SOEN 331: Introduction to Formal Methods for Software Engineering

Assignment 4

Temporal Logic

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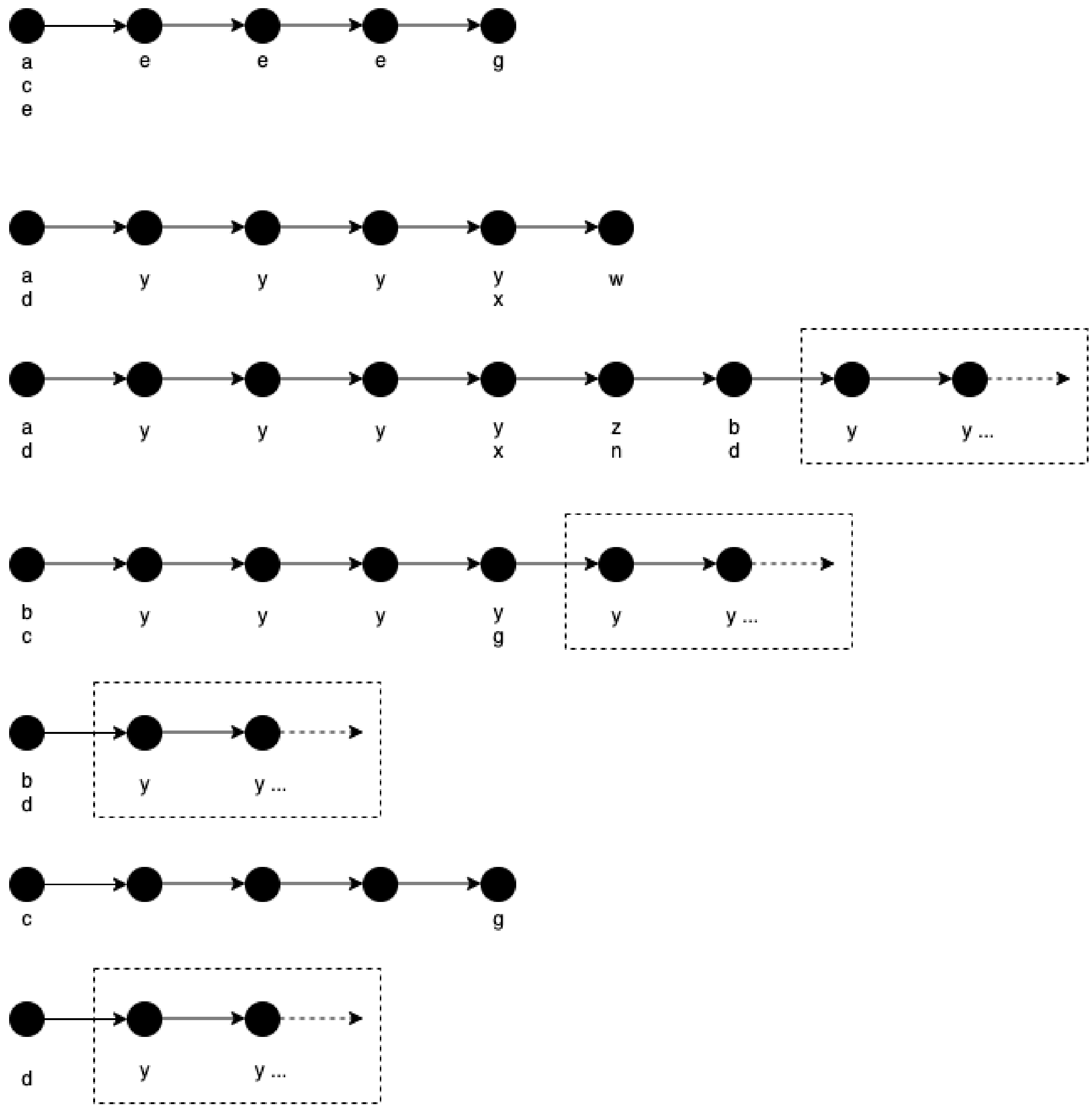
1 Problem 1: Analyzing program behavior

The behavior of a program is expressed by the following temporal formula:

$$\Box \begin{bmatrix} \textbf{start} \rightarrow \neg a \vee \neg b \\ \\ \textbf{start} \rightarrow c \oplus d \\ \\ b \vee d \rightarrow \bigcirc(x \,\mathcal{R}\, y) \\ \\ (a \wedge d \wedge \bigcirc y) \rightarrow \bigcirc^4 x \\ \\ (x \wedge y) \rightarrow \bigcirc(w \oplus z) \\ \\ (a \wedge c) \rightarrow (e \,\mathcal{W}\, g)) \\ \\ c \rightarrow \bigcirc^4 g \\ \\ (x \wedge y \wedge \bigcirc z) \rightarrow \bigcirc n \\ \\ x \wedge \bigcirc(z \wedge n) \rightarrow \bigcirc^2(b \wedge d) \end{bmatrix}$$

1.1 Visualize

Visualize all models of behavior.



1.2 Specify conditions

There are three models whereby the program terminates:

1. $\langle (a \wedge c \wedge e), e, e, e, g \rangle$
2. $\langle (a \wedge d), y, y, y, (y \wedge x), w \rangle$
3. $\langle c, g \rangle$

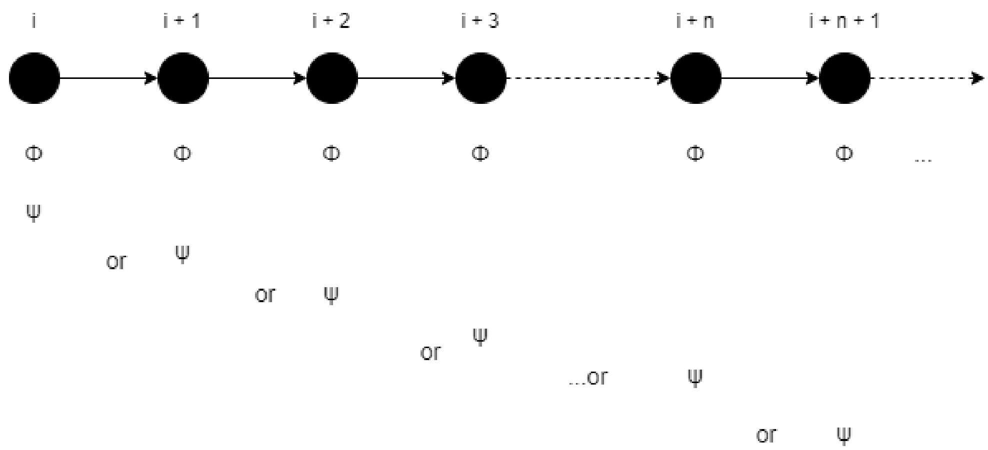
2 Problem 2: Visualizing temporal expressions

Provide a description and a visualization of each of the following expressions:

2.1

$$\Box\phi \rightarrow \diamond\psi$$

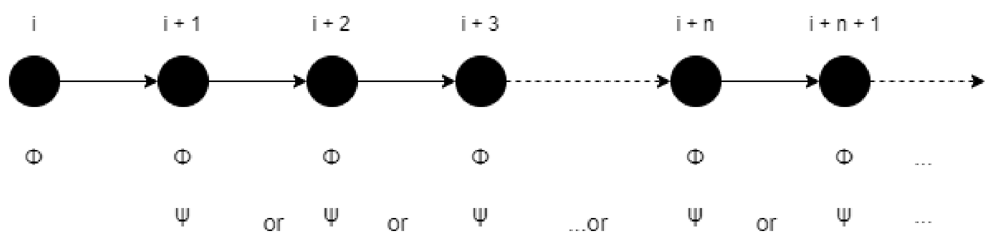
ϕ is always true at all times, while ψ will eventually become true (it can be now or in some future moment).



2.2

$$\Box\phi \rightarrow \bigcirc\Box \diamond \psi$$

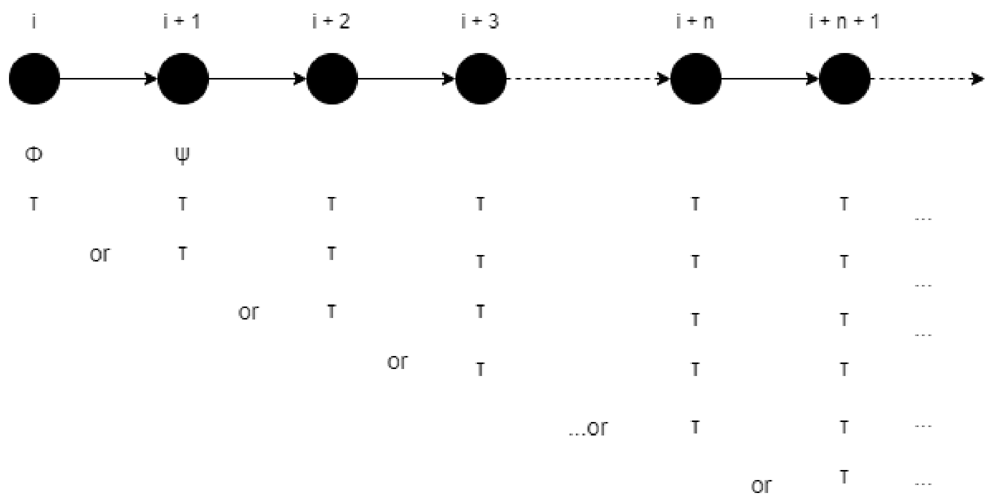
ϕ is always true at all times, while in the next moment of time ψ will always eventually become true (it can be happen next or in some future moment).



2.3

$$(\phi \wedge \bigcirc\psi) \rightarrow \diamond\Box\tau$$

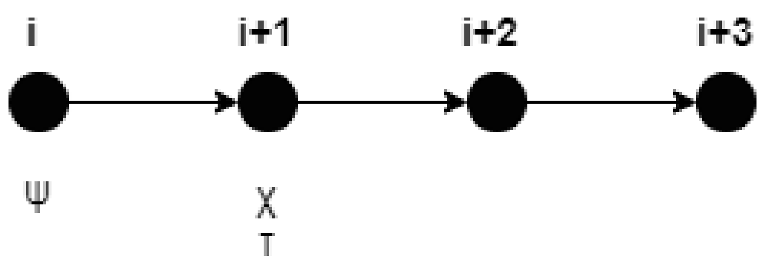
Right after ϕ there is ψ, and τ will eventually always become true (it can be now or in some future moment).



2.4

$$(\psi \wedge \bigcirc\chi) \rightarrow \bigcirc\tau$$

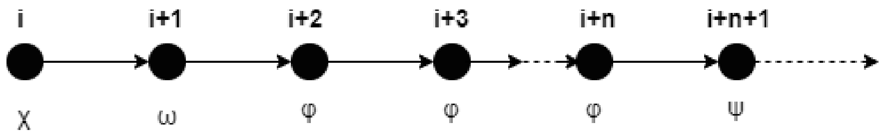
ψ is true at time i and χ will be true at the very next state (i+1), while τ will be true at the next state (i+1).



2.5

$$(\chi \wedge \bigcirc\omega) \rightarrow \bigcirc^2(\phi\mathbf{U}\psi)$$

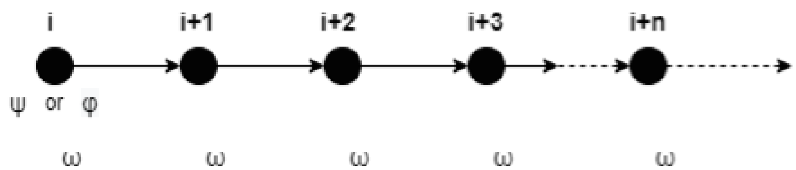
χ is true at time i and ω will be true at the next state (i+1), while in two states (i+2), ϕ will be true until (but not up to) ψ becomes true (at some unknown state in the future).



2.6

$(\phi \oplus \psi) \to \Box\omega$

Either ϕ or ψ are true at time i (but never both), while ω is always true.



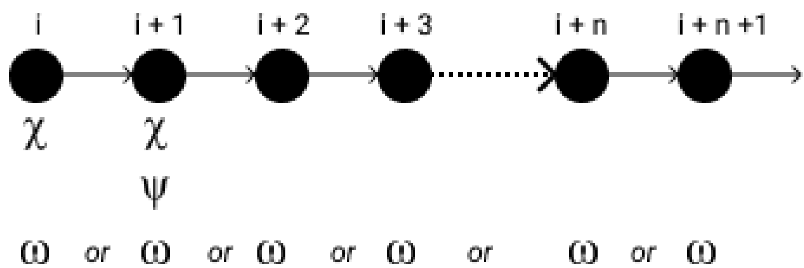
2.7

$\chi \wedge \bigcirc(\chi \wedge \psi) \rightarrow \diamond\omega$

2.7.1 Description

If χ is true at time i and both χ and ϕ are true at the next state in timeline (i+1), then eventually ω will become true.

2.7.2 Visualization



2.8

$$\begin{bmatrix} (\chi \wedge \bigcirc^2\psi) \rightarrow \bigcirc^2(\tau \mathbf{W} \omega) \\ \mu \rightarrow \bigcirc^5\omega \end{bmatrix}$$

2.8.1 Description

- The first statement reads as if at time i, χ is true and in the next 2 states, ψ is true, then from (i+2), τ is true unless ω is true (however, there's no guarantee that ω will be true in the future).
- The second statement reads as if at time i, μ is true then at (i+5), ω must be true

  $\rightarrow$ Combining the 2 statements logically, we have: if χ and μ are true at time i, and ψ is true at time i+2, then τ is true at (i+2), (i+3), (i+4) and ω is true at (i+5)

2.8.2 Visualization