2100	0.05	0.15	Yes
2120	0	0.15	No
2125	0.05	0.15	Yes
2150	0	0.1	No
2175	0	0.05	No
2200	0	0.05	<i>Not considered following two zero bids</i>
2225	0.05	0.1	
2250	0	0.05	
.	.	.	.

- Finally, select **both** the put and call with strike price K_0 . Notice that two options are selected at K_0 , while a single option, either a put or a call, is used for every other strike price.

The following table contains the options used to calculate the VIX in this example. VIX uses the average of quoted bid and ask, or mid-quote, prices for each option selected. The K_0 put and call prices are averaged to produce a single value. The price used for the 1960 strike in the near-term is, therefore, $(24.25 + 21.30)/2 = 22.775$; and the price used in the next-term is $(27.30 + 24.90)/2 = 26.10$.

Near term Strike	Option Type	Mid-quote Price		Next term Strike	Option Type	Mid-quote Price
1370	Put	0.2		1275	Put	0.075
1375	Put	0.125		1325	Put	0.15
1380	Put	0.15		1350	Put	0.15
.
1950	Put	18.25		1950	Put	21.60
1955	Put	19.75		1955	Put	23.20
1960	Put/Call Average	22.775		1960	Put/Call Average	26.1
1965	Call	21.05		1965	Call	24.15
1970	Call	18.1		1970	Call	21.10
.
2095	Call	0.2		2125	Call	0.1
2100	Call	0.1		2150	Call	0.1
2125	Call	0.1		2200	Call	0.08

STEP 2: Calculate volatility for both near-term and next-term options

Applying the VIX formula (1) to the near-term and next-term options with time to expiration of T_1 and T_2 , respectively, yields:

$$\sigma_1^2 = \frac{2}{T_1} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_i T_1} Q(K_i) - \frac{1}{T_1} \left[\frac{F_1}{K_0} - 1 \right]^2$$

$$\sigma_2^2 = \frac{2}{T_2} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_i T_2} Q(K_i) - \frac{1}{T_2} \left[\frac{F_2}{K_0} - 1 \right]^2$$

VIX is an amalgam of the information reflected in the prices of all of the selected options. The contribution of a single option to the VIX value is proportional to ΔK and the price of that option, and inversely proportional to the square of the option's strike price.

Generally, ΔK_i is half the difference between the strike prices on either side of K_i . For example, the ΔK for the next-term 1325 Put is 37.5: $\Delta K_{1325 \text{ Put}} = (1350 - 1275)/2$. At the upper and lower edges of any given strip of options, ΔK_i is simply the difference between K_i and the adjacent strike price. In this example, the 1370 Put is the lowest strike in the strip of near-term options and 1375 is the adjacent strike. Therefore, $\Delta K_{1370 \text{ Put}} = 5$ (i.e., $1375 - 1370$).

The contribution of the near-term 1370 Put is given by:

$$\frac{\Delta K_{1370 \text{ Put}}}{K_{1370 \text{ Put}}^2} e^{R_1 T_1} Q(1370 \text{ Put})$$

$$\frac{\Delta K_{1370 \text{ Put}}}{K_{1370 \text{ Put}}^2} e^{R_1 T_1} Q(1370 \text{ Put}) = \frac{5}{1370^2} e^{.000305 (0.0683486)} (0.20) = 0.0000005328$$

A similar calculation is performed for each option. The resulting values for the near-term options are then summed and multiplied by $2/T_1$. Likewise, the resulting values for the next-term options are summed and multiplied by $2/T_2$. The table below summarizes the results for each strip of options.

Near term Strike	Option Type	Mid-quote Price	Contribution by Strike	Next term Strike	Option Type	Mid-quote Price	Contribution by Strike
1370	Put	0.2	0.0000005328	1275	Put	0.075	0.0000023069
1375	Put	0.125	0.0000003306	1325	Put	0.15	0.0000032041
1380	Put	0.15	0.0000003938	1350	Put	0.15	0.0000020577
.
1950	Put	18.25	0.0000239979	1950	Put	21.6	0.0000284031
1955	Put	19.75	0.0000258376	1955	Put	23.2	0.0000303512
1960	Put/Call Average	22.775	0.0000296432	1960	Put/Call Average	26.1	0.0000339711
1965	Call	21.05	0.0000272588	1965	Call	24.15	0.0000312732
1970	Call	18.1	0.0000233198	1970	Call	21.1	0.0000271851
.
2095	Call	0.2	0.0000002278	2125	Call	0.1	0.0000005536
2100	Call	0.1	0.0000003401	2150	Call	0.1	0.0000008113
2125	Call	0.1	0.0000005536	2200	Call	0.075	0.0000007748
$\frac{2}{T_1} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_1 T_1} Q(K_i)$			0.018495	$\frac{2}{T_2} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_2 T_2} Q(K_i)$			0.018838

Next, calculate $\frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2$ for the near-term (T_1) and next-term (T_2):

$$\frac{1}{T_1} \left[\frac{F_1}{K_0} - 1 \right]^2 = \frac{1}{0.0683486} \left[\frac{1962.89996}{1960} - 1 \right]^2 = 0.00003203$$

$$\frac{1}{T_2} \left[\frac{F_2}{K_0} - 1 \right]^2 = \frac{1}{0.0882686} \left[\frac{1962.40006}{1960} - 1 \right]^2 = 0.00001699$$

Now calculate σ_1^2 and σ_2^2 :

$$\sigma_1^2 = \frac{2}{T_1} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_1 T_1} Q(K_i) - \frac{1}{T_1} \left[\frac{F_1}{K_0} - 1 \right]^2 = 0.018495 - 0.00003203 = \mathbf{0.01846292}$$

$$\sigma_2^2 = \frac{2}{T_2} \sum_i \frac{\Delta K_i}{K_i^2} e^{R_2 T_2} Q(K_i) - \frac{1}{T_2} \left[\frac{F_2}{K_0} - 1 \right]^2 = 0.018838 - 0.00001699 = \mathbf{0.01882101}$$

CBOE publishes the near-term and next-term VIX “components,” σ_1 and σ_2 , under ticker symbols “VIN” (CBOE Near-Term VIX) and “VIF” (CBOE Far-Term VIX) every 15 seconds during each CBOE trading day.

STEP 3: Calculate the 30-day weighted average of σ_1^2 and σ_2^2 . Then take the square root of that value and multiply by 100 to get VIX.

$$VIX = 100 \times \sqrt{\left\{ T_1 \sigma_1^2 \left[\frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right] + T_2 \sigma_2^2 \left[\frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \right\} \times \frac{N_{365}}{N_{30}}}$$

The inclusion of SPX Weeklys in the VIX calculation means that the near-term options will always have more than 23 days to expiration and the next-term options always have less than 37 days to expiration, so the resulting VIX value will always reflect an interpolation of σ_1^2 and σ_2^2 ; i.e., each individual weight is less than or equal to 1 and the sum of the weights equals 1.

Returning to the example...

N_{T_1} = number of minutes to settlement of the near-term options (35,924)

N_{T_2} = number of minutes to settlement of the next-term options (46,394)

N_{30} = number of minutes in a 30 days ($30 \times 1,440 = 43,200$)

N_{365} = number of minutes in a 365-day year ($365 \times 1,440 = 525,600$)

$$VIX = 100 \times \sqrt{\left\{ 0.0683486 \times 0.0184629 \times \left[\frac{46,394 - 43,200}{46,394 - 35,924} \right] + 0.0882686 \times 0.018821 \times \left[\frac{43,200 - 35,924}{46,394 - 35,924} \right] \right\} \times \frac{525,600}{43,200}}$$

$$VIX = 100 \times 0.13685821 = 13.69$$

www.cboe.com/VIX

NOTES ON CALCULATING OTHER CBOE VOLATILITY INDEXES

CBOE SHORT-TERM VOLATILITY INDEX (VXST)

On October 1, 2013, CBOE introduced the CBOE Short-Term Volatility Index (VXSTSM), the first volatility index to incorporate weekly options. Whereas the VIX calculation is a measure of thirty-day expected volatility, the VXST calculation uses shorter-dated S&P 500 Index options than those used in the VIX calculation to reflect that the VXST calculation is a measure of nine-day expected volatility. The universe of S&P 500 Index options used in the VXST calculation includes SPX options with “standard” 3rd Friday expiration dates and “weekly” SPX options that expire every Friday, except on the 3rd Friday of each month. VXST futures began trading on CFE in February 2014; CBOE began trading VXST options in April 2014. More information on VXST may be found on the CBOE website at www.cboe.com/VXST.

BROAD-BASED VOLATILITY INDEXES

CBOE calculates volatility indexes on three other broad-based indexes representing different segments of the U.S. stock market:

- CBOE DJIA Volatility Index (VXD) based on options on the Dow Jones Industrial Average (DJX);
- CBOE Nasdaq-100 Volatility Index (VXN) based on Nasdaq-100 Index (NDX) options; and
- CBOE Russell 2000 Volatility Index (RVX) based on Russell 2000 Index (RUT) options.
- CBOE S&P 500 3-Month Volatility Index (VXV) & CBOE S&P 500 6-Month Volatility Index (VXMT) based on S&P 500 Index (SPX) option

For each of these indexes, the calculation is identical to the method detailed in the previous example, except that CBOE includes only “standard” (i.e., 3rd Friday expiration) option series in the calculation.

The CBOE S&P 500 3-Month Volatility Index (VXV) and CBOE S&P 500 6-Month Volatility Index (VXMT) measures the market’s expectation of 3- and 6-month volatility implied by SPX options that bracket a 93- and 186-day maturity, respectively. Comparing VIX, VXV and VXMT provides investors with useful information about the SPX volatility term structure in the most active contract months.

COMMODITY, CURRENCY, INTERNATIONAL & SECTOR VOLATILITY INDEXES

CBOE calculates three commodity volatility indexes, one currency volatility index, four international volatility indexes and two sector volatility indexes:

- CBOE Crude Oil ETF Volatility Index (OVX) based on United States Oil Fund, LP (USO) options;
- CBOE Gold ETF Volatility Index (GVZ) based on the, SPDR Gold Shares (GLD) options;
- CBOE Silver ETF Volatility Index (VXSLV) based on iShares Silver Trust (SLV) options
- CBOE EuroCurrency ETF Volatility Index (EVZ) based on CurrencyShares Euro Trust (FXE) options
- CBOE Emerging Markets ETF Volatility Index (VXEEM) based on iShares MSCI Emerging Markets Index Fund (EEM) options
- CBOE EFA ETF Volatility Index (VXEFA) based on iShares MSCI EAFE Index Fund (EFA) options
- CBOE Brazil ETF Volatility Index (VXEWFZ) based on iShares MSCI Brazil Index Fund (EWZ) options
- CBOE China ETF Volatility Index (VXFXI) based on iShares Trust FTSE China 25 Index Fund (FXI) options
- CBOE Gold Miners ETF Volatility Index (VXGDX) based on Market Vectors Gold Miners Fund (GDX) options
- CBOE U.S. Energy ETF Sector Volatility Index (VXXLE) based on Energy Select Sector SPDR (XLE) options

Each of these volatility indexes are calculated using exchange traded fund, or “ETF”, options that trade like options on individual stocks - they may be exercised prior to their expiration date; exercise results in the delivery of ETF shares rather than cash; and they settle at the close of trading rather than at the open.

For each of the commodity, currency, international and sector volatility indexes, the formula is identical to that used for the VIX calculation. However, as with the other broad-based volatility indexes described above, only “standard” 3rd Friday expiring series are selected as component options.

Moreover, there is a slight difference in the methodology that accounts for the fact that ETF options expire at the close rather than at the open. Specifically, the “time to expiration” used to calculate volatility indexes varies depending on the settlement type (A.M.-settlement, P.M.-settlement) of the constituent option series and the trading hours of the constituent option series on their expiration date. As before, the “time to expiration” is given by the following expression:

$$T = \{M_{\text{Current day}} + M_{\text{Settlement day}} + M_{\text{Other days}}\} / \text{Minutes in a year}$$

WHERE...

$M_{\text{Current day}}$ = minutes remaining until midnight of the current day

$M_{\text{Other day}}$ = total minutes in the days between current day and settlement day

But now, adjusting for p.m. settlement...

$M_{\text{Settlement day}}$ = minutes from midnight until 3:00 p.m. on expiration day
= 900 minutes for 3:00 p.m.-expiration ETF options and

$M_{\text{Settlement day}}$ = minutes from midnight until 3:15 p.m. on expiration day
= 915 minutes for 3:15 p.m.-expiration ETF options (e.g., EEM, EFA and XLE options)

For example, assuming near- and next-term options with 9 and 37 days to expiration and 8:30 a.m. as the time of the calculation, T for the near-term and next-term options for the 3:00 p.m. expiration ETF options, T_1 and T_2 , respectively, is:

$$T_1 = \{930 + 900 + 11,520\} / 535,600 = 0.0253995$$

$$T_2 = \{930 + 900 + 51,840\} / 525,600 = 0.1021118$$

EQUITY VIX® VOLATILITY INDEXES

CBOE calculates five Equity VIX indexes based on the prices of options on individual stocks:

- CBOE Equity VIX® on Apple (VXAPL)
- CBOE Equity VIX® on Amazon (VXAZN)
- CBOE Equity VIX® on Goldman Sachs (VXGS)
- CBOE Equity VIX® on Google (VXGOG)
- CBOE Equity VIX® on IBM (VXIBM)

Equity VIX values are calculated using the standard VIX formula. However, Equity VIX indexes use only “standard” 3rd Friday expiring options in order to calculate these values.

Like the commodity, currency, international and sector volatility indexes, the “time to expiration” for the Equity VIX indexes reflect the fact that options on individual stocks expire at the close, and thus have more time to trade, than options (such as standard SPX options in the VIX calculation) that expire at the open on their expiration day.

Special Note: All CBOE volatility indexes are calculated using option price quotes from CBOE exclusively.

The information in this document is provided for information purposes only, and is not intended to provide, and should not be relied on for financial or legal advice. The CBOE Volatility Index® (VIX® index) and all other information provided by Chicago Board Options Exchange, Incorporated (CBOE) and its affiliates and their respective directors, officers, employees, agents, representatives and third party providers of information (the "Parties") in connection with the VIX® Index (collectively "Data") are presented "as is" and without representations or warranties of any kind. The Parties shall not be liable for loss or damage, direct, indirect or consequential, arising from any use of the Data or action taken in reliance upon the Data.

Options involve risk and are not suitable for all investors. Prior to buying or selling an option, a person must receive a copy of Characteristics and Risks of Standardized Options. Copies are available from your broker, by calling 1-888-OPTIONS or from The Options Clearing Corporation at www.theocc.com. Futures trading is not suitable for all investors, and involves risk of loss. No statement within this document should be construed as a recommendation to buy or sell a security or futures contract or to provide investment advice. It is not possible to invest directly in an index.

The VIX® index methodology is the property of CBOE. CBOE®, Chicago Board Options Exchange®, CFE®, CBOE Volatility Index®, OEX® and VIX® are registered trademarks and CBOE Futures ExchangeSM, WeeklysSM, CBOE Short-Term Volatility Index®, VXSTSM, SPXWSM, VXMTSM, VXXLESM, VXEEMSM, VXEFASM, VXGDSM, VXSLVSM, VXEWSSM, VAFXISM, VXAPLSM, VXAZNSM, VXGSSM, VXGOGSM, VXIBMSM, EVZSM, GVZSM, OVXSM, RVXSM, SPXSM, VXDSM, VXNSM, VXOSM and VXVSM are service marks of CBOE. CBOE and its affiliates do not sponsor, endorse, sell or promote any third party investment product that is or may be based on the VIX® Index. Standard & Poor's®, S&P®, S&P 100® and S&P 500® are registered trademarks of Standard & Poor's Financial Services, LLC and have been licensed for use by CBOE. Financial products based on S&P indices are not sponsored, endorsed, sold or promoted by Standard & Poor's, and Standard & Poor's makes no representation regarding the advisability of investing in such products. DJIA® and Dow Jones Industrial AverageSM are trademarks or service marks of Dow Jones Trademark Holdings, LLC and have been licensed to CME Group Index Services, LLC and sublicensed for use for certain purposes by CBOE. Nasdaq-100 Index®, Nasdaq-100® and Nasdaq® are trademark or service marks of The Nasdaq Stock Market, Inc. (with which its affiliates are the "Corporations"). These marks are licensed for use by CBOE in connection with the trading of products based on the Nasdaq-100 Index. The products have not been passed on by the Corporations as to their legality or suitability. The products are not issued, endorsed, sold or promoted by the Corporations. THE CORPORATIONS MAKE NO WARRANTIES AND BEAR NO LIABILITY WITH RESPECT TO THE PRODUCT(S). Russell 2000® is a registered trademark of Russell Investments, used under license. All other trademarks and service marks are the property of their respective owners. Redistribution, reproduction and/or photocopying in whole or in part are prohibited without the written permission of CBOE.

Copyright © 2014 CBOE. All rights reserved.

APPENDIX 1 - Complete SPX Option Data Used in Sample VIX Calculation

Option Series included in the VIX calculation are highlighted.

Near-Term Options					Next-Term Options				
Strike	Calls		Puts		Strike	Calls		Puts	
	Bid	Ask	Bid	Ask		Bid	Ask	Bid	Ask
800	1160.90	1164.40	0.00	0.10					
900	1060.90	1064.50	0.00	0.10					
1000	961.00	964.50	0.00	0.10					
1050	911.00	914.50	0.00	0.10					
1100	861.00	864.60	0.00	0.05					
1125	836.00	839.60	0.00	0.05					
1150	811.00	814.60	0.00	0.05					
1175	786.10	789.60	0.00	0.05					
1200	761.10	764.60	0.00	0.05					
1220	741.10	744.60	0.00	0.10					
1225	736.10	739.60	0.00	0.05	1225	735.90	738.80	0.00	0.10
1240	721.10	724.60	0.00	0.10	1250	710.80	713.80	0.00	0.10
1250	711.10	714.60	0.00	0.05	1275	686.00	688.70	0.05	0.10
1260	701.10	704.60	0.00	0.10	1300	660.90	663.80	0.00	0.10
1270	691.10	694.60	0.00	0.10	1325	635.90	638.60	0.10	0.20
1275	686.10	689.60	0.00	0.10	1350	610.90	613.60	0.10	0.20
1280	681.10	684.60	0.00	0.10	1375	585.90	588.70	0.10	0.25
1290	671.10	674.70	0.00	0.10	1400	561.00	563.70	0.15	0.25
1300	661.10	664.70	0.05	0.10	1425	536.00	538.80	0.20	0.30
1305	656.10	659.70	0.00	0.10	1450	511.10	513.80	0.25	0.35
1310	651.10	654.70	0.00	0.10	1475	486.10	488.90	0.30	0.40
1315	646.10	649.70	0.00	0.10	1500	461.20	464.00	0.35	0.45
1320	641.20	644.70	0.00	0.10	1510	451.30	454.00	0.35	0.50
1325	636.20	639.70	0.05	0.10	1520	441.30	444.00	0.40	0.50
1330	631.20	634.70	0.00	0.10	1525	436.30	439.10	0.40	0.55
1335	626.20	629.70	0.00	0.15	1530	431.30	434.10	0.45	0.55
1340	621.20	624.70	0.00	0.15	1540	421.40	424.10	0.45	0.60
1345	616.20	619.70	0.00	0.15	1550	411.40	414.20	0.50	0.60
1350	611.20	614.70	0.05	0.15	1555	406.40	409.20	0.50	0.65
1355	606.20	609.70	0.05	0.35	1560	401.40	404.20	0.55	0.65
1360	601.20	604.70	0.00	0.35	1565	396.50	399.20	0.55	0.70
1365	596.20	599.70	0.00	0.35	1570	391.20	394.00	0.60	0.70
1370	591.20	594.70	0.05	0.35	1575	386.50	389.30	0.60	0.75
1375	586.20	589.70	0.10	0.15	1580	381.50	384.30	0.60	0.75
1380	581.20	584.70	0.10	0.20	1585	376.60	379.30	0.65	0.75
1385	576.20	579.70	0.10	0.35	1590	371.30	374.10	0.65	0.80

1390	571.20	574.70	0.10	0.35	1595	366.60	369.40	0.70	0.80
1395	566.20	569.70	0.10	0.15	1600	361.60	364.40	0.70	0.85
1400	561.20	564.80	0.10	0.15	1605	356.70	359.40	0.75	0.85
1405	556.20	559.80	0.00	0.35	1610	351.70	354.50	0.75	0.90
1410	551.20	554.80	0.05	0.40	1615	346.70	349.50	0.80	0.90
1415	546.20	549.80	0.00	0.40	1620	341.80	344.50	0.80	0.95
1420	541.20	544.80	0.05	0.40	1625	336.80	339.50	0.85	0.95
1425	536.30	539.80	0.15	0.20	1630	331.80	334.60	0.90	1.00
1430	531.30	534.80	0.05	0.40	1635	326.90	329.60	0.90	1.05
1435	526.30	529.80	0.15	0.40	1640	321.90	324.70	0.95	1.05
1440	521.30	524.80	0.05	0.30	1645	316.90	319.70	0.95	1.10
1445	516.30	519.80	0.05	0.40	1650	312.00	314.70	1.00	1.15
1450	511.30	514.80	0.15	0.25	1655	307.00	309.80	1.05	1.15
1455	506.30	509.80	0.05	0.45	1660	302.10	304.80	1.10	1.20
1460	501.30	504.80	0.05	0.45	1665	297.10	299.90	1.15	1.25
1465	496.30	499.80	0.05	0.45	1670	292.20	294.90	1.15	1.30
1470	491.30	494.80	0.05	0.45	1675	287.20	289.90	1.20	1.35
1475	486.30	489.90	0.15	0.25	1680	282.30	285.00	1.25	1.40
1480	481.30	484.90	0.05	0.45	1685	277.30	280.10	1.30	1.45
1485	476.30	479.90	0.20	0.50	1690	272.40	275.10	1.35	1.50
1490	471.30	474.90	0.05	0.30	1695	267.40	270.20	1.40	1.55
1495	466.40	469.90	0.05	0.50	1700	262.50	265.20	1.45	1.60
1500	461.40	464.90	0.25	0.40	1705	257.50	260.30	1.50	1.70
1505	456.40	459.90	0.30	0.35	1710	252.60	255.30	1.60	1.75
1510	451.40	454.90	0.05	0.55	1715	247.70	250.40	1.65	1.80
1515	446.40	449.90	0.05	0.55	1720	242.70	245.50	1.70	1.90
1520	441.40	445.00	0.10	0.60	1725	237.80	240.60	1.75	1.95
1525	436.40	440.00	0.30	0.40	1730	232.90	235.60	1.85	2.00
1530	431.40	435.00	0.05	0.60	1735	228.00	230.70	1.90	2.10
1535	426.40	430.00	0.10	0.65	1740	223.40	225.30	2.00	2.20
1540	421.40	425.00	0.10	0.65	1745	218.50	220.40	2.10	2.25
1545	416.50	420.00	0.10	0.65	1750	213.60	215.50	2.20	2.35
1550	411.50	415.00	0.30	0.70	1755	208.70	210.60	2.30	2.45
1555	406.50	410.10	0.15	0.70	1760	203.80	205.70	2.40	2.55
1560	401.50	405.10	0.15	0.70	1765	198.90	200.80	2.50	2.65
1565	396.50	400.10	0.15	0.70	1770	194.00	195.90	2.65	2.80
1570	391.50	395.10	0.20	0.75	1775	189.20	191.10	2.75	2.90
1575	386.50	390.10	0.35	0.75	1780	184.30	185.80	2.90	3.10
1580	381.50	385.10	0.25	0.80	1785	179.40	180.90	3.00	3.20

1585	376.60	380.20	0.25	0.80	1790	174.60	176.10	3.10	3.40
1590	371.60	375.20	0.25	0.80	1795	169.70	171.20	3.30	3.60
1595	366.60	370.20	0.25	0.80	1800	164.90	166.40	3.50	3.70
1600	361.60	365.20	0.50	0.85	1805	160.10	161.60	3.70	3.90
1605	356.60	360.30	0.30	0.85	1810	155.30	156.70	3.80	4.10
1610	351.60	355.30	0.35	0.90	1815	150.50	152.00	4.10	4.30
1615	346.70	350.30	0.35	0.90	1820	145.70	147.20	4.30	4.50
1620	341.70	345.30	0.35	0.90	1825	140.90	142.40	4.50	4.80
1625	336.70	340.40	0.40	0.95	1830	136.20	137.70	4.80	5.00
1630	331.70	335.40	0.40	0.95	1835	131.50	132.90	5.00	5.30
1635	326.70	330.40	0.45	1.00	1840	126.80	128.20	5.30	5.60
1640	321.80	325.40	0.45	1.00	1845	122.10	123.50	5.60	5.90
1645	316.80	320.50	0.50	1.05	1850	117.40	118.80	5.90	6.20
1650	311.80	315.50	0.50	0.85	1855	112.80	114.20	6.30	6.60
1655	306.80	310.50	0.55	1.10	1860	108.20	109.60	6.60	6.90
1660	301.90	305.60	0.55	1.10	1865	103.60	105.00	7.00	7.30
1665	296.90	300.60	0.60	1.15	1870	99.00	100.40	7.50	7.80
1670	291.90	295.70	0.60	1.15	1875	94.50	95.90	8.00	8.30
1675	287.00	290.70	0.65	1.20	1880	90.00	91.40	8.40	8.80
1680	282.00	285.70	0.70	1.25	1885	85.50	86.90	9.00	9.40
1685	277.00	280.80	0.75	1.30	1890	81.10	82.50	9.50	10.00
1690	272.10	275.80	0.75	1.30	1895	76.80	78.10	10.20	10.60
1695	267.10	270.90	0.80	1.35	1900	72.40	73.70	10.90	11.30
1700	262.10	265.90	0.85	1.40	1905	68.20	69.40	11.60	12.00
1705	257.20	261.00	0.85	1.40	1910	64.00	65.20	12.40	12.80
1710	252.20	256.00	0.90	1.45	1915	59.80	61.10	13.20	13.70
1715	247.30	251.10	0.95	1.50	1920	55.70	57.00	14.20	14.60
1720	242.30	246.10	1.00	1.55	1925	51.70	53.00	15.20	15.60
1725	237.40	241.20	1.05	1.60	1930	47.80	49.10	16.20	16.60
1730	232.40	236.30	1.10	1.65	1935	44.60	45.10	17.40	17.80
1735	227.50	231.30	1.15	1.70	1940	40.80	41.30	18.60	19.00
1740	222.50	226.40	1.20	1.75	1945	37.20	37.70	20.00	20.40
1745	217.60	221.50	1.25	1.85	1950	33.70	34.40	21.40	21.80
1750	212.60	216.60	1.30	1.90	1955	30.30	30.90	23.00	23.40
1755	207.70	211.60	1.40	1.95	1960	27.00	27.60	24.70	25.10
1760	202.80	206.70	1.45	2.05	1965	23.80	24.50	26.50	27.30
1765	197.80	201.80	1.50	2.15	1970	20.80	21.40	28.50	29.40
1770	192.90	196.90	1.60	2.20	1975	18.00	18.60	30.50	31.60
1775	188.00	192.00	1.65	2.35	1980	15.50	15.90	33.00	34.00

1780	183.10	187.10	1.75	2.40	1985	13.10	13.50	35.50	36.60
1785	178.20	182.20	1.85	2.50	1990	10.90	11.30	38.40	39.50
1790	173.30	177.30	1.90	2.60	1995	9.00	9.30	41.30	42.50
1795	168.40	172.40	2.00	2.75	2000	7.20	7.60	44.50	45.80
1800	163.50	167.50	2.15	2.90	2005	5.70	6.00	48.10	49.30
1805	158.60	162.60	2.25	3.00	2010	4.50	4.80	51.70	53.00
1810	153.80	157.80	2.35	3.20	2015	3.40	3.70	55.80	57.00
1815	148.90	152.90	2.50	3.40	2020	2.60	2.80	59.90	61.70
1820	144.10	148.10	2.65	3.50	2025	1.95	2.15	64.10	66.10
1825	139.20	143.30	3.00	3.60	2030	1.45	1.65	68.60	70.60
1830	134.40	138.40	3.00	3.90	2035	1.05	1.25	73.30	75.20
1835	129.60	133.60	3.20	4.10	2040	0.80	0.95	78.00	80.00
1840	124.80	128.80	3.40	4.40	2045	0.60	0.75	82.00	84.80
1845	120.10	124.10	3.60	4.60	2050	0.50	0.65	86.90	89.60
1850	115.40	119.30	3.80	4.90	2060	0.30	0.40	96.60	99.40
1855	110.60	114.60	4.10	5.20	2070	0.20	0.30	106.70	109.50
1860	105.90	109.90	4.40	5.50	2075	0.15	0.25	111.70	114.50
1865	101.30	105.20	4.70	5.80	2100	0.10	0.20	136.30	139.10
1870	96.60	100.50	5.00	6.20	2125	0.05	0.15	161.50	164.30
1875	92.00	95.90	5.40	6.60	2150	0.05	0.15	186.30	189.00
1880	87.40	91.30	5.80	7.00	2175	0.00	0.10	211.30	214.00
1885	82.90	86.70	6.20	7.50	2200	0.05	0.10	236.30	239.00
1890	78.40	82.20	6.70	8.00	2225	0.00	0.10	261.30	264.00
1895	74.00	77.70	7.20	8.60	2250	0.00	0.10	286.30	289.00
1900	69.60	73.20	7.80	8.80					
1905	66.00	68.50	8.50	9.50					
1910	61.60	64.10	9.10	10.20					
1915	57.40	59.80	9.90	11.30					
1920	53.30	55.60	10.70	12.10					
1925	49.10	51.20	11.60	12.60					
1930	45.20	47.30	12.50	14.00					
1935	41.20	43.40	13.60	14.70					
1940	37.40	39.50	14.70	15.80					
1945	33.70	35.70	15.90	17.20					
1950	30.10	32.10	17.70	18.80					
1955	26.70	28.50	19.00	20.50					
1960	23.40	25.10	20.60	22.00					
1965	20.30	21.80	22.30	24.00					
1970	17.40	18.80	24.30	25.80					

1975	14.60	15.90	26.50	28.10
1980	12.20	13.30	28.90	30.60
1985	9.90	11.00	31.40	33.20
1990	7.90	9.00	34.30	36.50
1995	6.20	7.10	37.40	39.70
2000	4.70	5.20	40.70	43.20
2005	3.40	4.20	44.00	47.70
2010	2.65	3.10	48.00	51.40
2015	1.75	2.30	52.20	56.00
2020	1.20	1.70	56.60	60.40
2025	1.00	1.25	61.20	65.00
2030	0.45	1.00	65.90	69.70
2035	0.25	0.80	70.70	74.40
2040	0.35	0.65	75.60	79.30
2045	0.20	0.60	80.50	84.10
2050	0.20	0.30	85.40	89.00
2055	0.15	0.50	90.40	94.00
2060	0.15	0.30	95.30	98.90
2065	0.15	0.20	100.30	103.90
2070	0.10	0.20	105.30	108.90
2075	0.10	0.20	110.30	113.80
2080	0.05	0.45	115.30	118.80
2085	0.05	0.40	120.30	123.80
2090	0.05	0.15	125.30	128.80
2095	0.05	0.35	130.30	133.80
2100	0.05	0.15	135.30	138.80
2120	0.00	0.15	155.30	158.80
2125	0.05	0.15	160.30	163.80
2150	0.00	0.10	185.20	188.80
2175	0.00	0.05	210.20	213.70
2200	0.00	0.05	235.20	238.70
2225	0.05	0.10	260.20	263.70
2250	0.00	0.05	285.20	288.70

Individual Contributions — $K_0 = 1960$

Near term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike	Next term Strike	Option Type	Mid-quote Price	Delta-K	Contribution by Strike
1370	Put	0.200	5	0.0000005328	1275	Put	0.075	50	0.0000023069
1375	Put	0.125	5	0.0000003306	1325	Put	0.150	37.5	0.0000032041
1380	Put	0.150	5	0.0000003938	1350	Put	0.150	25	0.0000020577
1385	Put	0.225	5	0.0000005865	1375	Put	0.175	25	0.0000023141
1390	Put	0.225	5	0.0000005823	1400	Put	0.200	25	0.0000025511
1395	Put	0.125	5	0.0000003212	1425	Put	0.250	25	0.0000030779
1400	Put	0.125	7.5	0.0000004783	1450	Put	0.300	25	0.0000035673
1410	Put	0.225	10	0.0000011318	1475	Put	0.350	25	0.0000040219
1420	Put	0.225	7.5	0.0000008369	1500	Put	0.400	17.5	0.0000031112
1425	Put	0.175	5	0.0000004309	1510	Put	0.425	10	0.0000018640
1430	Put	0.225	5	0.0000005502	1520	Put	0.450	7.5	0.0000014608
1435	Put	0.275	5	0.0000006677	1525	Put	0.475	5	0.0000010213
1440	Put	0.175	5	0.0000004220	1530	Put	0.500	7.5	0.0000016020
1445	Put	0.225	5	0.0000005388	1540	Put	0.525	10	0.0000022138
1450	Put	0.200	5	0.0000004756	1550	Put	0.550	7.5	0.0000017170
1455	Put	0.250	5	0.0000005905	1555	Put	0.575	5	0.0000011890
1460	Put	0.250	5	0.0000005864	1560	Put	0.600	5	0.0000012328
1465	Put	0.250	5	0.0000005824	1565	Put	0.625	5	0.0000012759
1470	Put	0.250	5	0.0000005785	1570	Put	0.650	5	0.0000013185
1475	Put	0.200	5	0.0000004596	1575	Put	0.675	5	0.0000013606
1480	Put	0.250	5	0.0000005707	1580	Put	0.675	5	0.0000013520
1485	Put	0.350	5	0.0000007936	1585	Put	0.700	5	0.0000013932
1490	Put	0.175	5	0.0000003941	1590	Put	0.725	5	0.0000014339
1495	Put	0.275	5	0.0000006152	1595	Put	0.750	5	0.0000014741
1500	Put	0.325	5	0.0000007222	1600	Put	0.775	5	0.0000015137
1505	Put	0.325	5	0.0000007174	1605	Put	0.800	5	0.0000015528
1510	Put	0.300	5	0.0000006579	1610	Put	0.825	5	0.0000015914
1515	Put	0.300	5	0.0000006535	1615	Put	0.850	5	0.0000016295
1520	Put	0.350	5	0.0000007575	1620	Put	0.875	5	0.0000016671
1525	Put	0.350	5	0.0000007525	1625	Put	0.900	5	0.0000017042
1530	Put	0.325	5	0.0000006942	1630	Put	0.950	5	0.0000017878
1535	Put	0.375	5	0.0000007958	1635	Put	0.975	5	0.0000018237
1540	Put	0.375	5	0.0000007906	1640	Put	1.000	5	0.0000018591
1545	Put	0.375	5	0.0000007855	1645	Put	1.025	5	0.0000018940
1550	Put	0.500	5	0.0000010406	1650	Put	1.075	5	0.0000019743
1555	Put	0.425	5	0.0000008788	1655	Put	1.100	5	0.0000020081
1560	Put	0.425	5	0.0000008732	1660	Put	1.150	5	0.0000020867
1565	Put	0.425	5	0.0000008676	1665	Put	1.200	5	0.0000021644
1570	Put	0.475	5	0.0000009635	1670	Put	1.225	5	0.0000021963
1575	Put	0.550	5	0.0000011086	1675	Put	1.275	5	0.0000022723

1580	Put	0.525	5	0.0000010515	1680	Put	1.325	5	0.0000023474
1585	Put	0.525	5	0.0000010449	1685	Put	1.375	5	0.0000024215
1590	Put	0.525	5	0.0000010384	1690	Put	1.425	5	0.0000024947
1595	Put	0.525	5	0.0000010319	1695	Put	1.475	5	0.0000025670
1600	Put	0.675	5	0.0000013184	1700	Put	1.525	5	0.0000026385
1605	Put	0.575	5	0.0000011161	1705	Put	1.600	5	0.0000027520
1610	Put	0.625	5	0.0000012056	1710	Put	1.675	5	0.0000028642
1615	Put	0.625	5	0.0000011982	1715	Put	1.725	5	0.0000029325
1620	Put	0.625	5	0.0000011908	1720	Put	1.800	5	0.0000030423
1625	Put	0.675	5	0.0000012781	1725	Put	1.850	5	0.0000031087
1630	Put	0.675	5	0.0000012703	1730	Put	1.925	5	0.0000032160
1635	Put	0.725	5	0.0000013561	1735	Put	2.000	5	0.0000033221
1640	Put	0.725	5	0.0000013478	1740	Put	2.100	5	0.0000034682
1645	Put	0.775	5	0.0000014320	1745	Put	2.175	5	0.0000035715
1650	Put	0.675	5	0.0000012397	1750	Put	2.275	5	0.0000037144
1655	Put	0.825	5	0.0000015060	1755	Put	2.375	5	0.0000038556
1660	Put	0.825	5	0.0000014970	1760	Put	2.475	5	0.0000039951
1665	Put	0.875	5	0.0000015782	1765	Put	2.575	5	0.0000041330
1670	Put	0.875	5	0.0000015688	1770	Put	2.725	5	0.0000043491
1675	Put	0.925	5	0.0000016485	1775	Put	2.825	5	0.0000044834
1680	Put	0.975	5	0.0000017273	1780	Put	3.000	5	0.0000047344
1685	Put	1.025	5	0.0000018051	1785	Put	3.100	5	0.0000048648
1690	Put	1.025	5	0.0000017944	1790	Put	3.250	5	0.0000050718
1695	Put	1.075	5	0.0000018709	1795	Put	3.450	5	0.0000053539
1700	Put	1.125	5	0.0000019464	1800	Put	3.600	5	0.0000055557
1705	Put	1.125	5	0.0000019350	1805	Put	3.800	5	0.0000058319
1710	Put	1.175	5	0.0000020092	1810	Put	3.950	5	0.0000060287
1715	Put	1.225	5	0.0000020825	1815	Put	4.200	5	0.0000063750
1720	Put	1.275	5	0.0000021549	1820	Put	4.400	5	0.0000066419
1725	Put	1.325	5	0.0000022265	1825	Put	4.650	5	0.0000069808
1730	Put	1.375	5	0.0000022972	1830	Put	4.900	5	0.0000073160
1735	Put	1.425	5	0.0000023670	1835	Put	5.150	5	0.0000076474
1740	Put	1.475	5	0.0000024360	1840	Put	5.450	5	0.0000080490
1745	Put	1.550	5	0.0000025452	1845	Put	5.750	5	0.0000084461
1750	Put	1.600	5	0.0000026123	1850	Put	6.050	5	0.0000088388
1755	Put	1.675	5	0.0000027192	1855	Put	6.450	5	0.0000093724
1760	Put	1.750	5	0.0000028248	1860	Put	6.750	5	0.0000097557
1765	Put	1.825	5	0.0000029292	1865	Put	7.150	5	0.0000102785
1770	Put	1.900	5	0.0000030324	1870	Put	7.650	5	0.0000109385
1775	Put	2.000	5	0.0000031740	1875	Put	8.150	5	0.0000115914
1780	Put	2.075	5	0.0000032746	1880	Put	8.600	5	0.0000121664
1785	Put	2.175	5	0.0000034132	1885	Put	9.200	5	0.0000129463

1790	Put	2.250	5	0.0000035112	1890	Put	9.750	5	0.0000136478
1795	Put	2.375	5	0.0000036856	1895	Put	10.400	5	0.0000144809
1800	Put	2.525	5	0.0000038967	1900	Put	11.100	5	0.0000153743
1805	Put	2.625	5	0.0000040286	1905	Put	11.800	5	0.0000162582
1810	Put	2.775	5	0.0000042353	1910	Put	12.600	5	0.0000172697
1815	Put	2.950	5	0.0000044776	1915	Put	13.450	5	0.0000183386
1820	Put	3.075	5	0.0000046417	1920	Put	14.400	5	0.0000195317
1825	Put	3.300	5	0.0000049541	1925	Put	15.400	5	0.0000207797
1830	Put	3.450	5	0.0000051511	1930	Put	16.400	5	0.0000220146
1835	Put	3.650	5	0.0000054200	1935	Put	17.600	5	0.0000235035
1840	Put	3.900	5	0.0000057598	1940	Put	18.800	5	0.0000249767
1845	Put	4.100	5	0.0000060224	1945	Put	20.200	5	0.0000266989
1850	Put	4.350	5	0.0000063551	1950	Put	21.600	5	0.0000284031
1855	Put	4.650	5	0.0000067568	1955	Put	23.200	5	0.0000303512
1860	Put	4.950	5	0.0000071542	1960	Put/Call Average	26.100	5	0.0000339711
1865	Put	5.250	5	0.0000075471					
1870	Put	5.600	5	0.0000080073	1965	Call	24.150	5	0.0000312732
1875	Put	6.000	5	0.0000085335	1970	Call	21.100	5	0.0000271851
1880	Put	6.400	5	0.0000090541	1975	Call	18.300	5	0.0000234584
1885	Put	6.850	5	0.0000096393	1980	Call	15.700	5	0.0000200240
1890	Put	7.350	5	0.0000102883	1985	Call	13.300	5	0.0000168776
1895	Put	7.900	5	0.0000109999	1990	Call	11.100	5	0.0000140152
1900	Put	8.300	5	0.0000114961	1995	Call	9.150	5	0.0000114952
1905	Put	9.000	5	0.0000124003	2000	Call	7.400	5	0.0000092502
1910	Put	9.650	5	0.0000132263	2005	Call	5.850	5	0.0000072763
1915	Put	10.600	5	0.0000144526	2010	Call	4.650	5	0.0000057550
1920	Put	11.400	5	0.0000154626	2015	Call	3.550	5	0.0000043718
1925	Put	12.100	5	0.0000163269	2020	Call	2.700	5	0.0000033086
1930	Put	13.250	5	0.0000177861	2025	Call	2.050	5	0.0000024997
1935	Put	14.150	5	0.0000188962	2030	Call	1.550	5	0.0000018807
1940	Put	15.250	5	0.0000202603	2035	Call	1.150	5	0.0000013885
1945	Put	16.550	5	0.0000218745	2040	Call	0.875	5	0.0000010513
1950	Put	18.250	5	0.0000239979	2045	Call	0.675	5	0.0000008070
1955	Put	19.750	5	0.0000258376	2050	Call	0.575	7.5	0.0000010262
1960	Put/Call Average	24.250	5	0.0000296432	2060	Call	0.350	10	0.0000008248
					2070	Call	0.250	7.5	0.0000004376
1965	Call	21.050	5	0.0000272588	2075	Call	0.200	15	0.0000006968
1970	Call	18.100	5	0.0000233198	2100	Call	0.150	25	0.0000008504
1975	Call	15.250	5	0.0000195486	2125	Call	0.100	25	0.0000005536
1980	Call	12.750	5	0.0000162614	2150	Call	0.100	37.5	0.0000008113
1985	Call	10.450	5	0.0000132609	2200	Call	0.075	50	0.0000007748
1990	Call	8.450	5	0.0000106691					

1995	Call	6.650	5	0.0000083544
2000	Call	4.950	5	0.0000061876
2005	Call	3.800	5	0.0000047264
2010	Call	2.875	5	0.0000035582
2015	Call	2.025	5	0.0000024938
2020	Call	1.450	5	0.0000017768
2025	Call	1.125	5	0.0000013718
2030	Call	0.725	5	0.0000008797
2035	Call	0.525	5	0.0000006339
2040	Call	0.500	5	0.0000006007
2045	Call	0.400	5	0.0000004782
2050	Call	0.250	5	0.0000002974
2055	Call	0.325	5	0.0000003848
2060	Call	0.225	5	0.0000002651
2065	Call	0.175	5	0.0000002052
2070	Call	0.150	5	0.0000001750
2075	Call	0.150	5	0.0000001742
2080	Call	0.250	5	0.0000002889
2085	Call	0.225	5	0.0000002588
2090	Call	0.100	5	0.0000001145
2095	Call	0.200	5	0.0000002278
2100	Call	0.100	15	0.0000003401
2125	Call	0.100	25	0.0000005536

Sum of Individual Contributions **0.0006320516**

$$\frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)$$

0.018494953

Sum of Individual Contributions **0.0008314022**

$$\frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i)$$

0.018837995