



# Graph Generation

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# 01 Generative Adversarial Networks (GANs)

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**Goal:** generate fake objects (e.g. images) similar to real ones

**Idea:** play an adversarial game with two agents

I. Goodfellow et al., *Generative Adversarial Nets*. in Proc. of NIPS, 2014, pp. 2672--2680.





# 01 Generative Adversarial Networks (GANs)

---

**Goal:** generate fake objects (e.g. images) similar to real ones

**Idea:** play an adversarial game with two agents



**Generator:** maps noise  $z$  to a fake object  $x$

**Discriminator:** maps object  $x$  to probability of real/fake

**Game:** The generator tries to fool the discriminator  
The discriminator tries to detect the fake objects

I. Goodfellow et al., *Generative Adversarial Nets*. in Proc. of NIPS, 2014, pp. 2672--2680.



# 01 Generative Adversarial Networks (GANs)

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**Idea:** play an adversarial game with two agents



**Generator:** maps noise  $z$  to a fake object  $x$

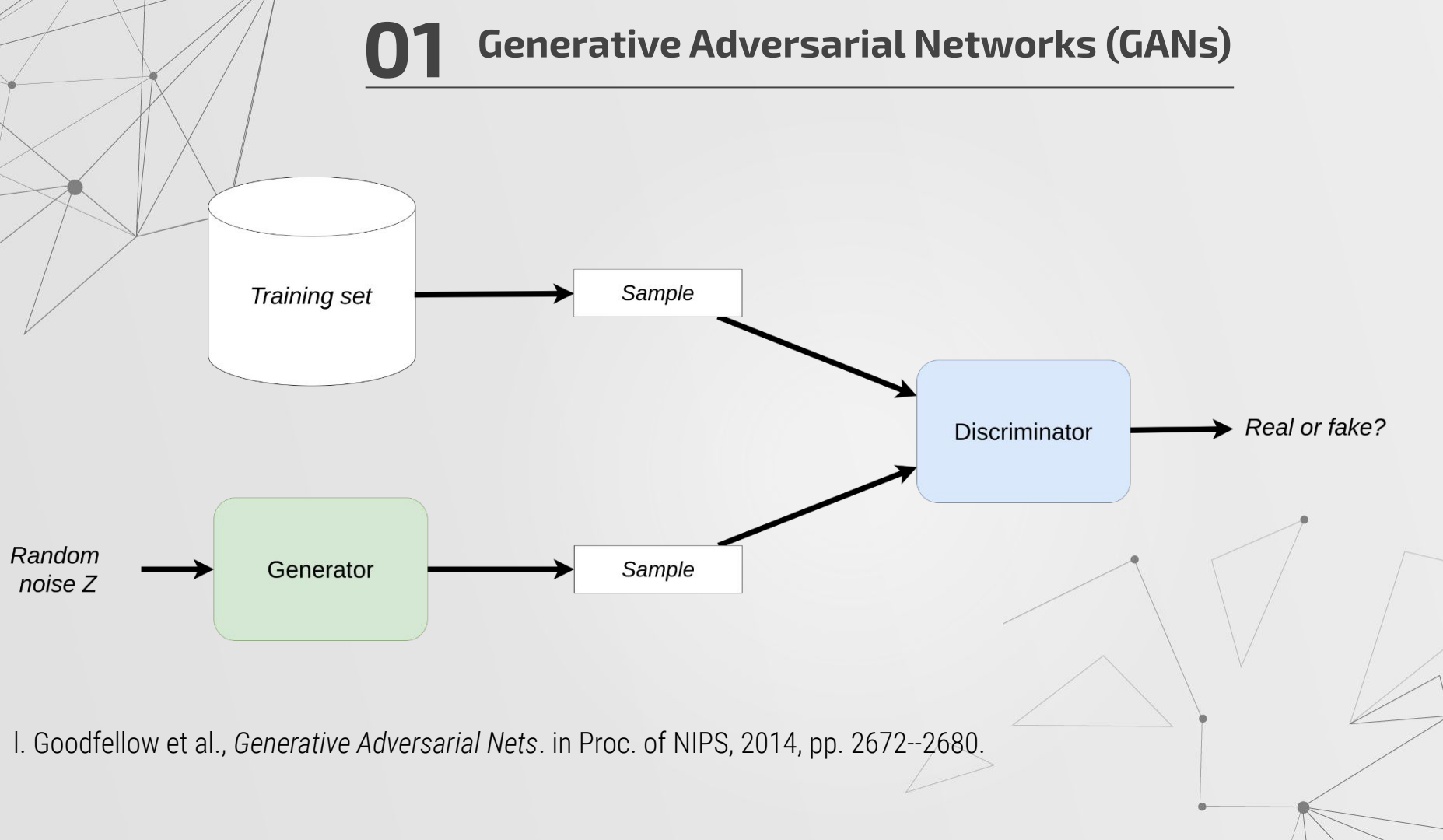
**Discriminator:** maps object  $x$  to probability of real/fake

**Game:** The generator tries to fool the discriminator  
The discriminator tries to detect the fake objects

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_z(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

I. Goodfellow et al., *Generative Adversarial Nets*. in Proc. of NIPS, 2014, pp. 2672--2680.

# 01 Generative Adversarial Networks (GANs)



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The **Discriminator** wants to **max**:

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The **Discriminator** wants to **max**:

- Recall that  $D(x)$  is in  $[0, 1]$
- **First term:**
  - large if  $D(x)$  is close to 1
  - assign high probability to real objects

I. Goodfellow et al., *Generative Adversarial Nets*. in Proc. of NIPS, 2014, pp. 2672--2680.

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$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

The **Discriminator** wants to **max**:

- Recall that  $D(x)$  is in  $[0, 1]$
- **First term:**
  - large if  $D(x)$  is close to 1
  - assign high probability to real objects
- **Second term:**
  - large if  $1 - D(G(z))$  is close to 1
  - large if  $D(G(z))$  is close to 0
  - assign low probability to fake objects

I. Goodfellow et al., *Generative Adversarial Nets*. in Proc. of NIPS, 2014, pp. 2672--2680.

# 01 Generative Adversarial Networks (GANs)

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

The **Generator** wants to **min**:

I. Goodfellow et al., *Generative Adversarial Nets*. in Proc. of NIPS, 2014, pp. 2672--2680.

# 01 Generative Adversarial Networks (GANs)

$$\min_G \max_D \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})} [\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})} [\log(1 - D(G(\mathbf{z})))]$$

The **Generator** wants to **min**:

- **Second term:**
  - small if  $1 - D(G(\mathbf{z}))$  is close to 0
  - small if  $D(G(\mathbf{z}))$  is close to 1
  - fool the discriminator into assigning high probability to fake objects

I. Goodfellow et al., *Generative Adversarial Nets*. in Proc. of NIPS, 2014, pp. 2672--2680.

# 02 Learning social Graph using topologies using GANs

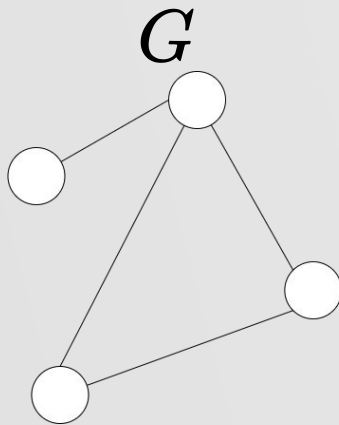
## Learning Social Graph Topologies using Generative Adversarial Neural Networks

Sahar Tavakoli<sup>1</sup>, Alireza Hajibagheri<sup>1</sup>, and Gita Sukthankar<sup>1</sup>

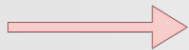
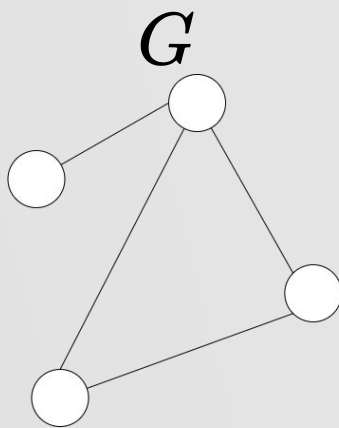
<sup>1</sup>University of Central Florida, Orlando, Florida

sahar@knights.ucf.edu, alireza@eecs.ucf.edu, gitars@eecs.ucf.edu

## 02 Learning social Graph using topologies using GANs



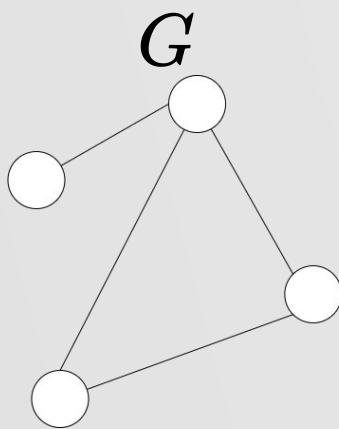
## 02 Learning social Graph using topologies using GANs



$Adj(G)$

0	1	0	0
1	0	1	1
0	1	0	1
0	1	1	0

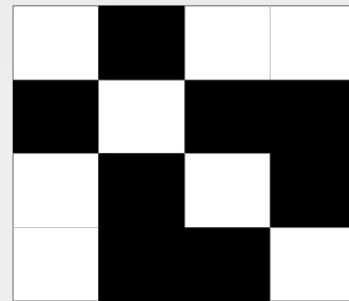
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$Adj(G)$

0	1	0	0
1	0	1	1
0	1	0	1
0	1	1	0

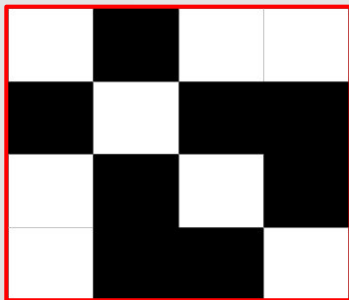
$Img$





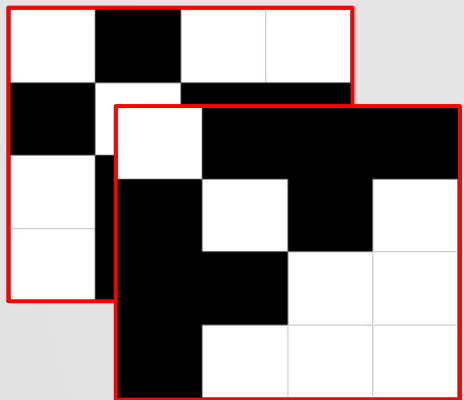
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*Img*



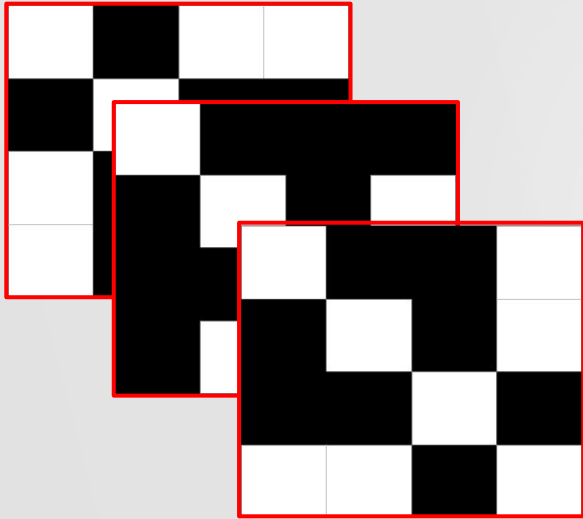
## 02 Learning social Graph using topologies using GANs

*Img*



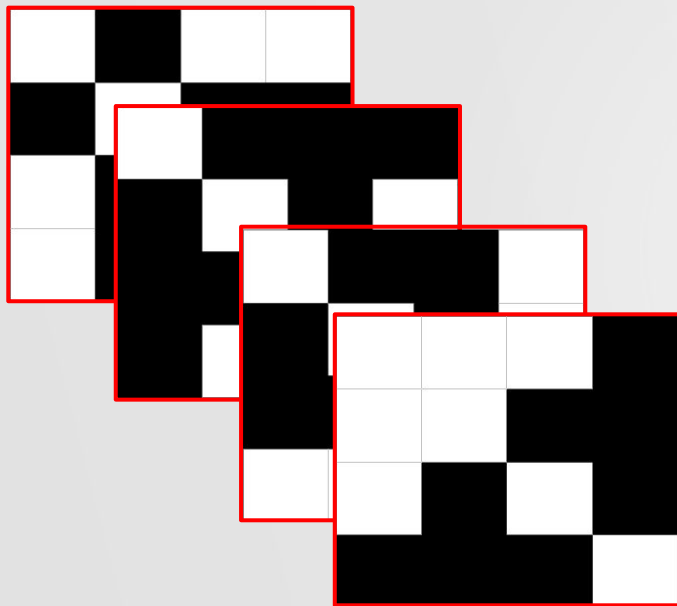
## 02 Learning social Graph using topologies using GANs

*Img*

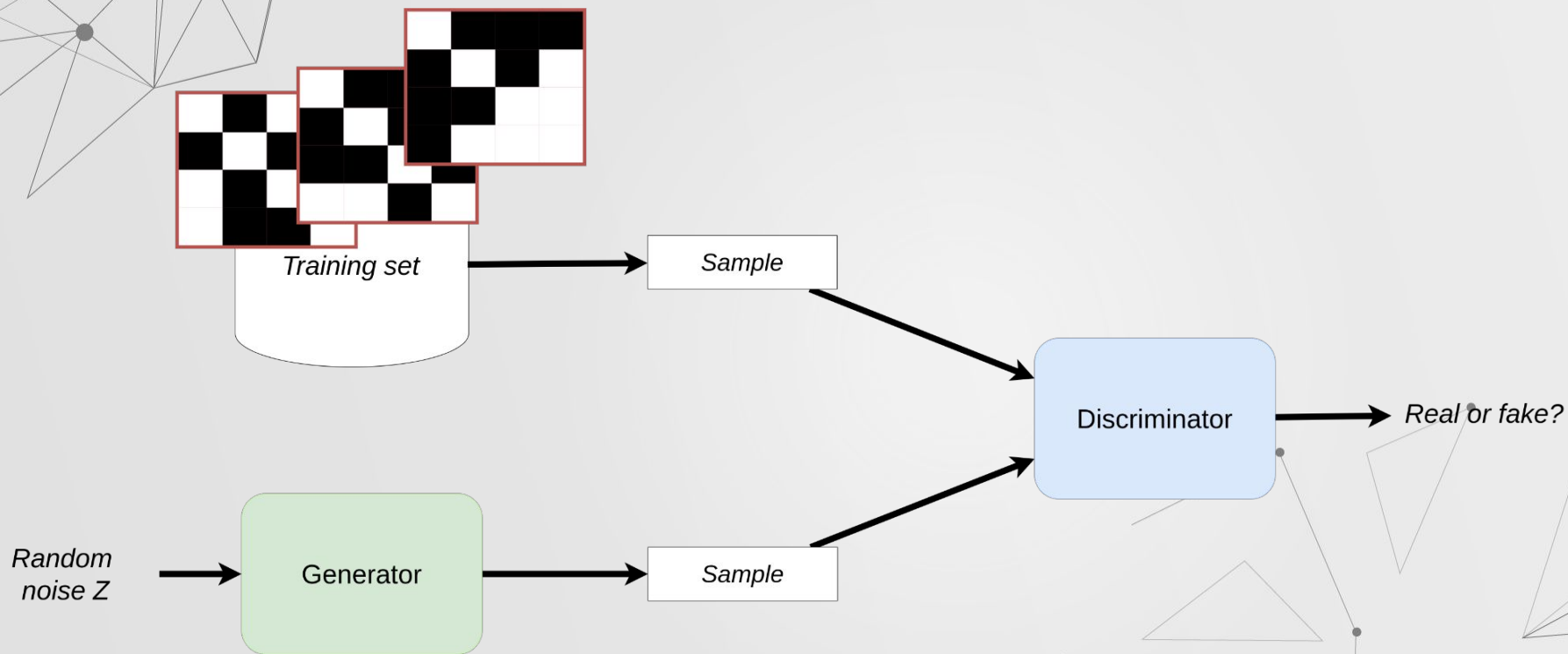


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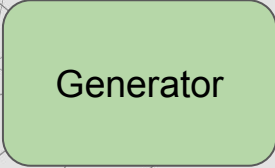
*Img*



## 02 Learning social Graph using topologies using GANs



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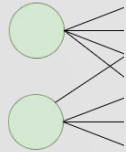


Generator

INPUT

$$I \in \mathbf{R}^{100}$$

$$I \sim N(0, 1)$$



100  
neurons

## 02 Learning social Graph using topologies using GANs

Generator

INPUT

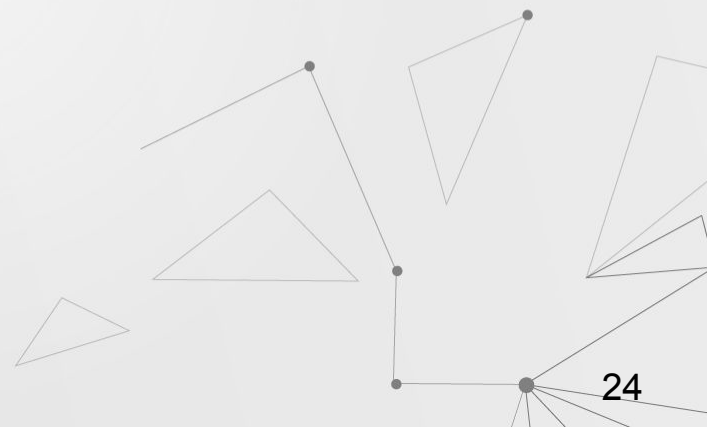
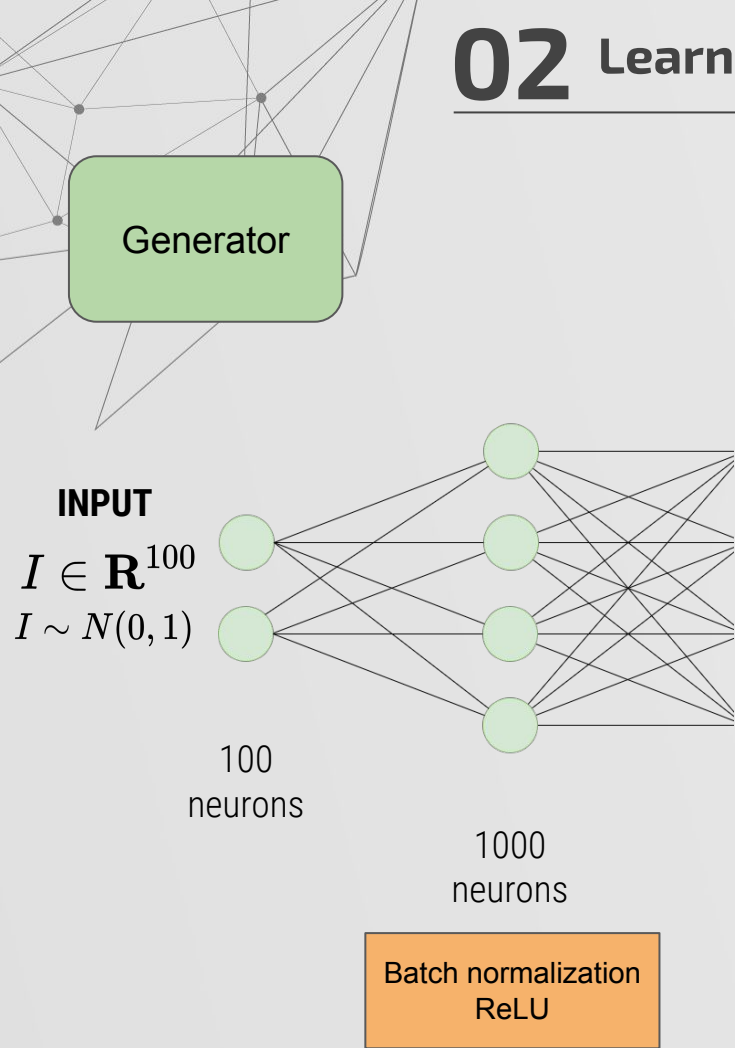
$$I \in \mathbf{R}^{100}$$

$$I \sim N(0, 1)$$

100  
neurons

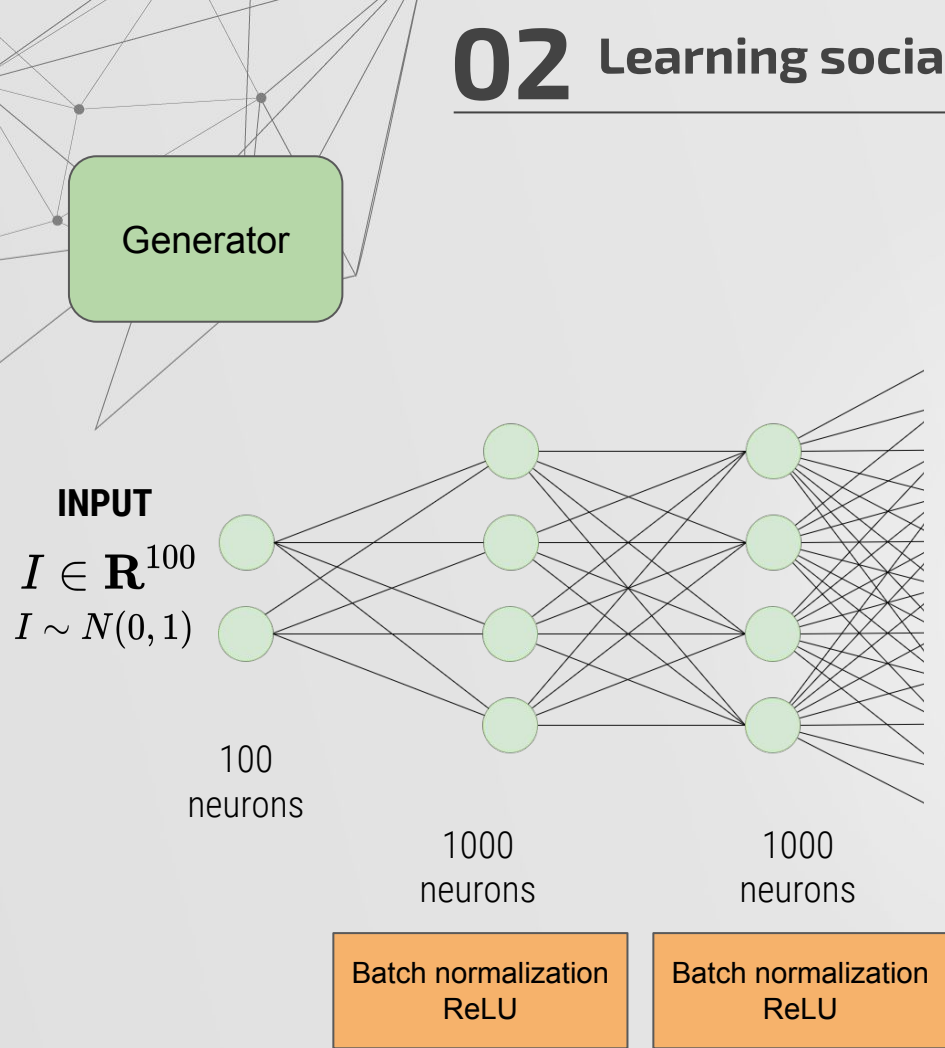
1000  
neurons

## 02 Learning social Graph using topologies using GANs

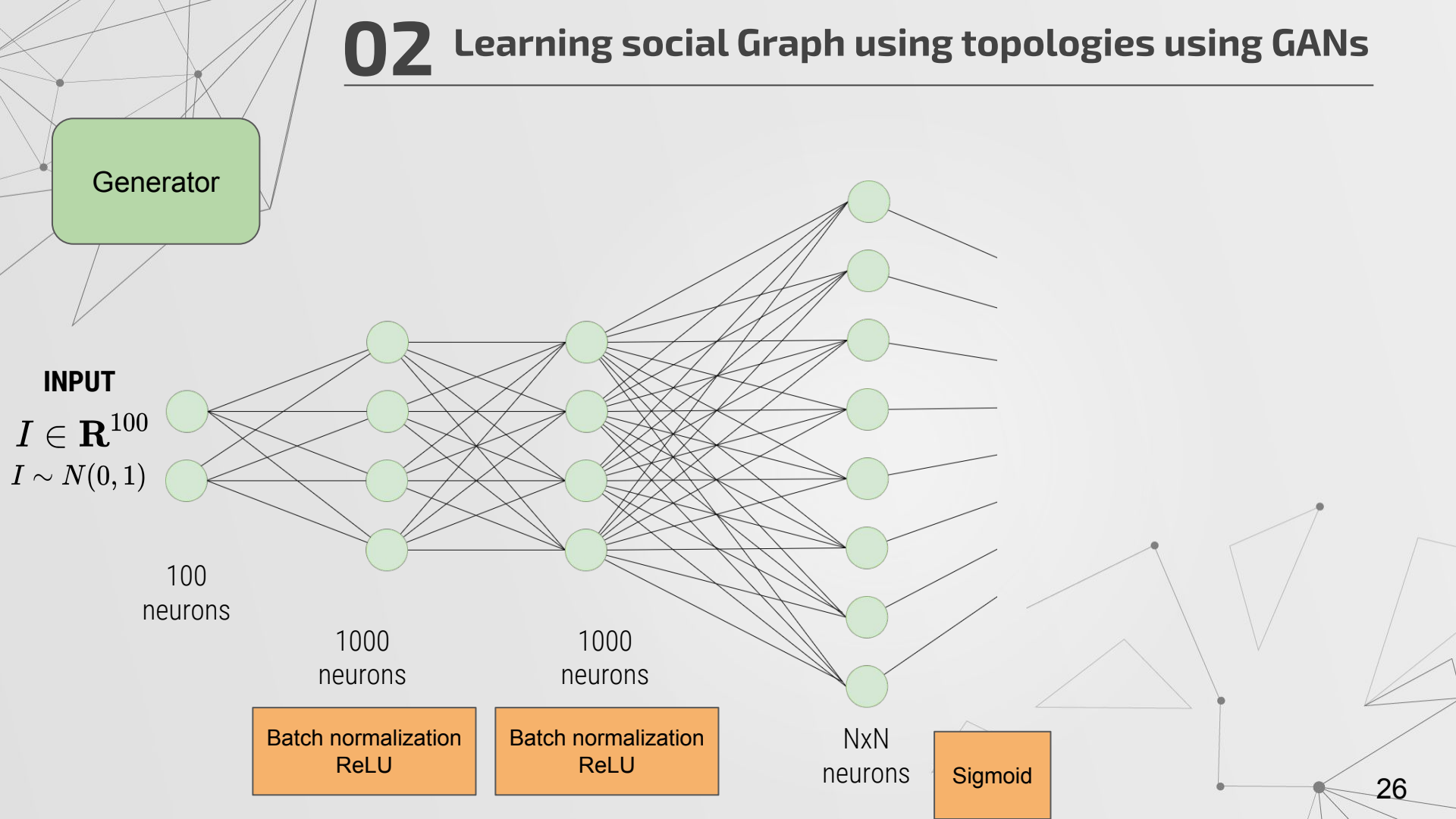




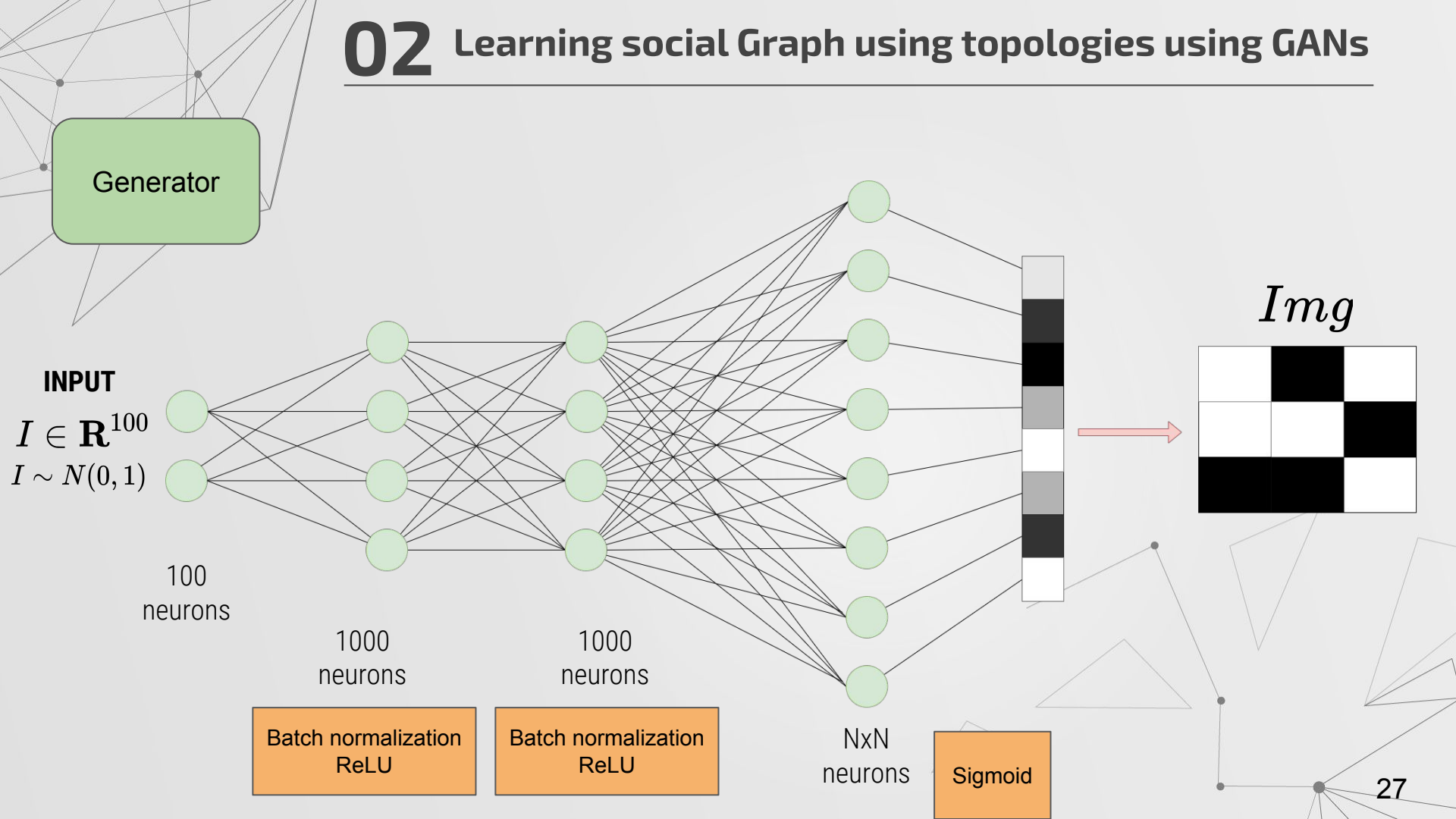
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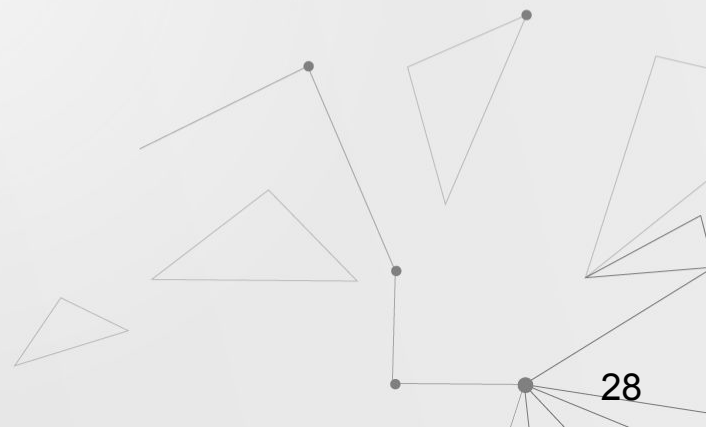
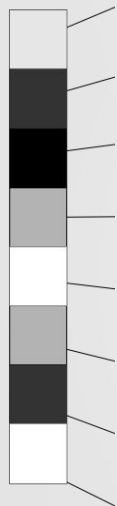
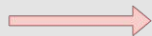
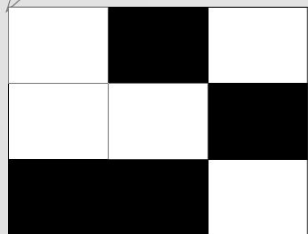
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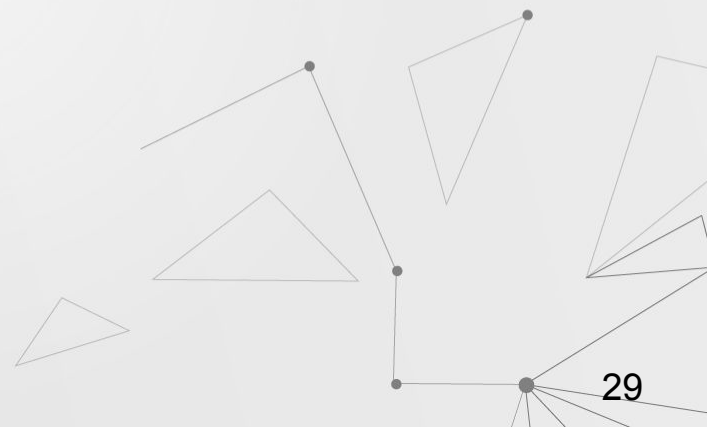
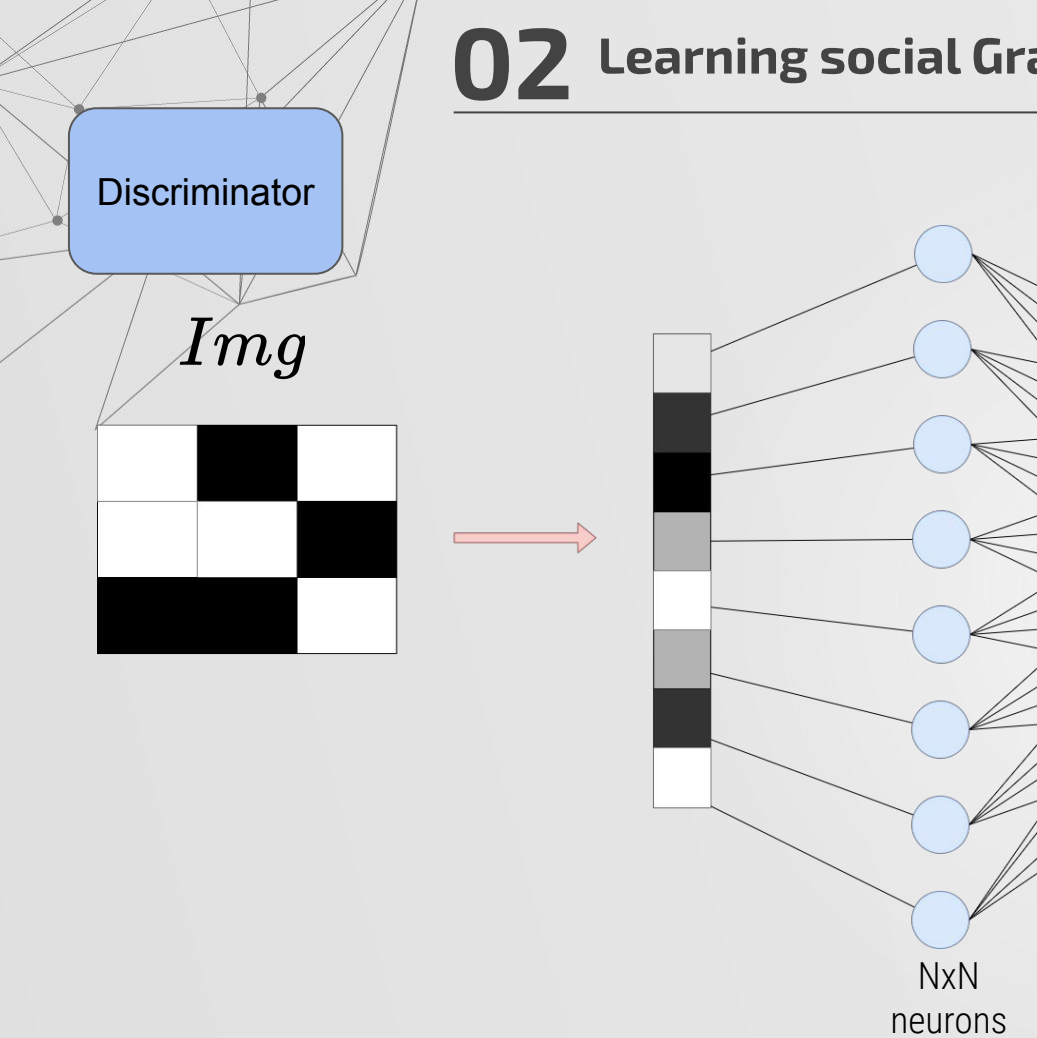
## 02 Learning social Graph using topologies using GANs

Discriminator

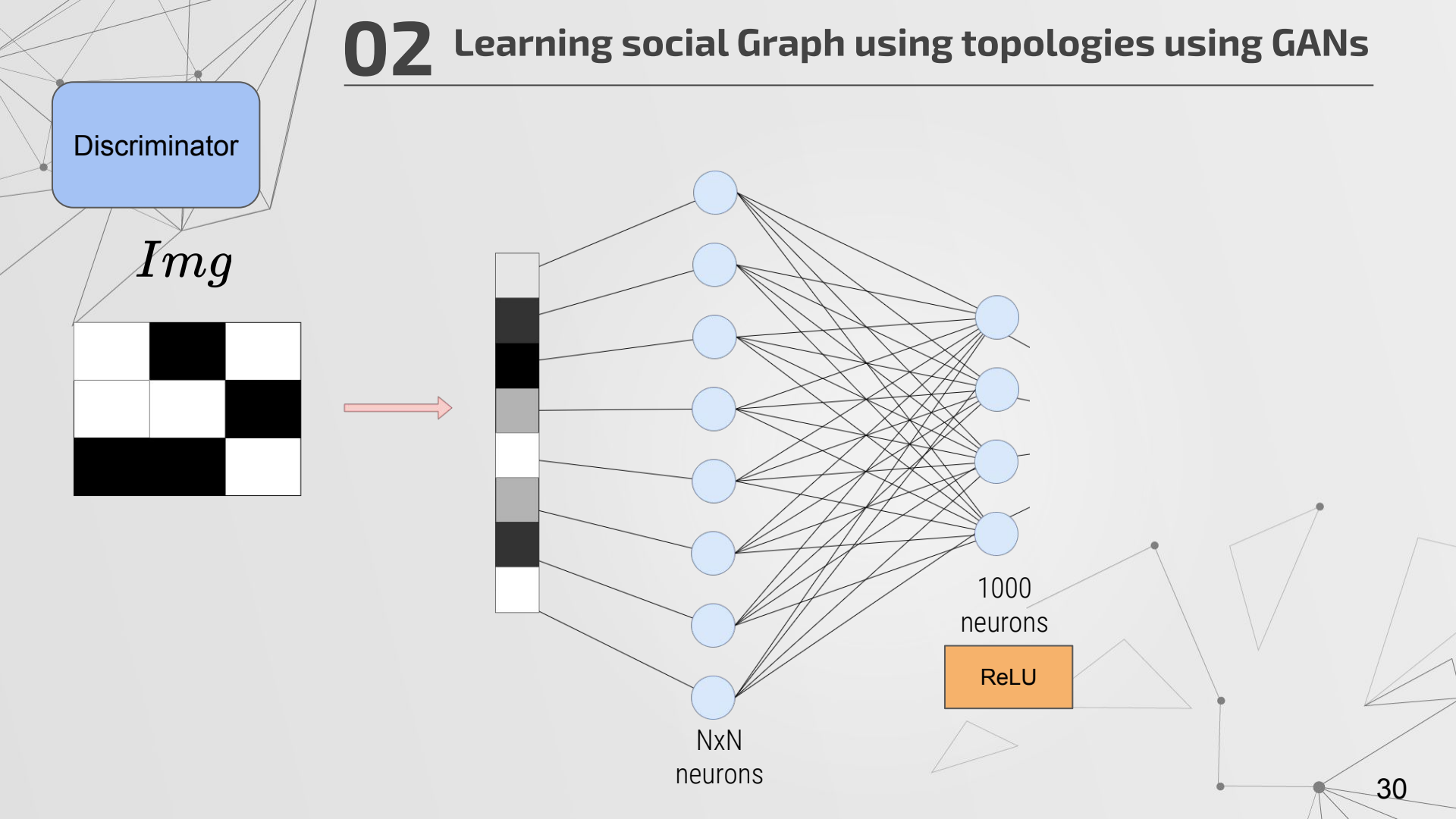
*Img*



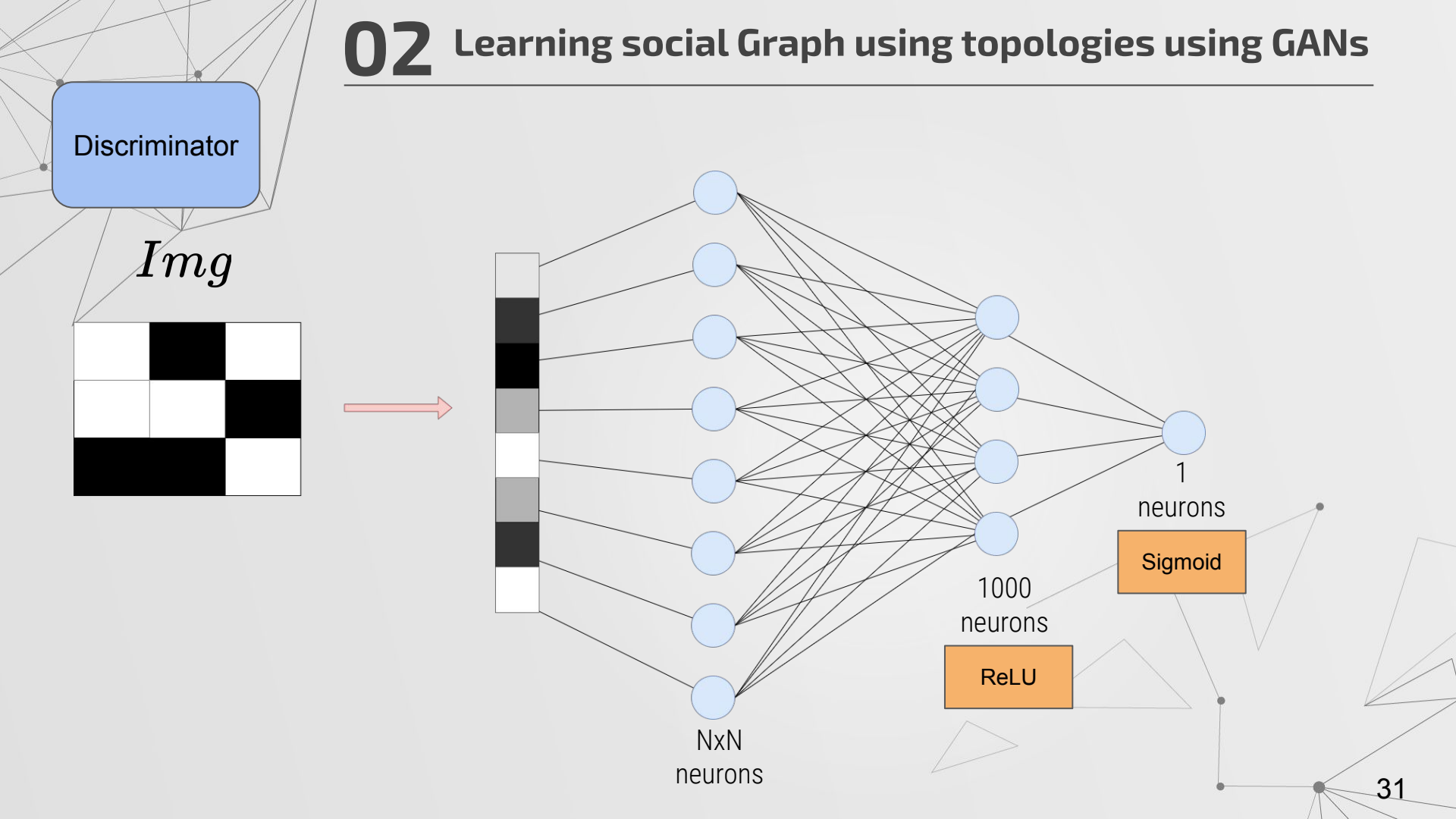
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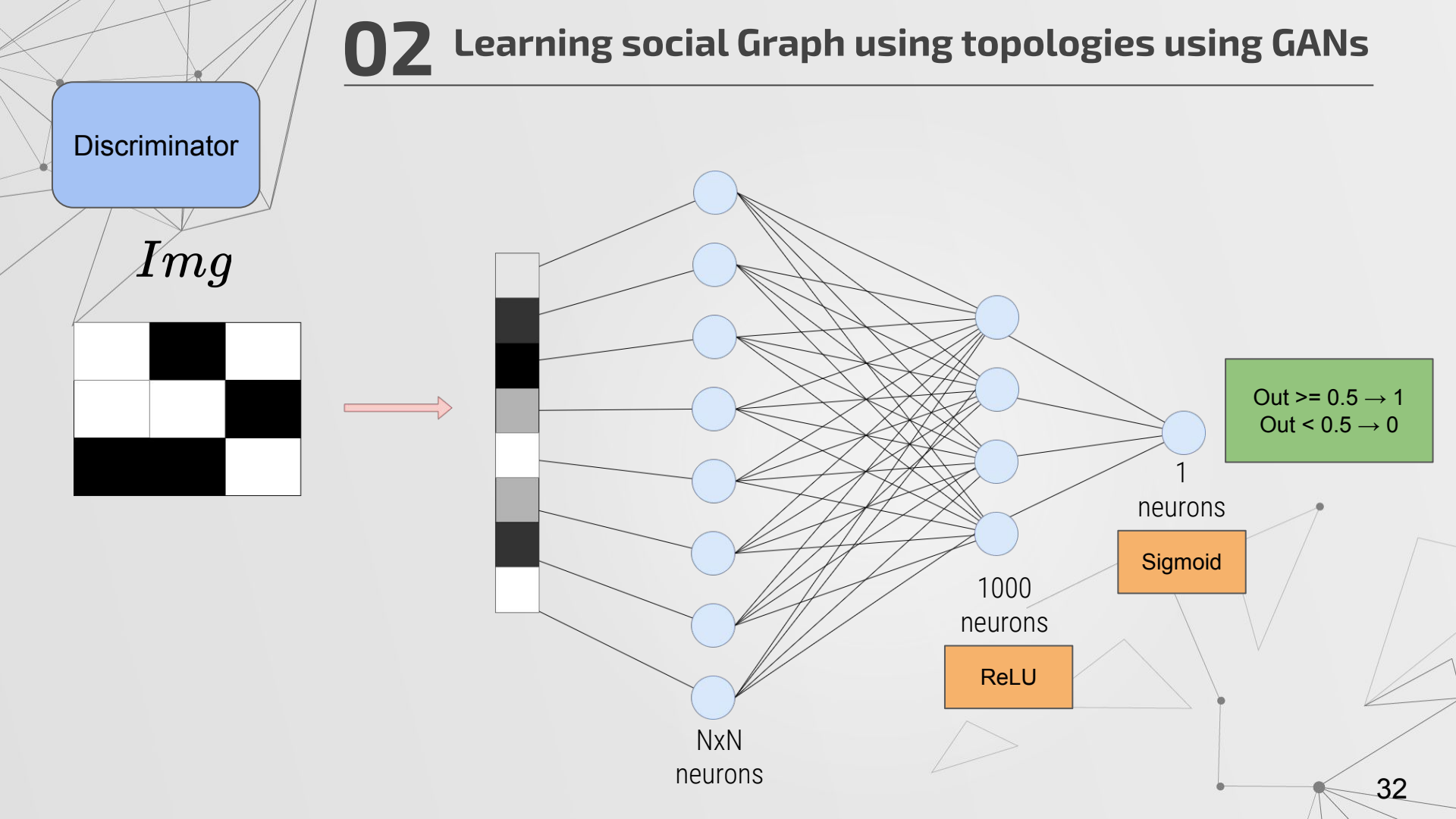
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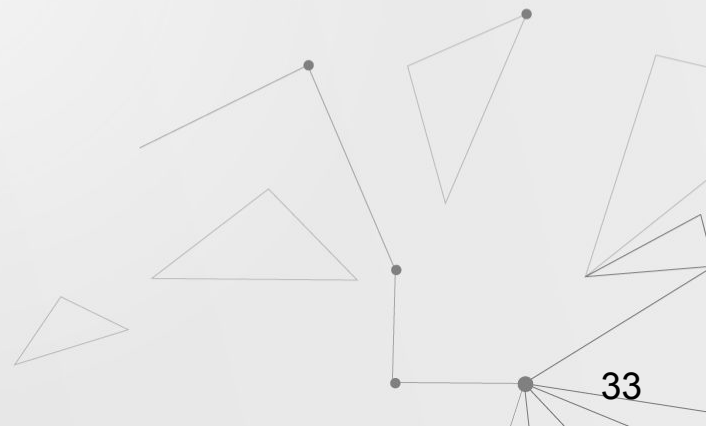


## 02 Learning social Graph using topologies using GANs

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Results:

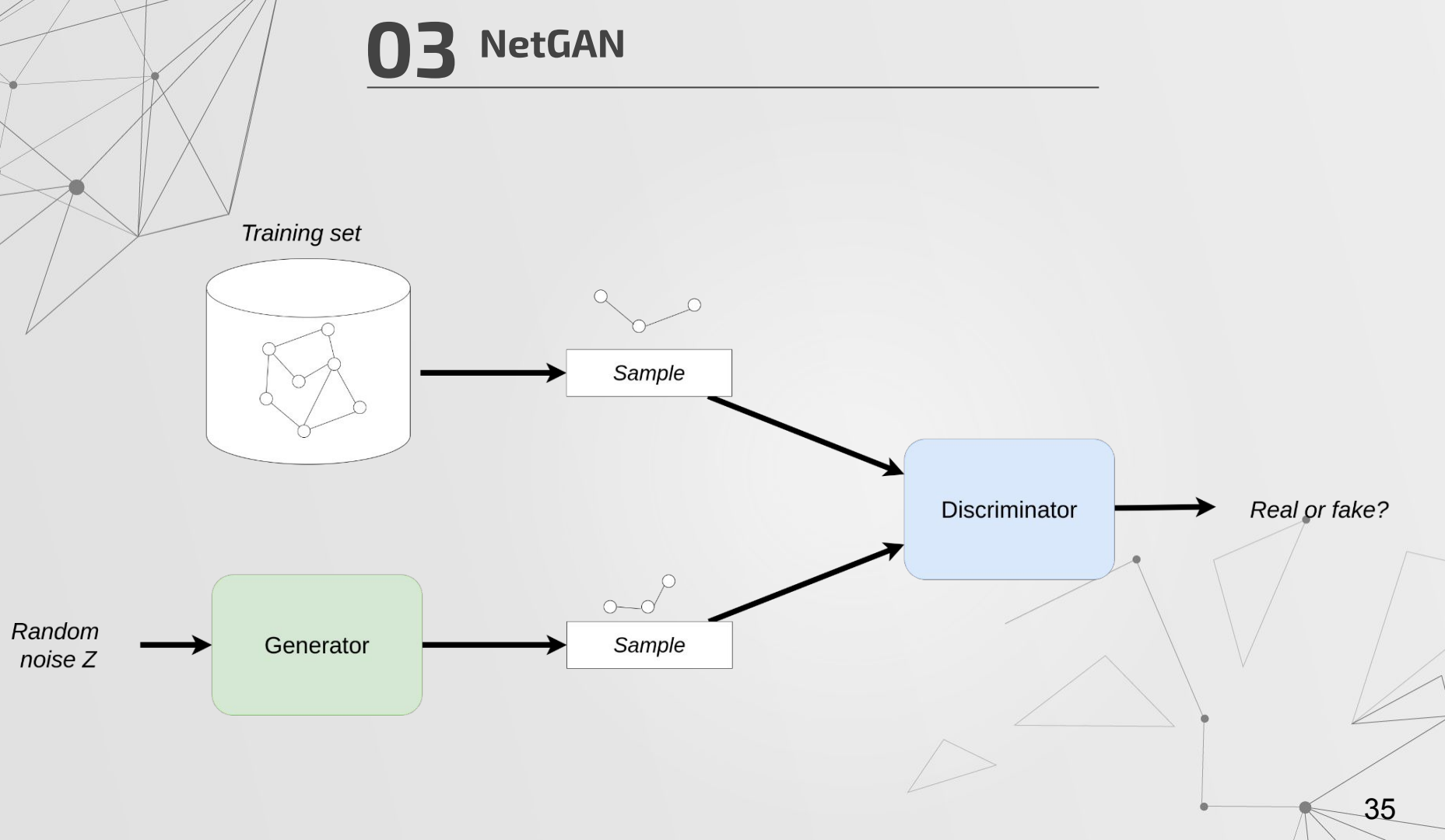
- **Comparison** of several **features** between original and generated graph. (Nb nodes, nb.edges, avg. degree, diameter, assortativity, etc ..)
- On several **social interaction networks** (Karate club, Football, Dolphins, Enron)

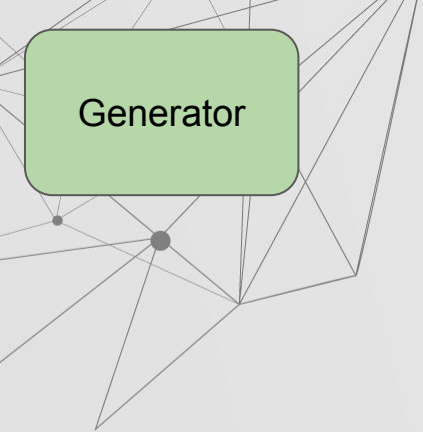


## NetGAN: Generating Graphs via Random Walks

Aleksandar Bojchevski<sup>\*1</sup> Oleksandr Shchur<sup>\*1</sup> Daniel Zügner<sup>\*1</sup> Stephan Günnemann<sup>1</sup>

# 03 NetGAN





Generator

# 03 NetGAN

---

z

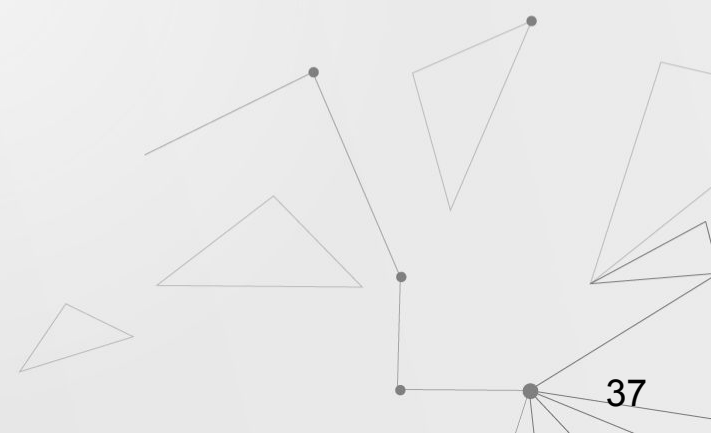
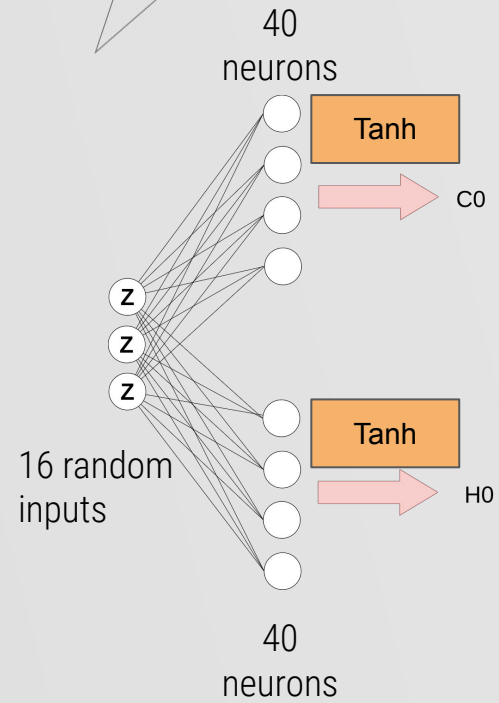
z

z

16 random  
inputs

# 03 NetGAN

Generator

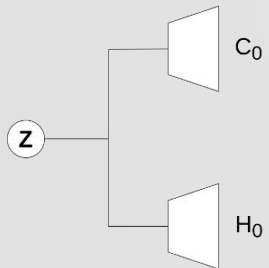




Generator

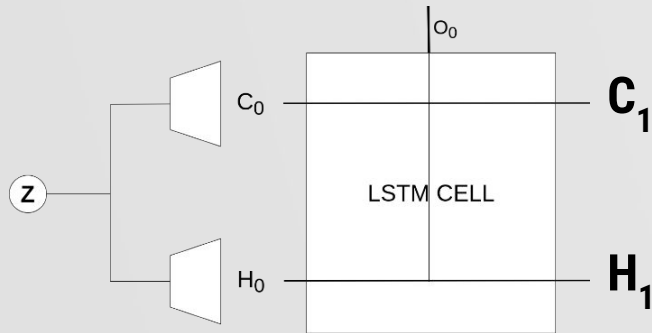
# 03 NetGAN

---



Generator

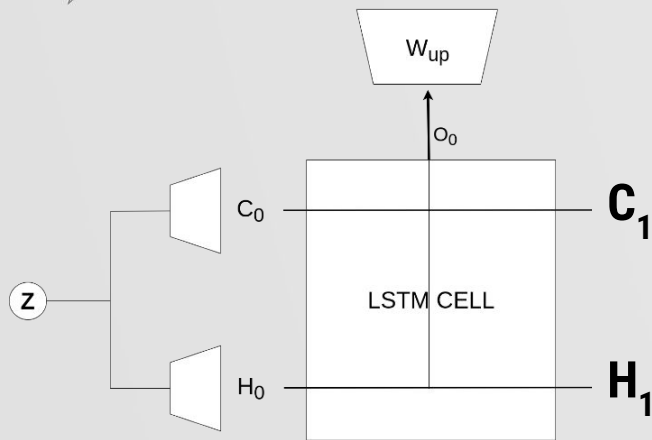
# 03 NetGAN



**LSTM cell**

Generator

# 03 NetGAN

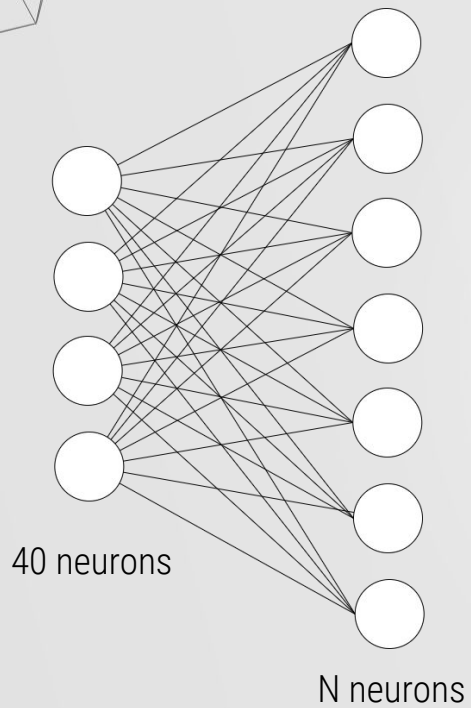


$W_{up}$



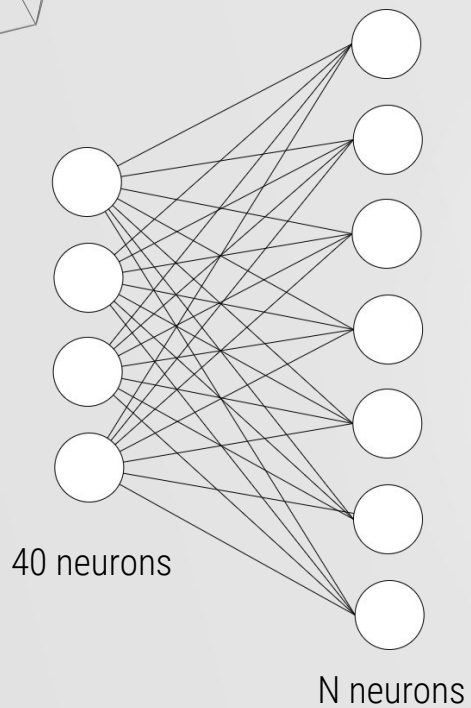
# 03 NetGAN

$W_{up}$



# 03 NetGAN

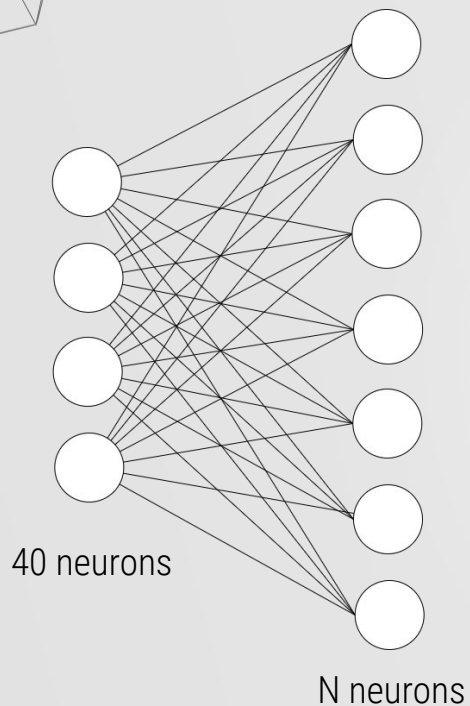
$W_{up}$



$p_1$
$p_2$
$p_N$

# 03 NetGAN

$W_{up}$



$p_1$
$p_2$
$p_N$

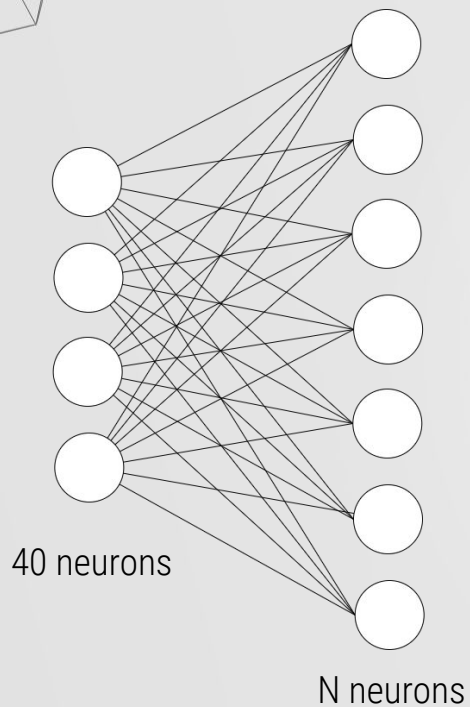
$$\sigma((p_1 + g)/\tau) = v_1^*$$

Temperature param

i.i.d. Sample from  
Gumbel dist.

# 03 NetGAN

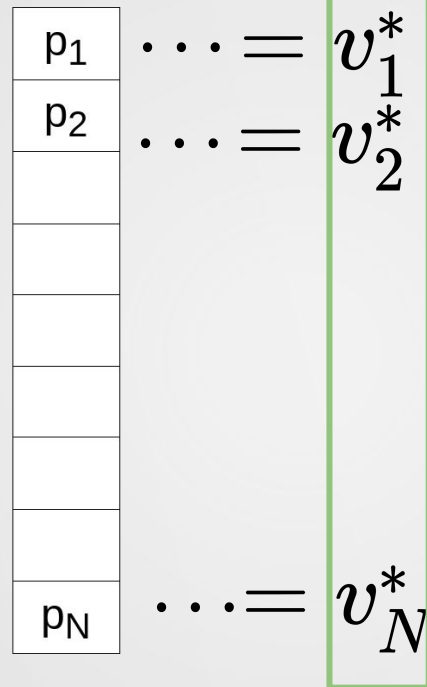
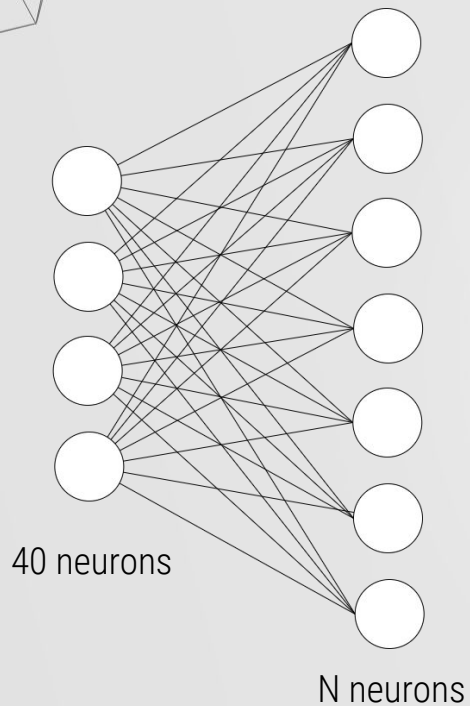
$W_{up}$



$p_1$	$\dots = v_1^*$
$p_2$	$\dots = v_2^*$
$p_N$	$\dots = v_N^*$

# 03 NetGAN

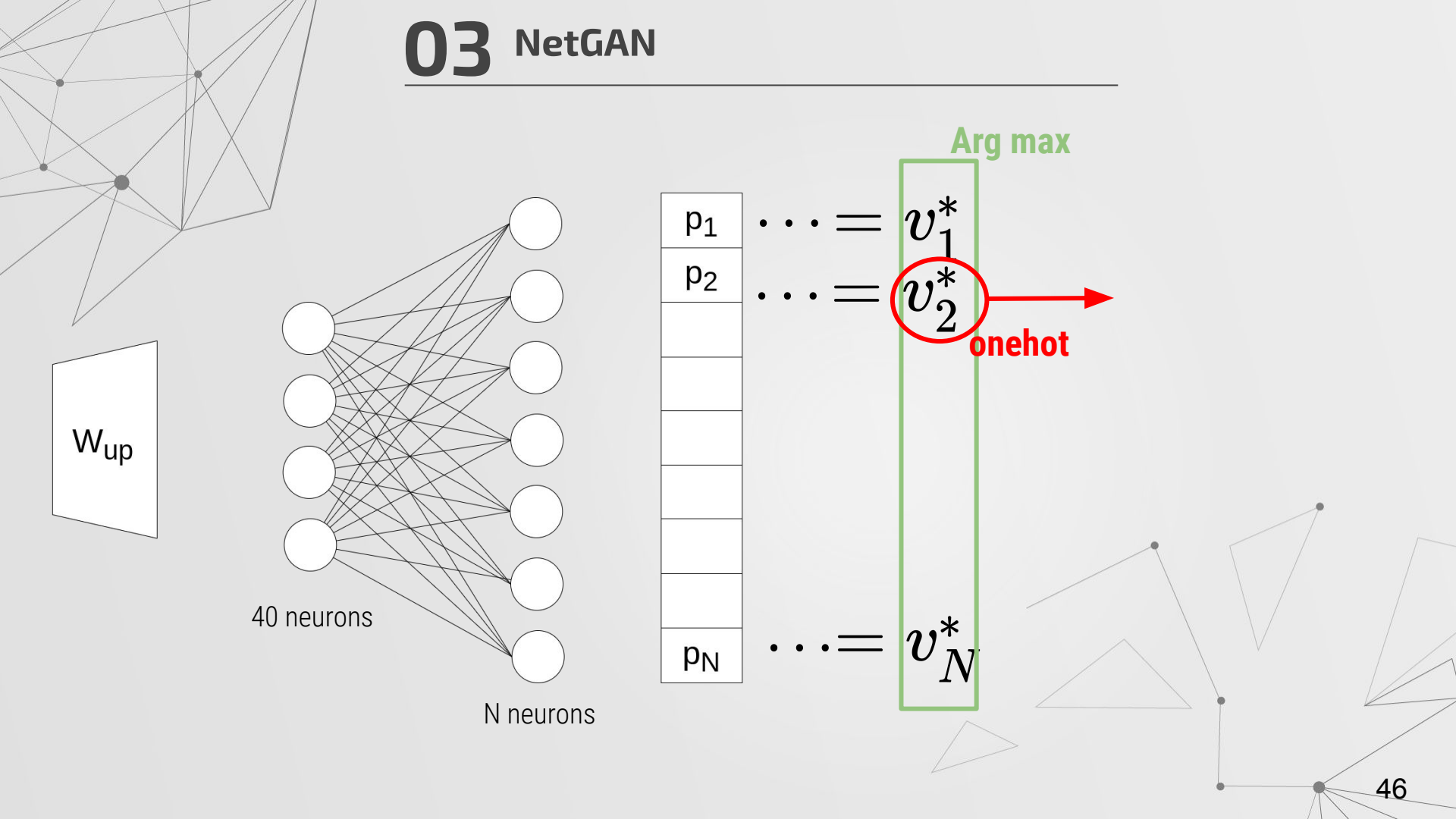
$W_{up}$



Arg max

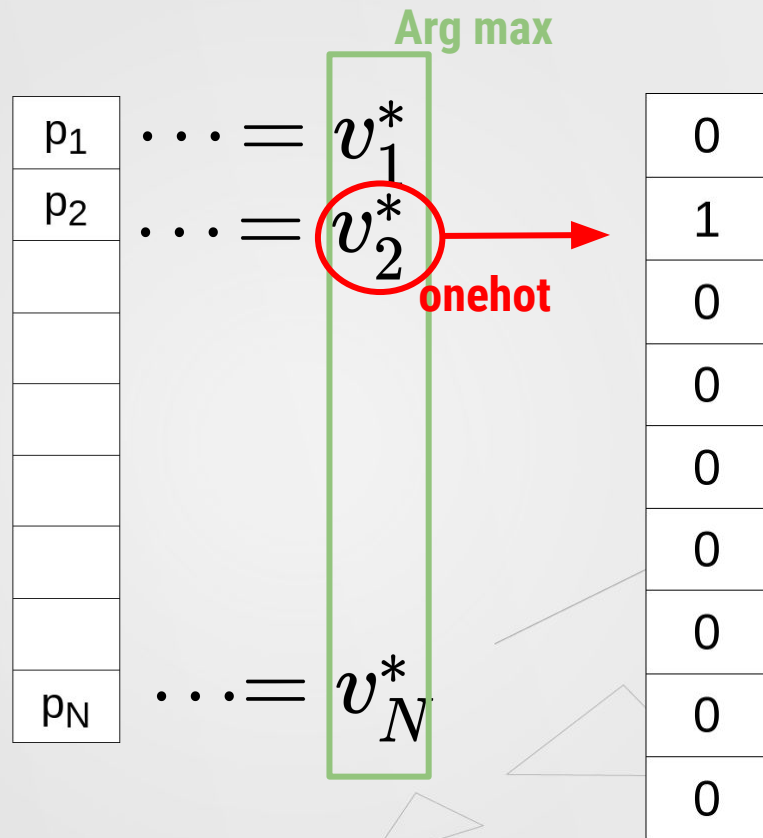
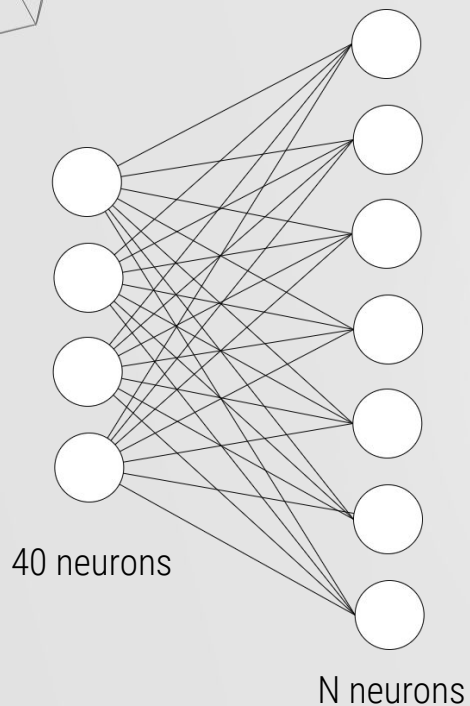


# 03 NetGAN



# 03 NetGAN

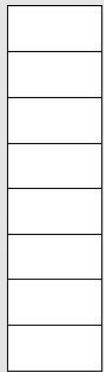
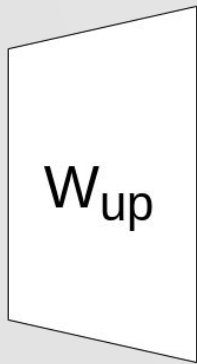
$W_{up}$



# 03 NetGAN

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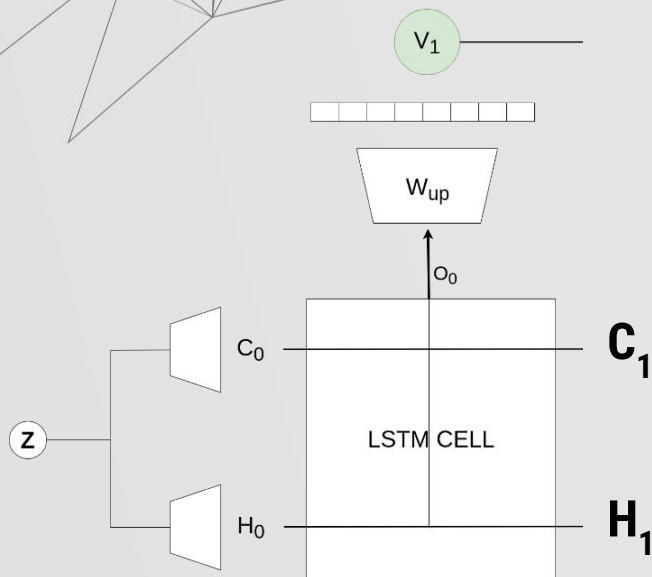
**H**





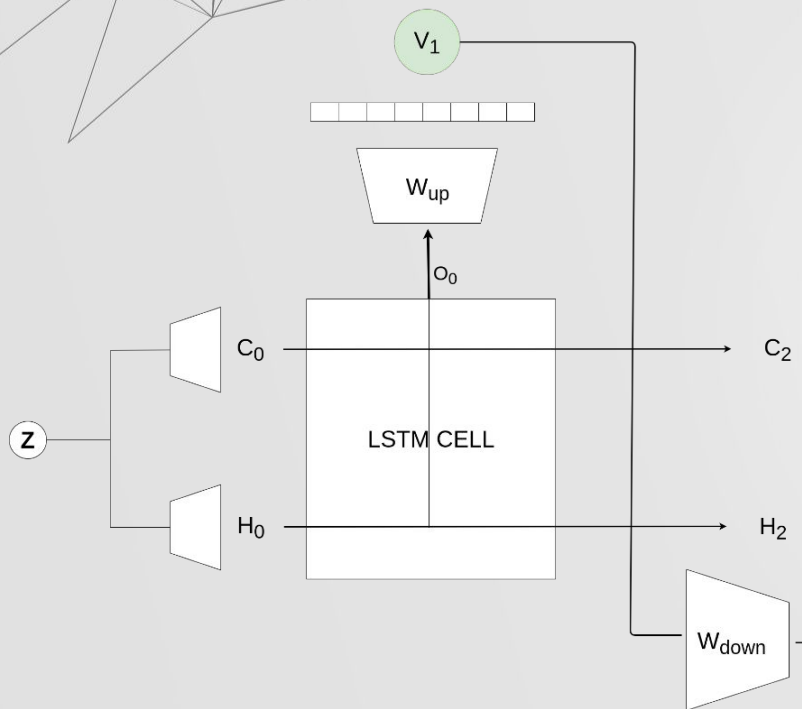
Generator

# 03 NetGAN



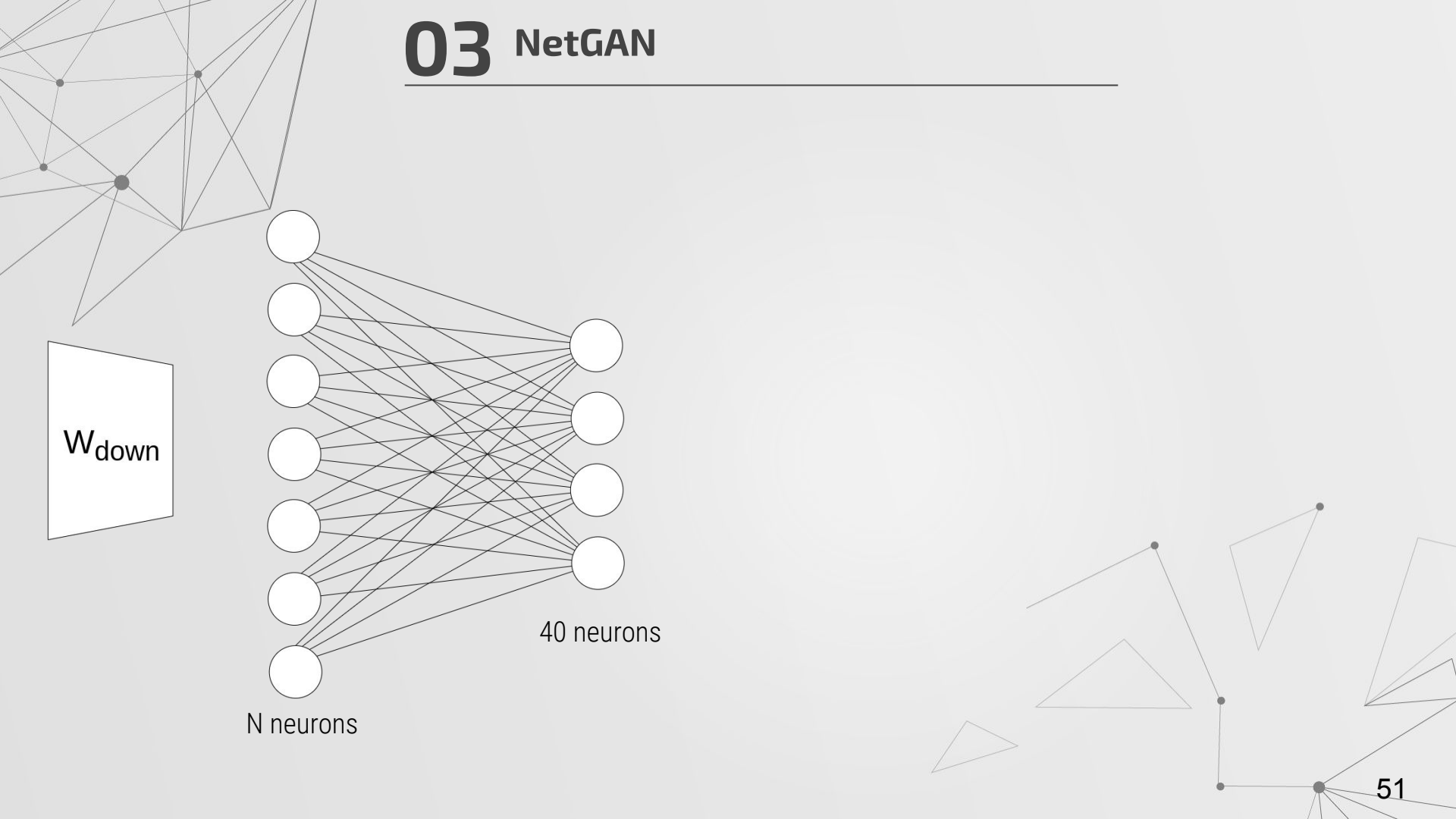
Generator

# 03 NetGAN



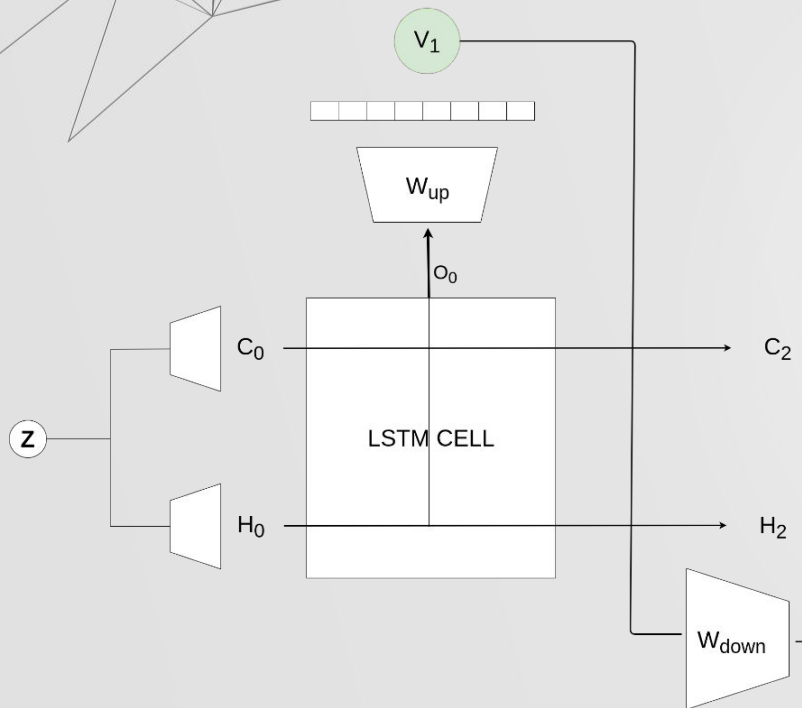
$W_{down}$

# 03 NetGAN



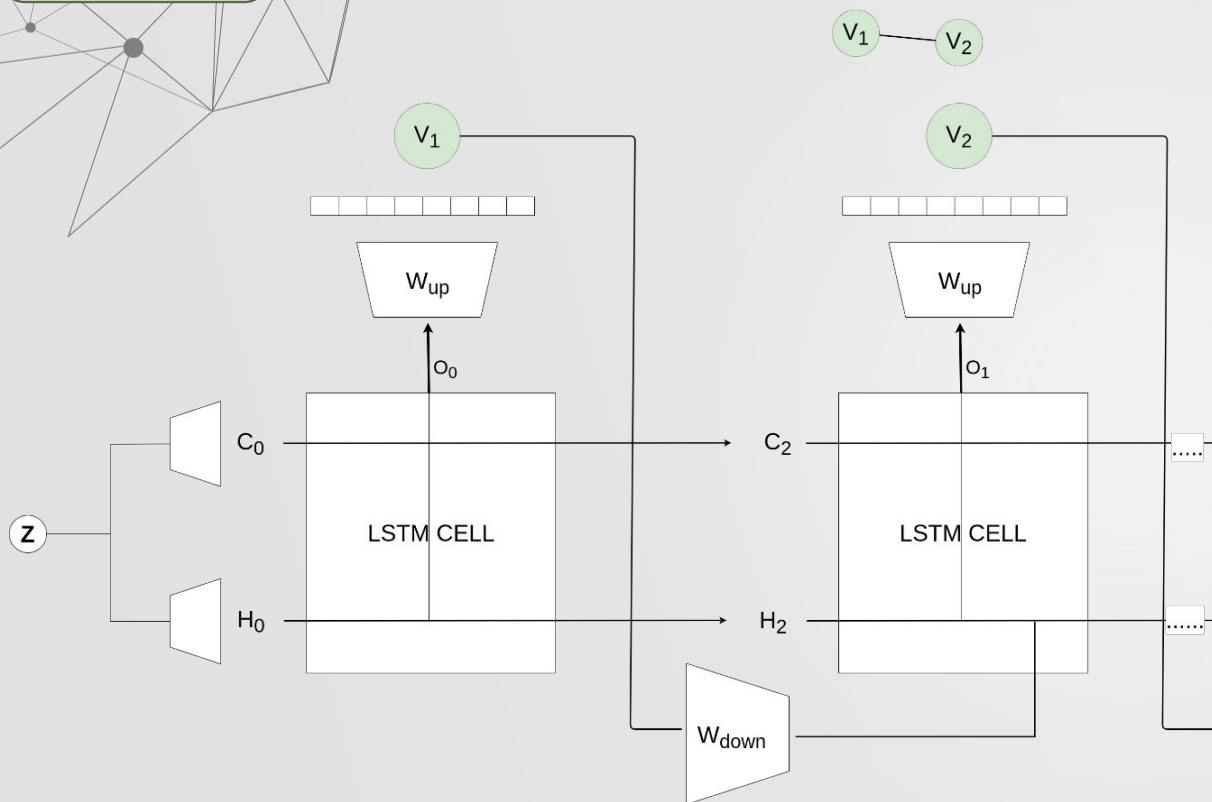
Generator

# 03 NetGAN

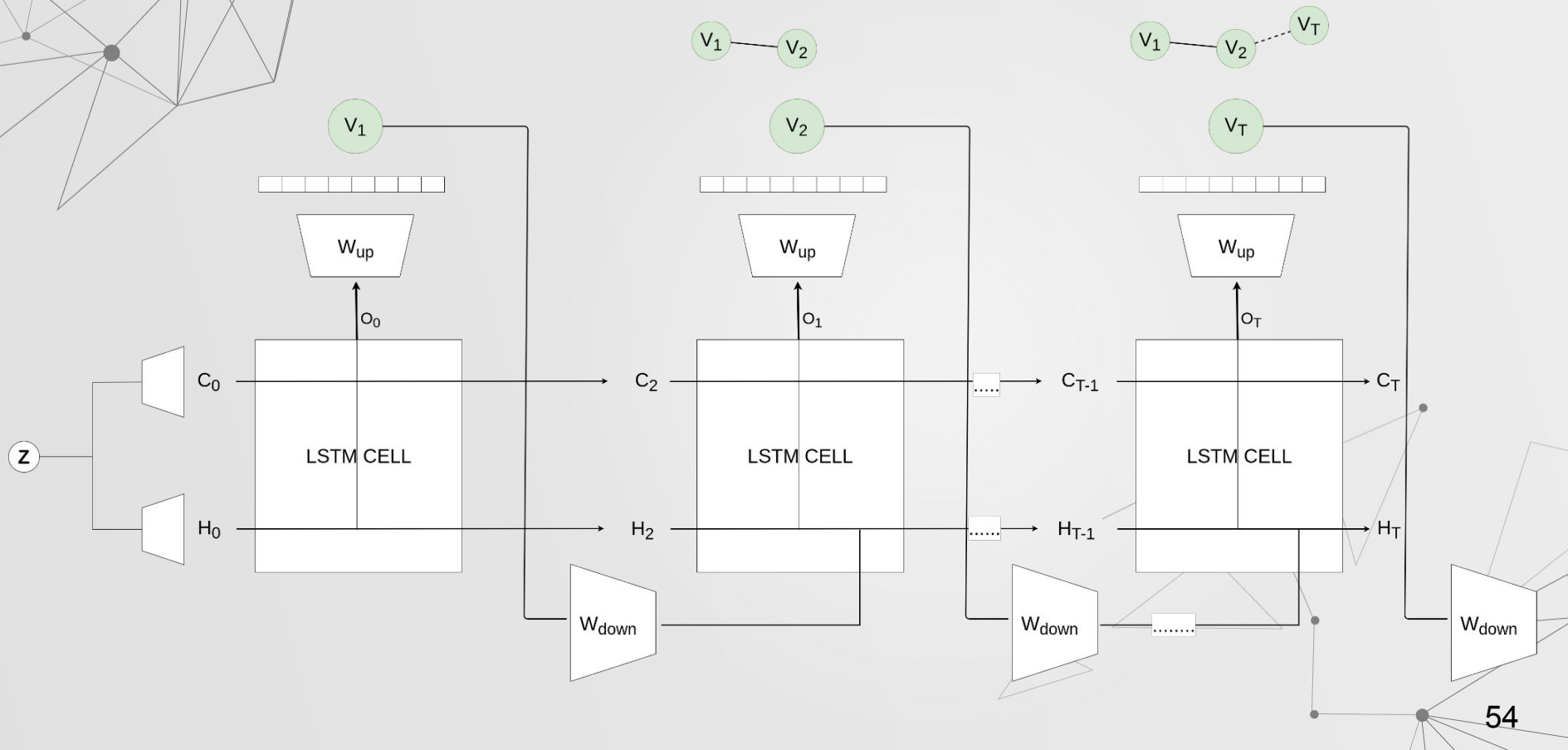


Generator

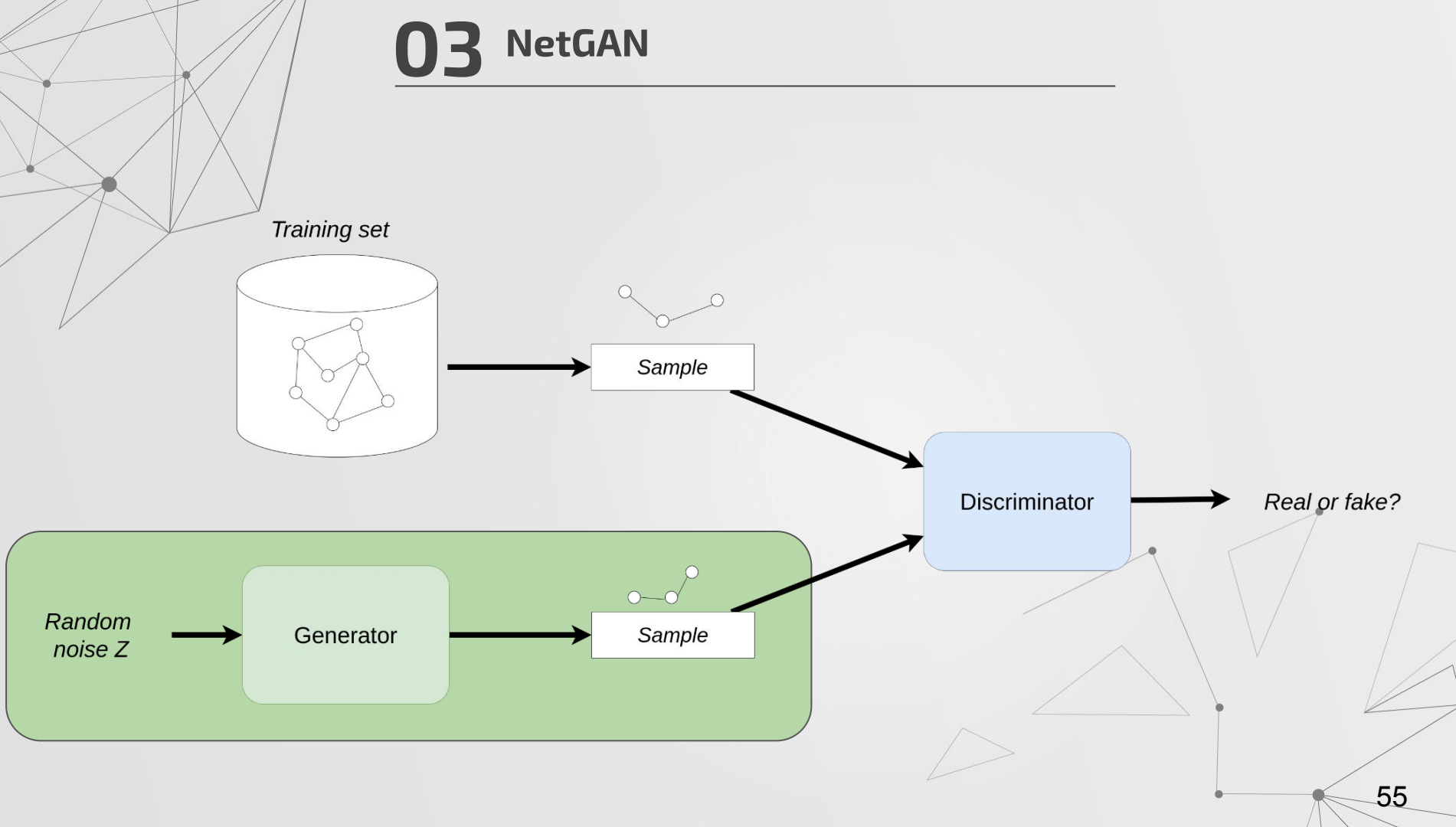
# 03 NetGAN



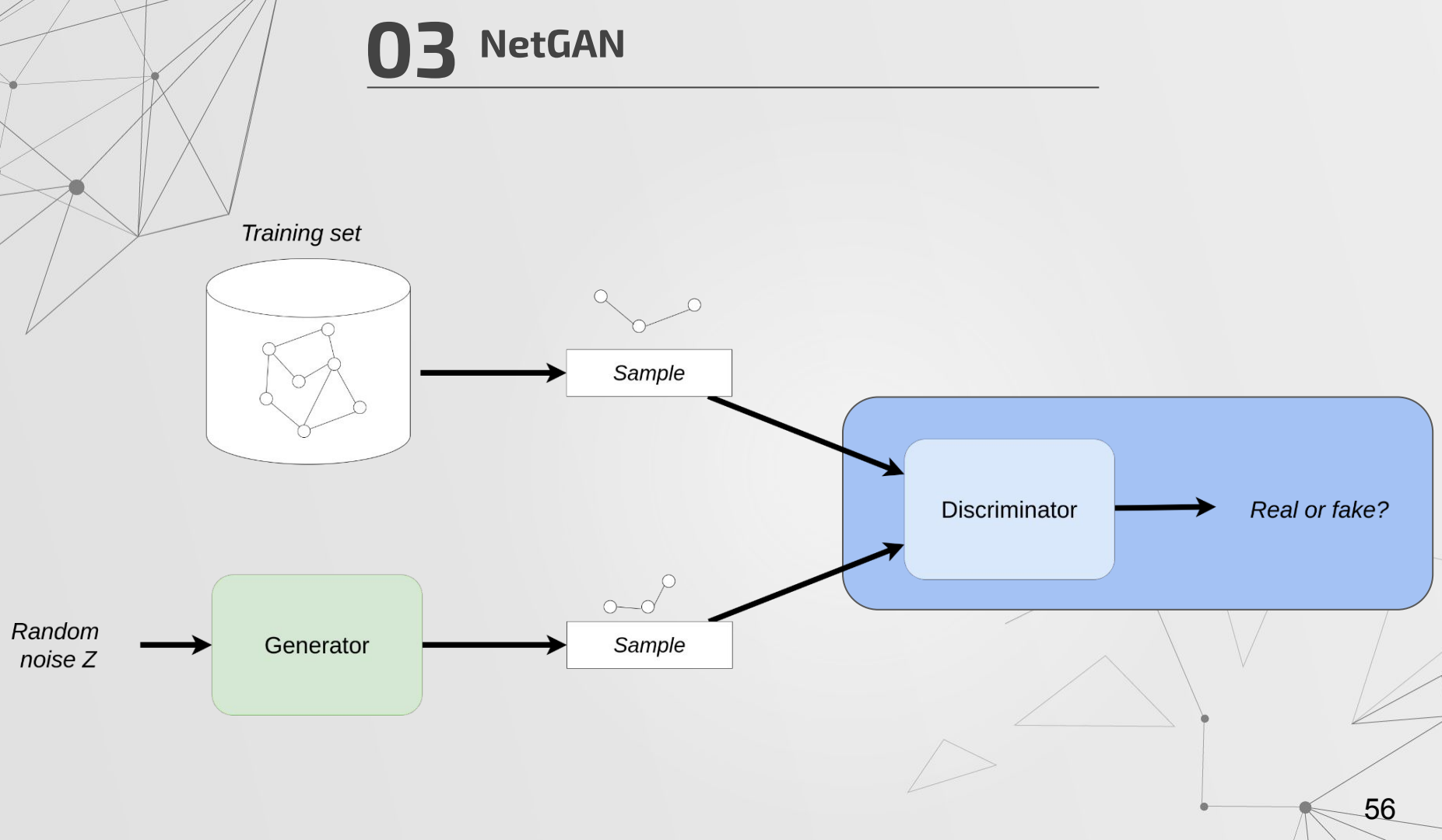
# 03 NetGAN



# 03 NetGAN



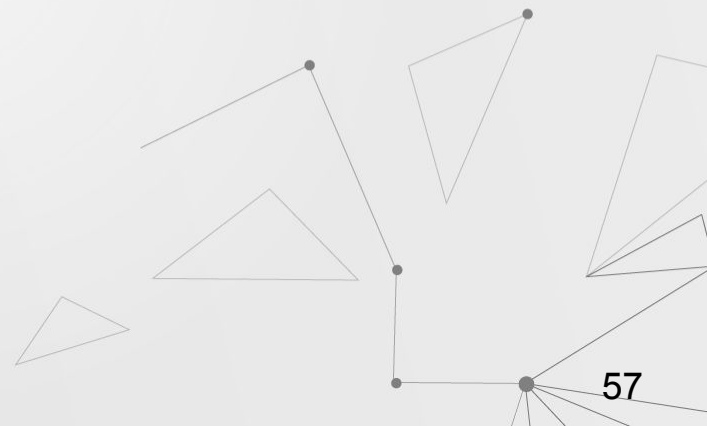
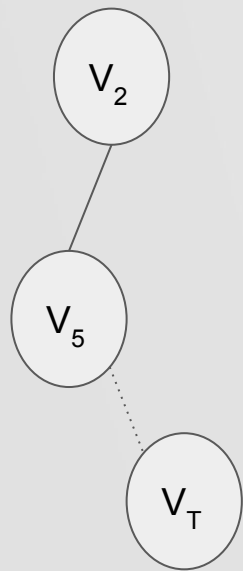
# 03 NetGAN





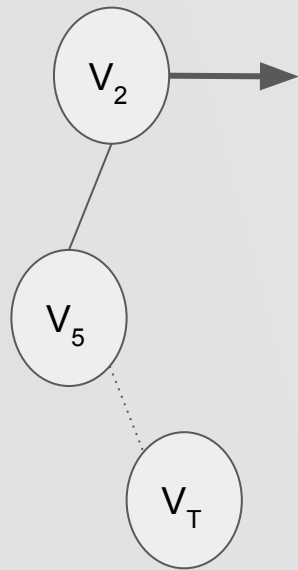
# 03 NetGAN

Discriminator



# 03 NetGAN

Discriminator

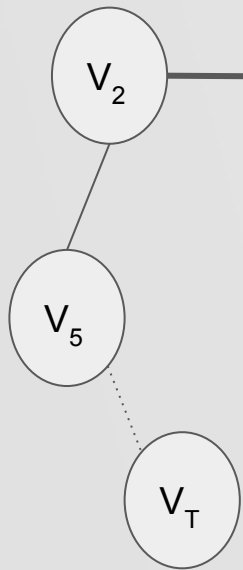


0
1
0
0
0
0
0
0
0



# 03 NetGAN

Discriminator

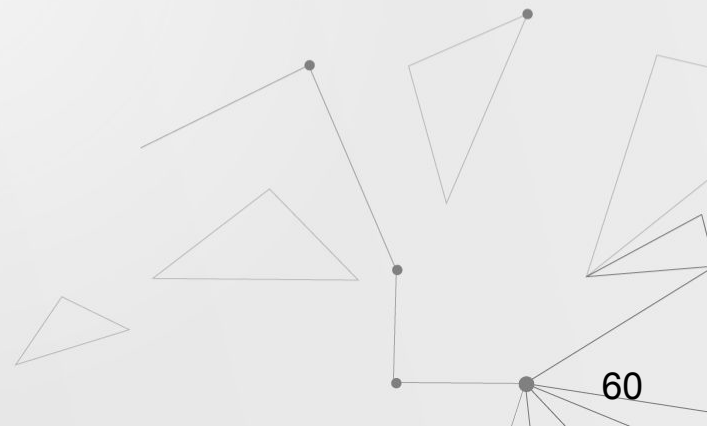
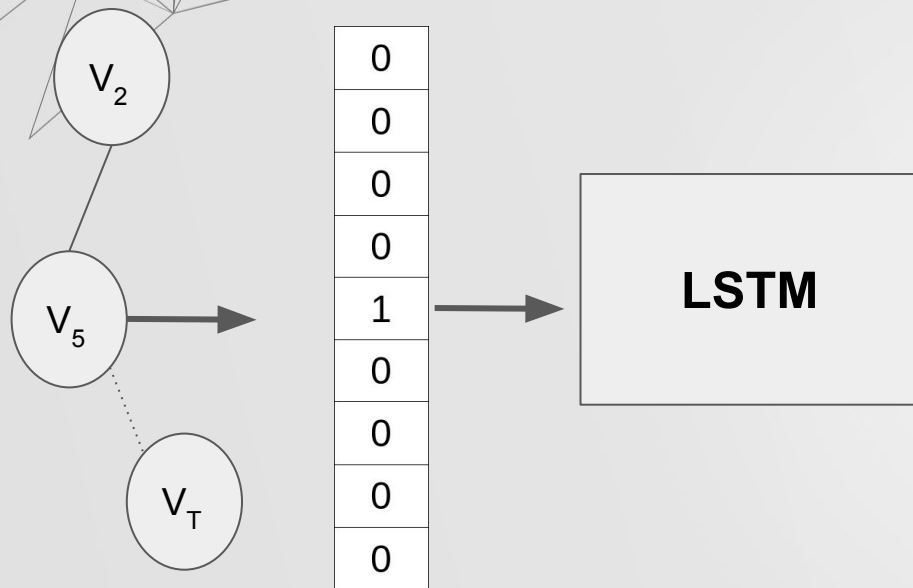


0
1
0
0
0
0
0
0
0
0

LSTM

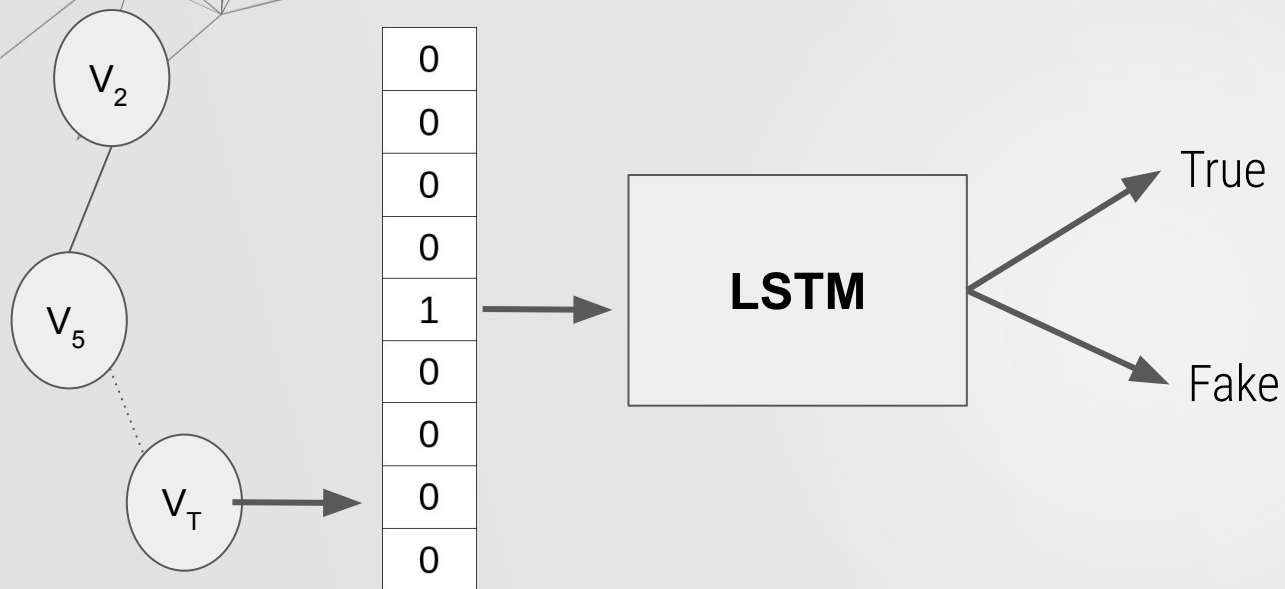
# 03 NetGAN

Discriminator

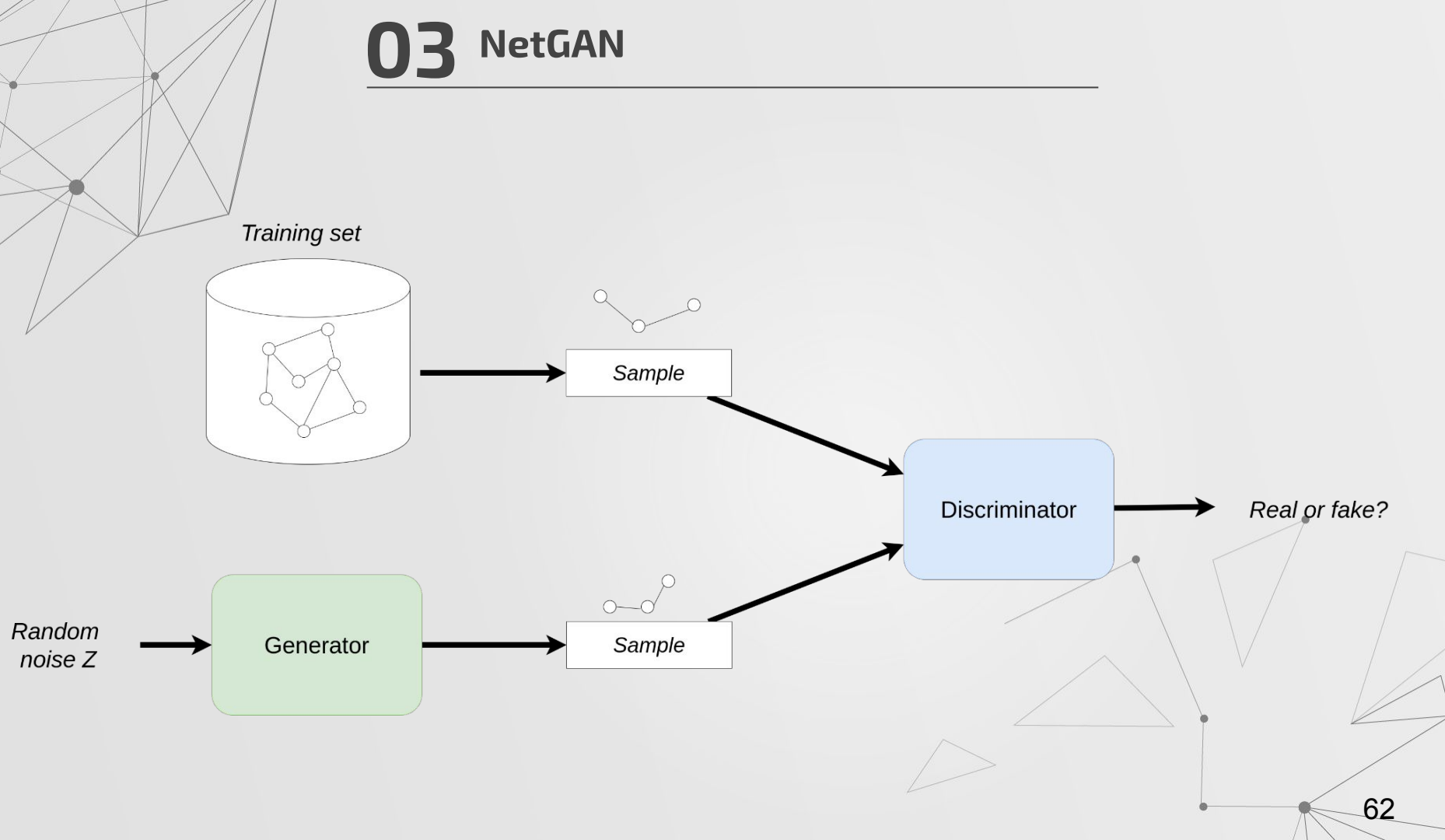


# 03 NetGAN

Discriminator

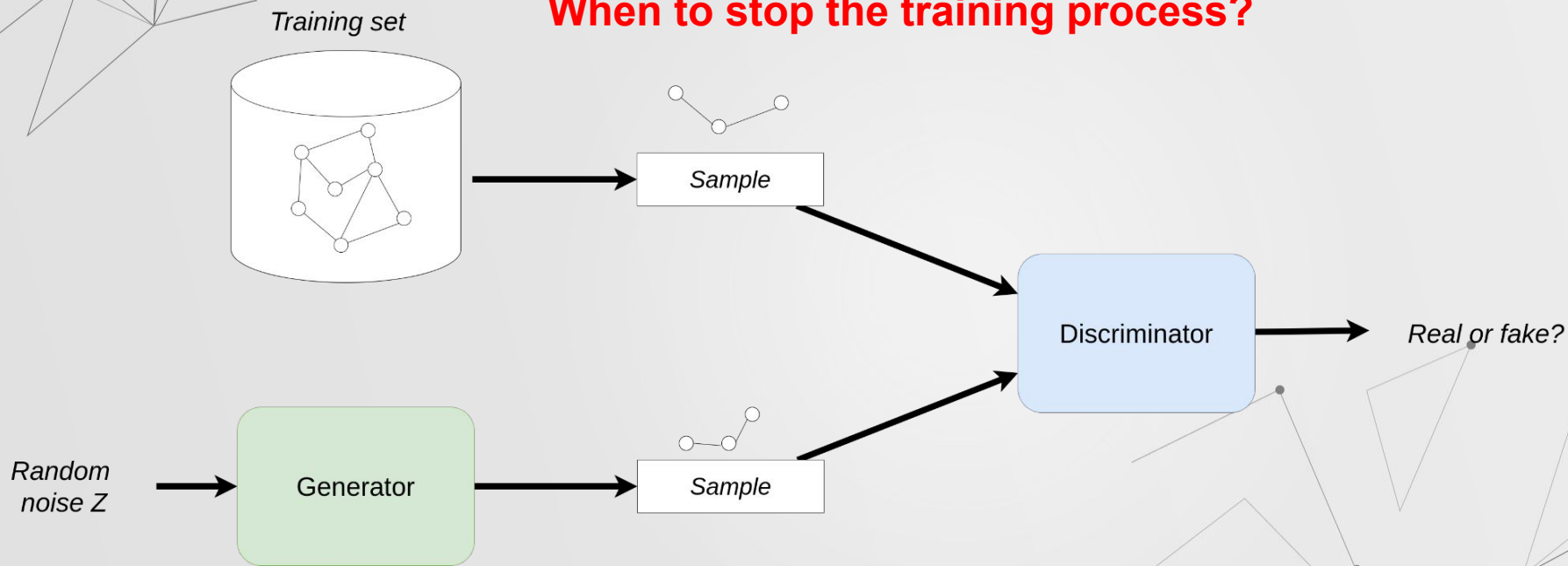


# 03 NetGAN



# 03 NetGAN

**When to stop the training process?**



**When to stop the training process?**

**EO-Criterion & VAL-Criterion**



**When to stop the training process?**

**EO-Criterion & VAL-Criterion**

### When to stop the training process?

#### EO-Criterion:

Stop the training process, when the input graph and the generated graph has an **edge overlapping** specified by the user.





# 03 NetGAN

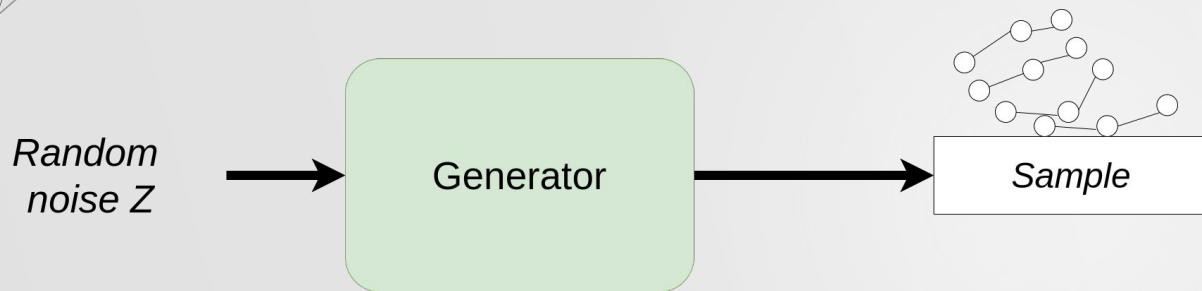
---

**How to build the graph?**



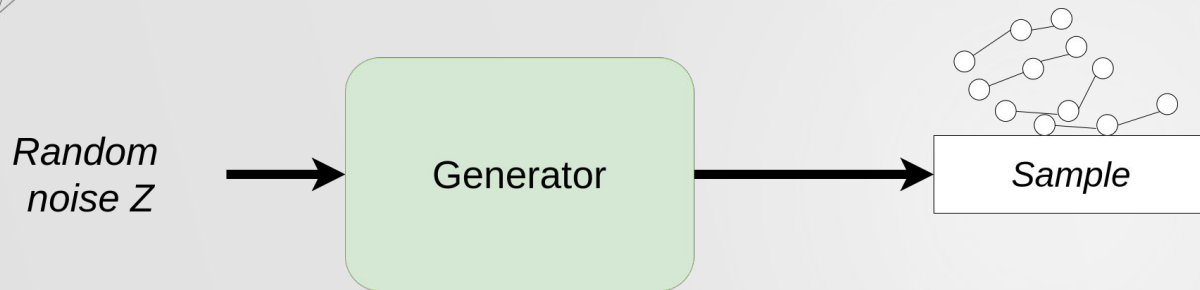
# 03 NetGAN

How to build the graph?



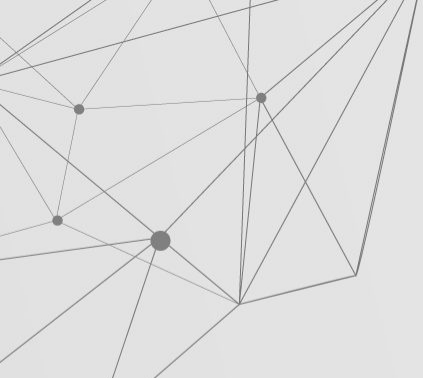
# 03 NetGAN

How to build the graph?



0	10	2	0	1
..	..	..	..	2
..	..	..	..	..
..	..	..	..	..
..	..	..	..	..

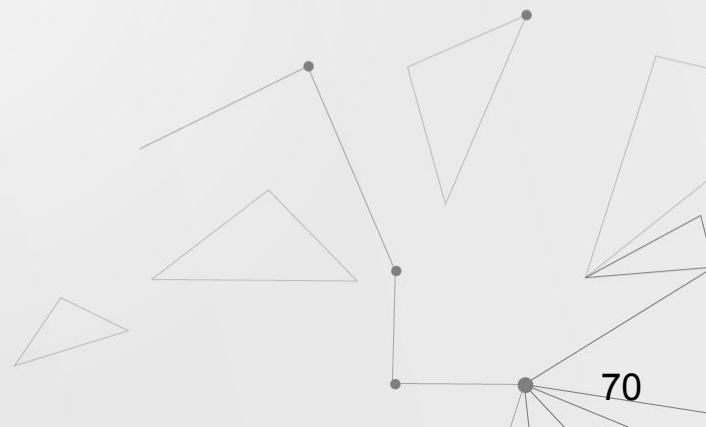
# 03 NetGAN



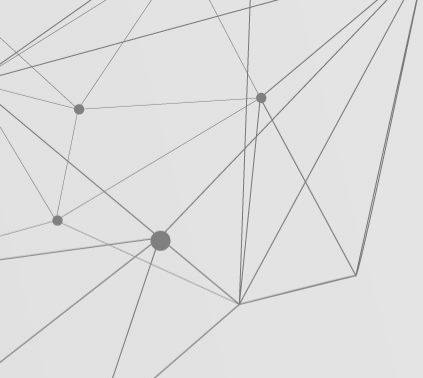
0	10	2	0	1
..	..	..	..	2
..	..	..	..	..
..	..	..	..	..
..	..	..	..	..

## How to build the graph?

1. Symmetrize  $s_{i,j} = s_{j,i} = \max(s_{i,j}, s_{j,i})$



# 03 NetGAN

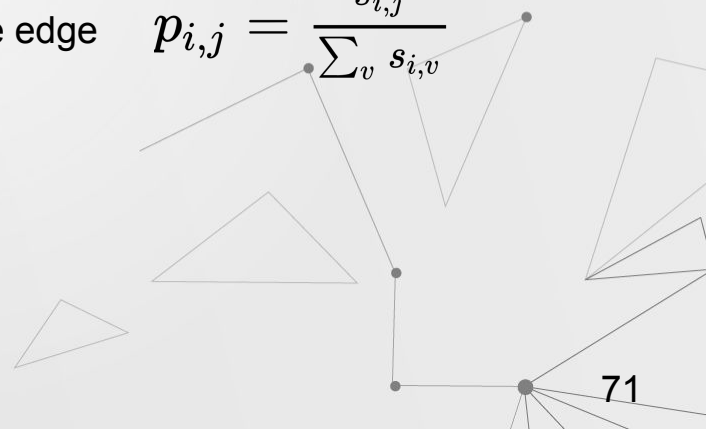


0	10	2	0	1
..	..	..	..	2
..	..	..	..	..
..	..	..	..	..
..	..	..	..	..

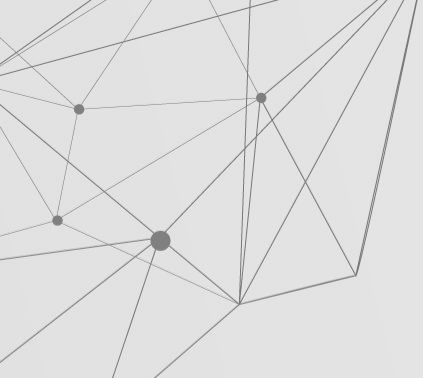
## How to build the graph?

1. Symmetrize  $s_{i,j} = s_{j,i} = \max(s_{i,j}, s_{j,i})$

2. Ensure every node i has at least one edge  $p_{i,j} = \frac{s_{i,j}}{\sum_v s_{i,v}}$



# 03 NetGAN



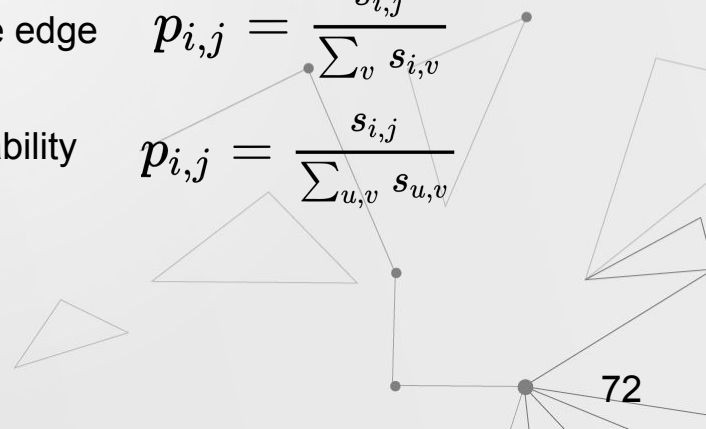
0	10	2	0	1
..	..	..	..	2
..	..	..	..	..
..	..	..	..	..
..	..	..	..	..

## How to build the graph?

1. Symmetrize  $s_{i,j} = s_{j,i} = \max(s_{i,j}, s_{j,i})$

2. Ensure every node  $i$  has at least one edge  $p_{i,j} = \frac{s_{i,j}}{\sum_v s_{i,v}}$

3. Continue sampling edges with probability  $p_{i,j} = \frac{s_{i,j}}{\sum_{u,v} s_{u,v}}$







# 04 NetGAN practice

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Jupyter Notebook

Code:

[https://github.com/mmiller96/netgan\\_pytorch](https://github.com/mmiller96/netgan_pytorch)