



# PyTorch Tutorial

## 03. Gradient Descent

# Revision

- What would be the best model for the data?
- Linear model?

x (hours)	y (points)
1	2
2	4
3	6
4	?



## Linear Model

$$\hat{y} = x * \omega$$

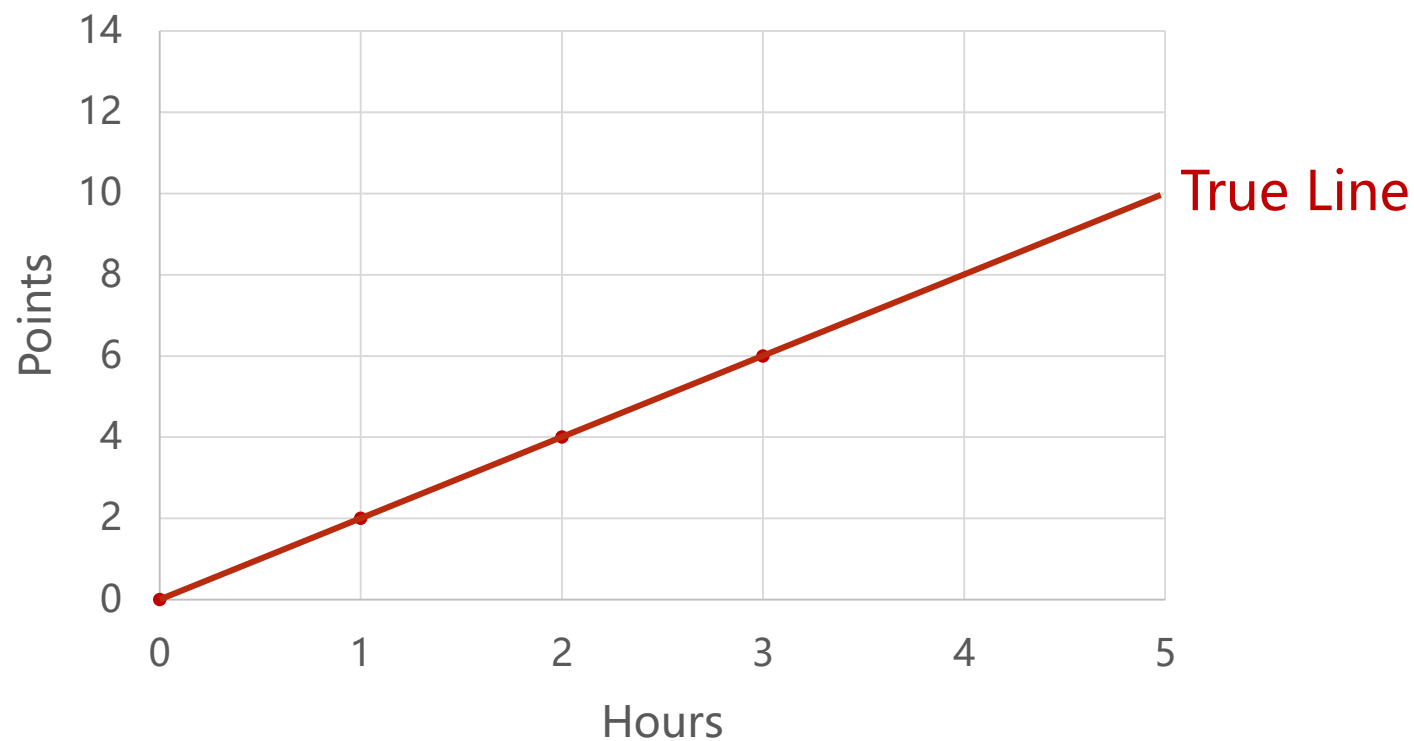
To simplify the model

# Revision

## Linear Model

$$\hat{y} = x * \omega$$

x (hours)	y (points)
1	2
2	4
3	6



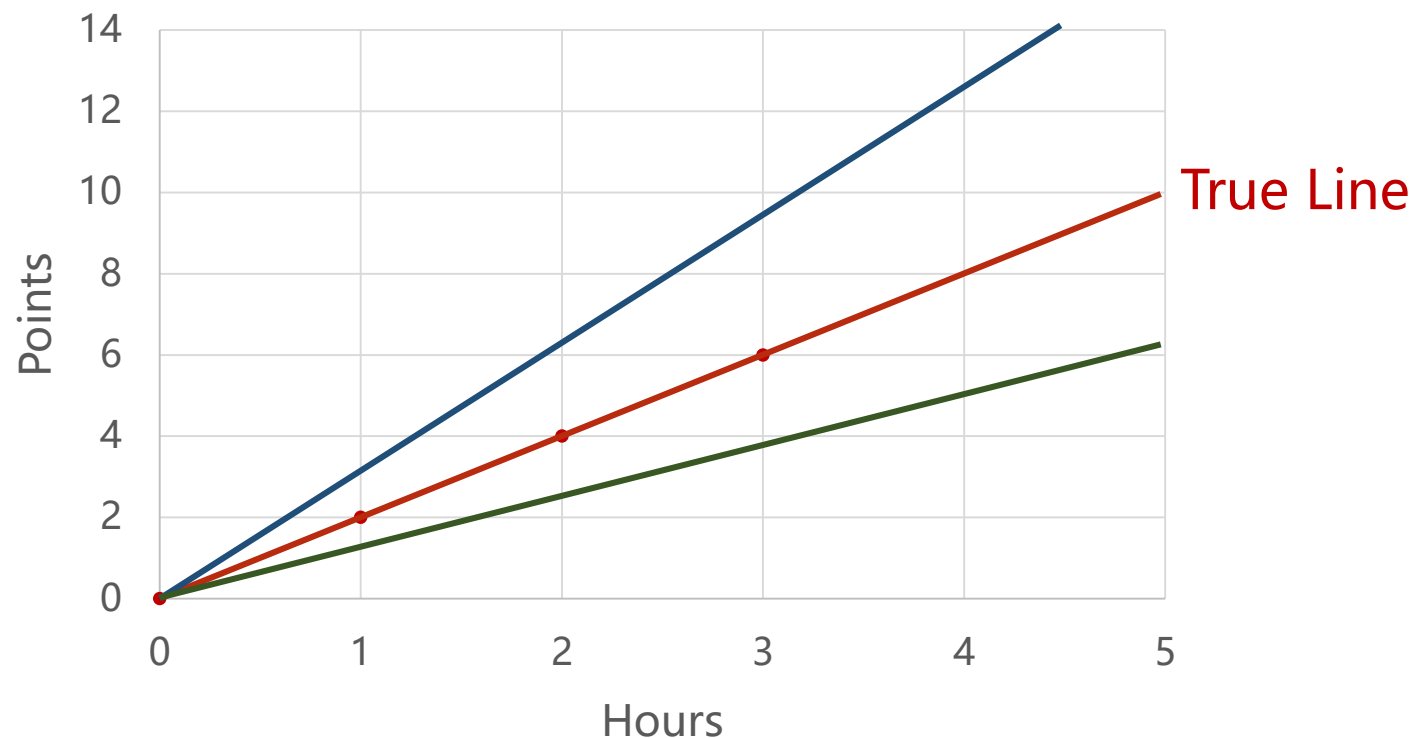
# Revision

## Linear Model

$$\hat{y} = x * \omega$$

x (hours)	y (points)
1	2
2	4
3	6

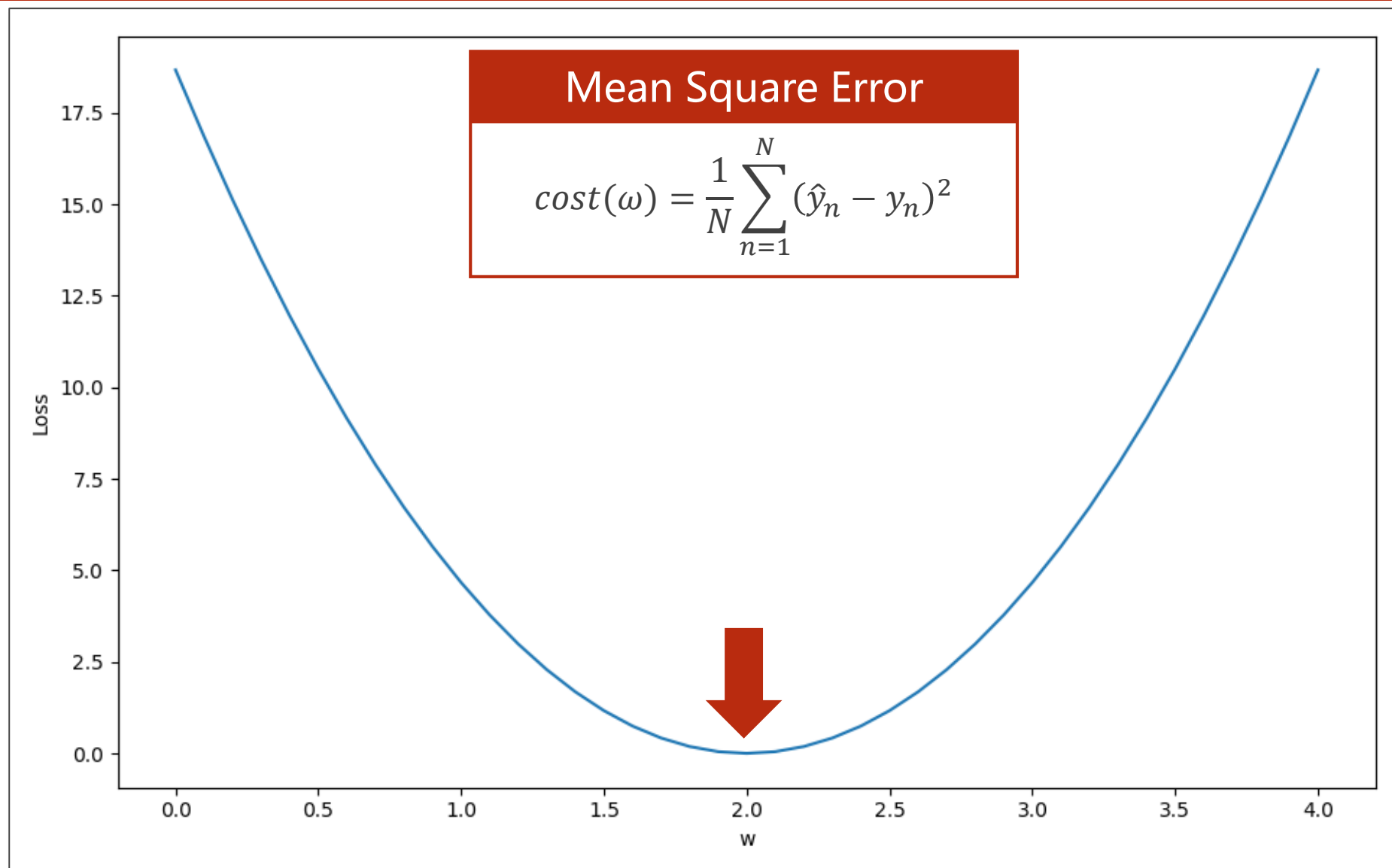
The machine starts with **a random guess**,  $\omega$  = random value



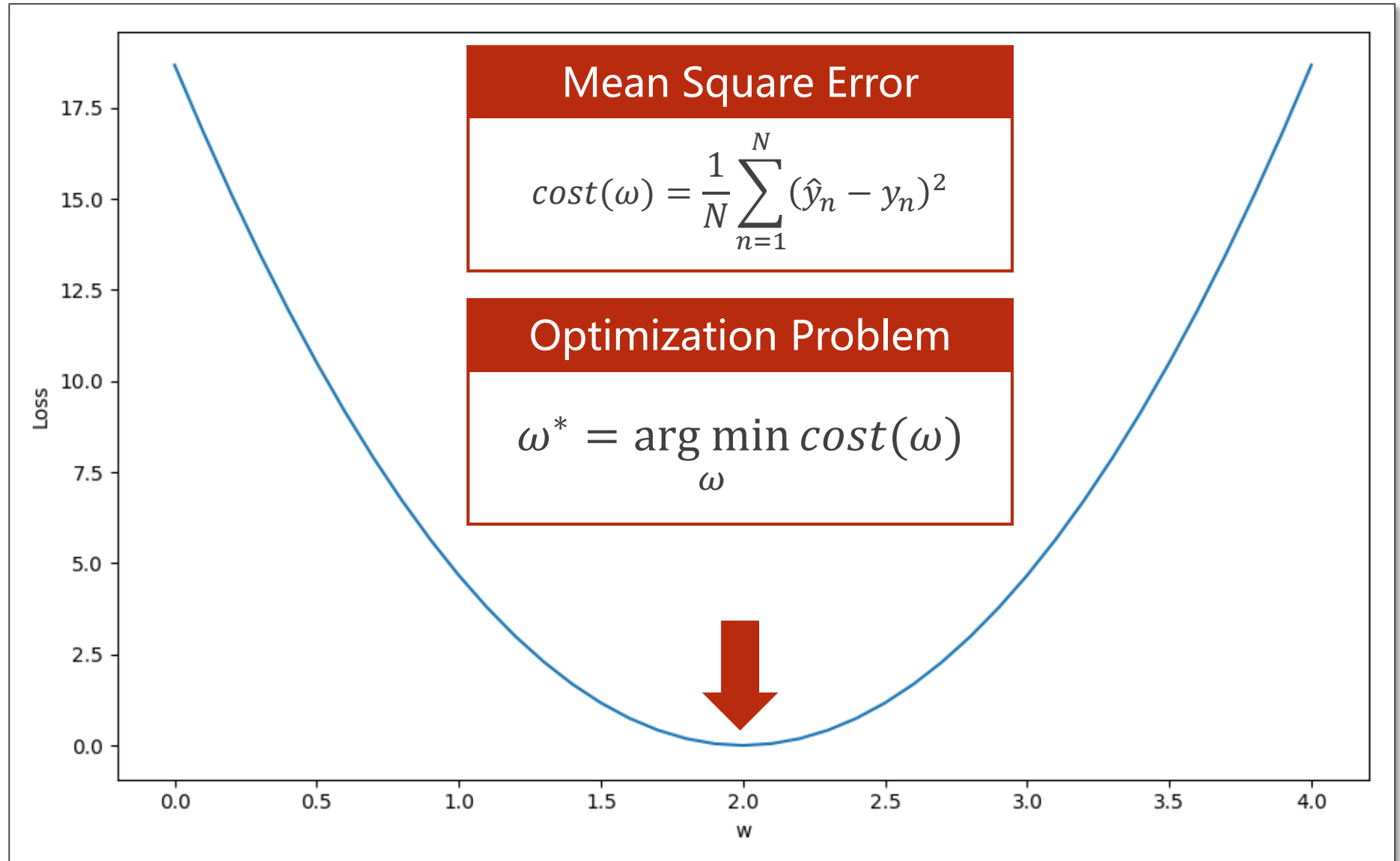
# Revision

It can be found that when  $\omega = 2$ , the cost will be minimal.

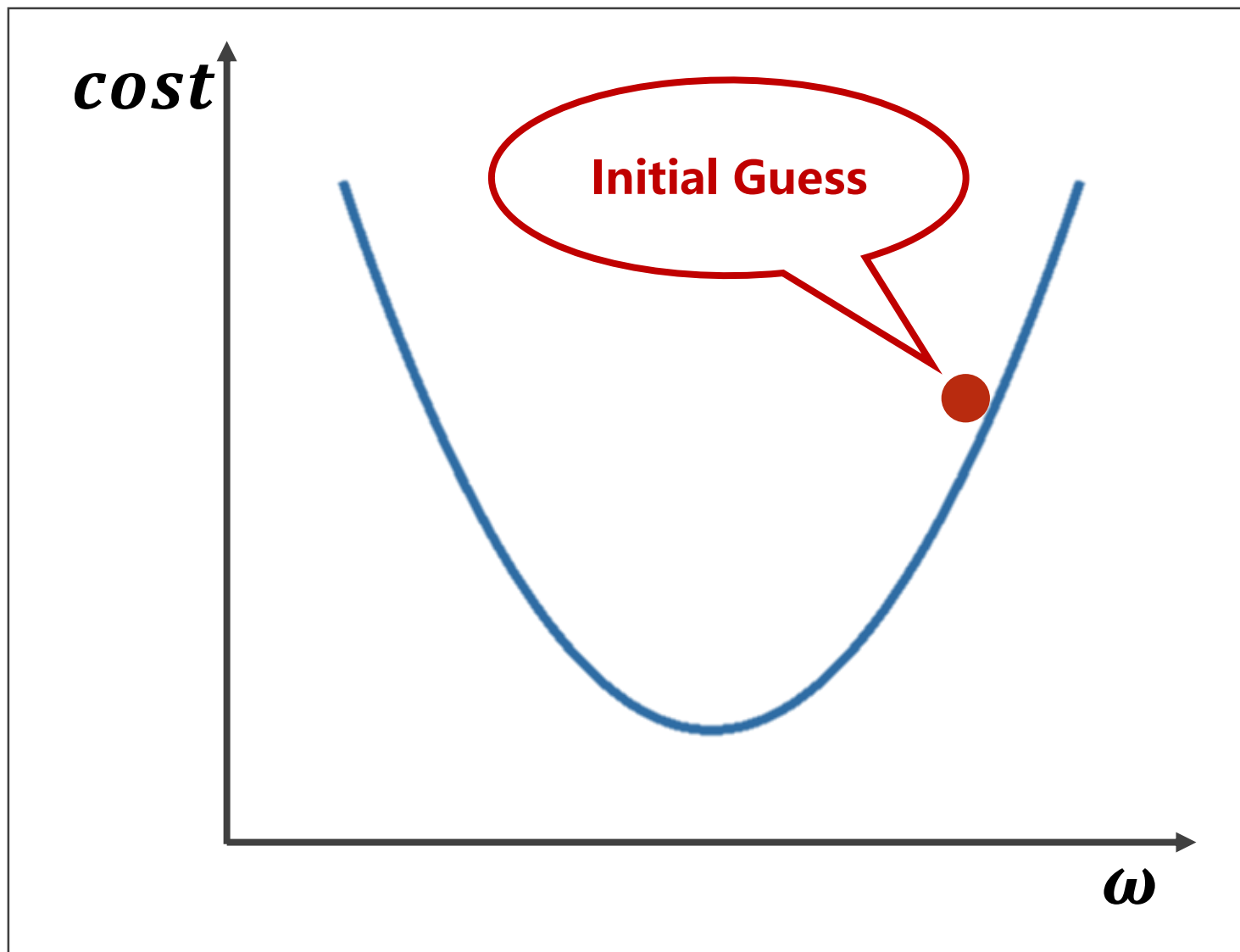
will be minimal.  
MUGU  $\omega = 2$  the cost



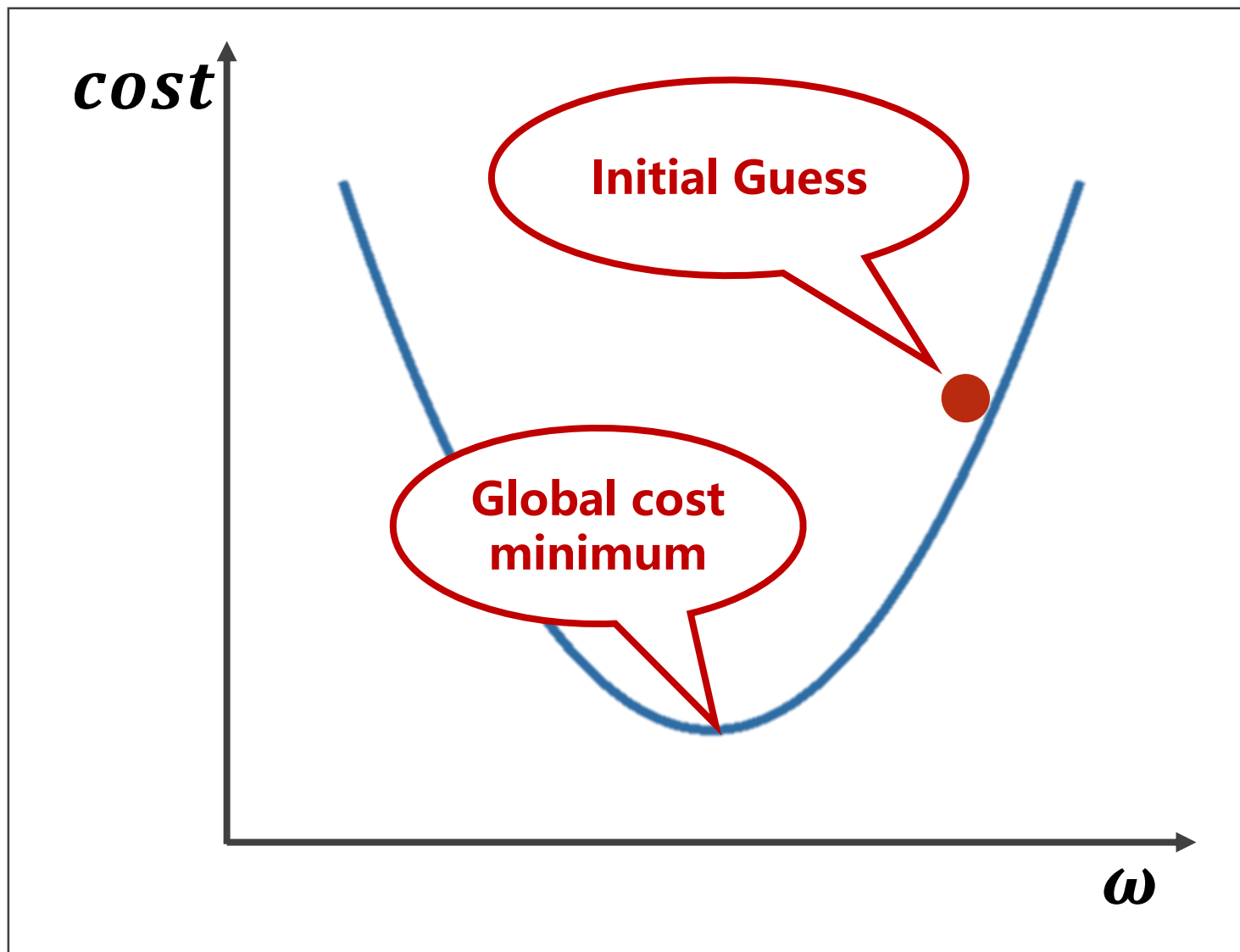
# Optimization Problem



# Gradient Descent Algorithm

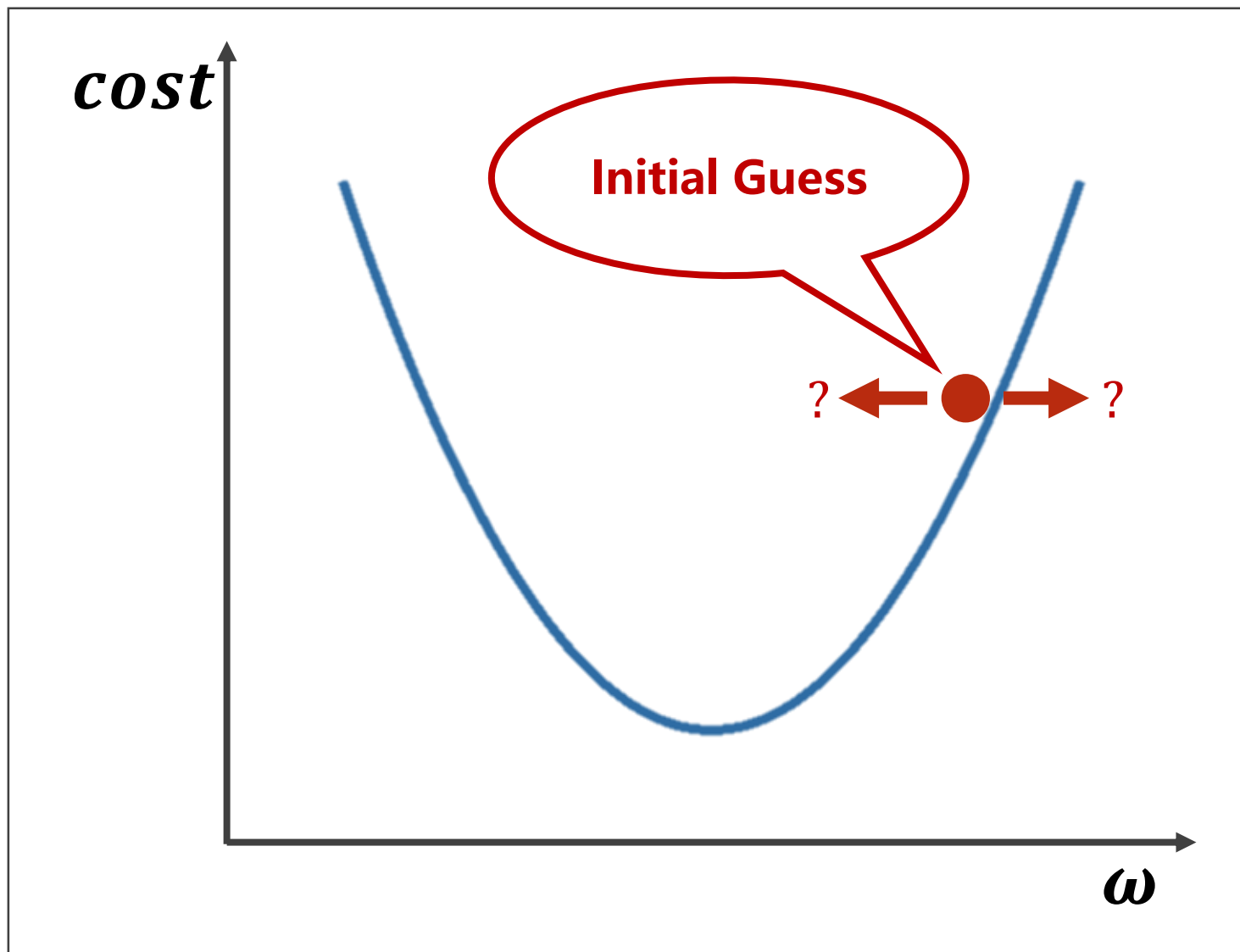


# Gradient Descent Algorithm

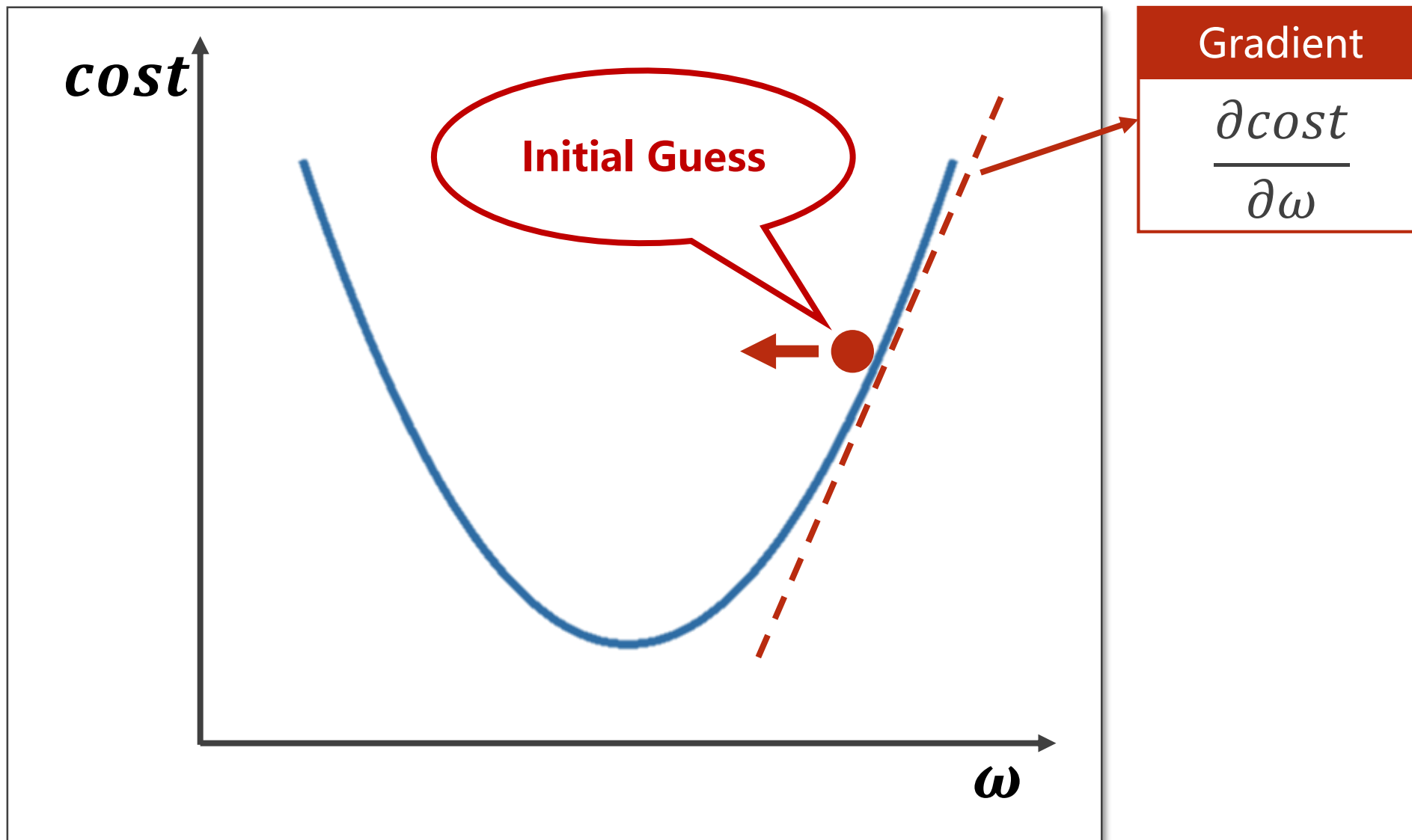




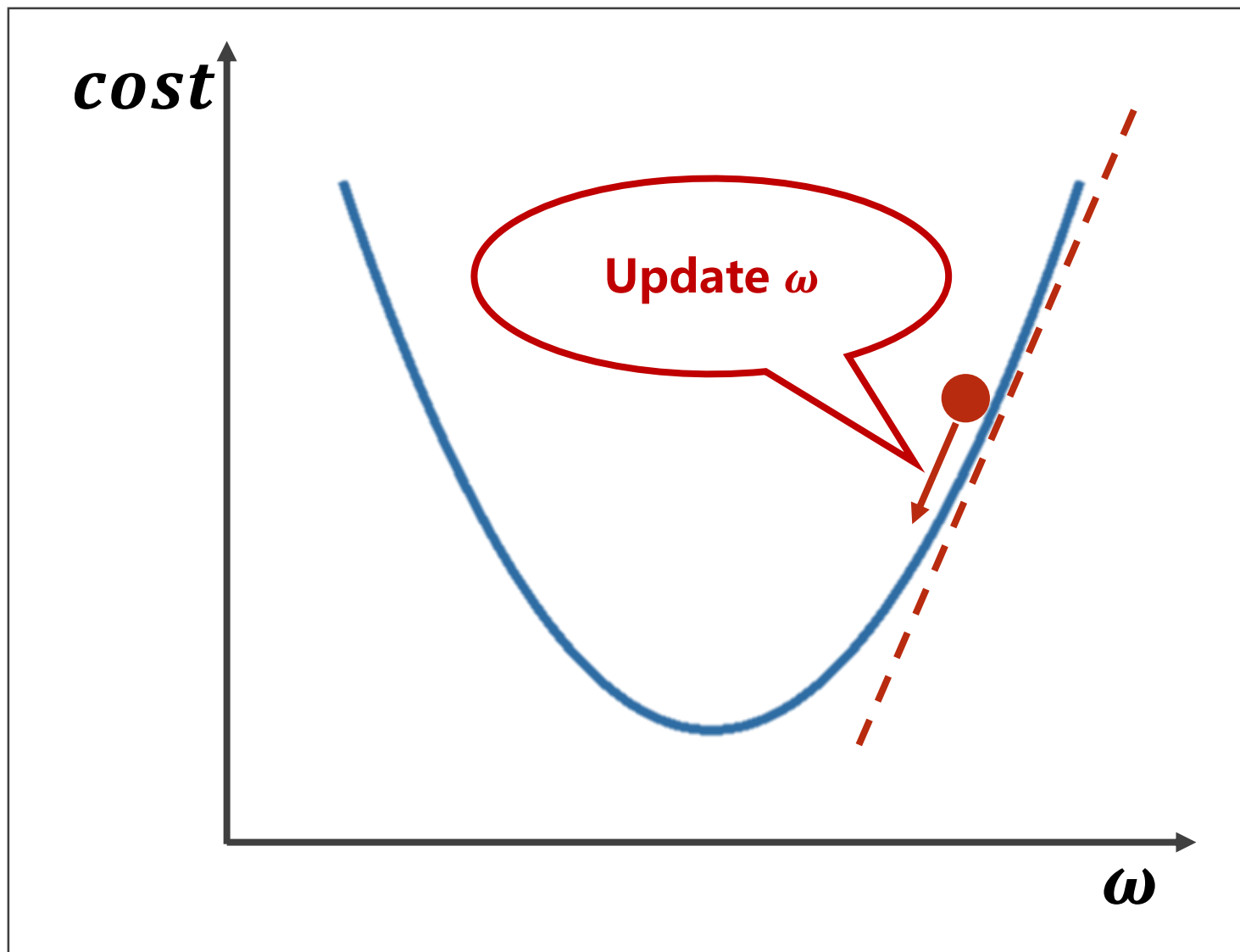
# Gradient Descent Algorithm



# Gradient Descent Algorithm



# Gradient Descent Algorithm



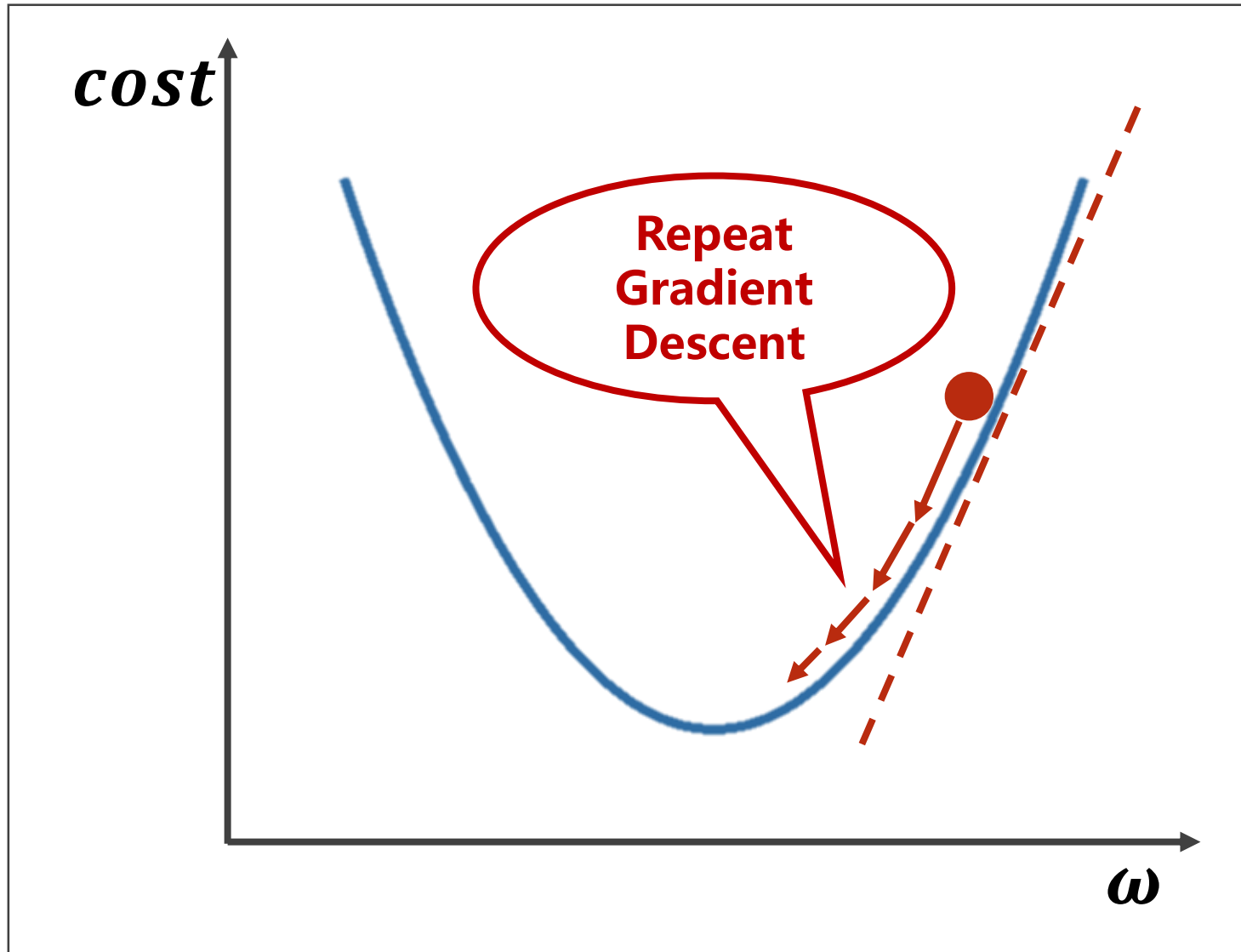
Gradient

$$\frac{\partial cost}{\partial \omega}$$

Update

$$\omega = \omega - \alpha \frac{\partial cost}{\partial \omega}$$

# Gradient Descent Algorithm



Gradient

$$\frac{\partial cost}{\partial \omega}$$

Update

$$\omega = \omega - \alpha \frac{\partial cost}{\partial \omega}$$

# Gradient Descent Algorithm

## Derivative

$$\begin{aligned}\frac{\partial cost(\omega)}{\partial \omega} &= \frac{\partial}{\partial \omega} \frac{1}{N} \sum_{n=1}^N (x_n \cdot \omega - y_n)^2 \\ &= \frac{1}{N} \sum_{n=1}^N \frac{\partial}{\partial \omega} (x_n \cdot \omega - y_n)^2 \\ &= \frac{1}{N} \sum_{n=1}^N 2 \cdot (x_n \cdot \omega - y_n) \frac{\partial (x_n \cdot \omega - y_n)}{\partial \omega} \\ &= \frac{1}{N} \sum_{n=1}^N 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)\end{aligned}$$

## Gradient

$$\frac{\partial cost}{\partial \omega}$$

## Update

$$\omega = \omega - \alpha \frac{\partial cost}{\partial \omega}$$

# Gradient Descent Algorithm

## Derivative

$$\begin{aligned}\frac{\partial cost(\omega)}{\partial \omega} &= \frac{\partial}{\partial \omega} \frac{1}{N} \sum_{n=1}^N (x_n \cdot \omega - y_n)^2 \\ &= \frac{1}{N} \sum_{n=1}^N \frac{\partial}{\partial \omega} (x_n \cdot \omega - y_n)^2 \\ &= \frac{1}{N} \sum_{n=1}^N 2 \cdot (x_n \cdot \omega - y_n) \frac{\partial (x_n \cdot \omega - y_n)}{\partial \omega} \\ &= \frac{1}{N} \sum_{n=1}^N 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)\end{aligned}$$

## Gradient

$$\frac{\partial cost}{\partial \omega}$$

## Update

$$\omega = \omega - \alpha \frac{\partial cost}{\partial \omega}$$

## Update

$$\omega = \omega - \alpha \frac{1}{N} \sum_{n=1}^N 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$

# Implementation

```
x_data = [1.0, 2.0, 3.0]
y_data = [2.0, 4.0, 6.0]
```

```
w = 1.0
```

```
def forward(x):
    return x * w
```

```
def cost(xs, ys):
    cost = 0
    for x, y in zip(xs, ys):
        y_pred = forward(x)
        cost += (y_pred - y) ** 2
    return cost / len(xs)
```

```
def gradient(xs, ys):
    grad = 0
    for x, y in zip(xs, ys):
        grad += 2 * x * (x * w - y)
    return grad / len(xs)
```

```
print('Predict (before training)', 4, forward(4))
for epoch in range(100):
    cost_val = cost(x_data, y_data)
    grad_val = gradient(x_data, y_data)
    w -= 0.01 * grad_val
    print('Epoch:', epoch, 'w=', w, 'loss=', cost_val)
print('Predict (after training)', 4, forward(4))
```

```
x_data = [1.0, 2.0, 3.0]
y_data = [2.0, 4.0, 6.0]
```

Prepare the training set.

# Implementation

```
x_data = [1.0, 2.0, 3.0]
y_data = [2.0, 4.0, 6.0]
```

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```

$w = 1.0$

Initial guess of weight.



# Implementation

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```

```
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def forward(x):
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```

Define the model:

Linear Model

$$\hat{y} = x * \omega$$

# Implementation

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```

## Define the cost function

### Mean Square Error

$$cost(\omega) = \frac{1}{N} \sum_{n=1}^N (\hat{y}_n - y_n)^2$$

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    return grad / len(xs)
```

Define the gradient function

Gradient

$$\frac{\partial cost}{\partial \omega} = \frac{1}{N} \sum_{n=1}^N 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$

# Implementation

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```
for epoch in range(100):
    cost_val = cost(x_data, y_data)
    grad_val = gradient(x_data, y_data)
    w -= 0.01 * grad_val
```

Do the update

Update

$$\omega = \omega - \alpha \frac{\partial cost}{\partial \omega}$$

# Implementation

```
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y_data = [2.0, 4.0, 6.0]

w = 1.0

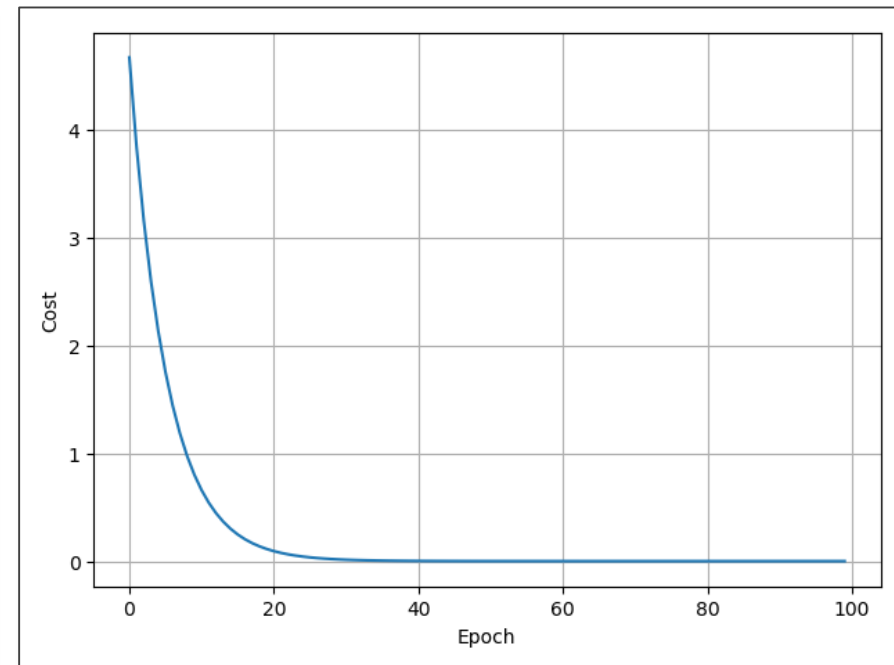
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    return x * w

def cost(xs, ys):
    cost = 0
    for x, y in zip(xs, ys):
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    w -= 0.01 * grad_val
    print('Epoch:', epoch, 'w=', w, 'loss=', cost_val)
print('Predict (after training)', 4, forward(4))
```

```
Predict (before training) 4 4.0
Epoch: 0 w= 1.09 cost= 4.67
Epoch: 1 w= 1.18 cost= 3.84
Epoch: 2 w= 1.25 cost= 3.15
Epoch: 3 w= 1.32 cost= 2.59
Epoch: 4 w= 1.39 cost= 2.13
Epoch: 5 w= 1.44 cost= 1.75
Epoch: 6 w= 1.50 cost= 1.44
Epoch: 7 w= 1.54 cost= 1.18
Epoch: 8 w= 1.59 cost= 0.97
Epoch: 9 w= 1.62 cost= 0.80
Epoch: 10 w= 1.66 cost= 0.66
.....
Epoch: 90 w= 2.00 cost= 0.00
Epoch: 91 w= 2.00 cost= 0.00
Epoch: 92 w= 2.00 cost= 0.00
Epoch: 93 w= 2.00 cost= 0.00
Epoch: 94 w= 2.00 cost= 0.00
Epoch: 95 w= 2.00 cost= 0.00
Epoch: 96 w= 2.00 cost= 0.00
Epoch: 97 w= 2.00 cost= 0.00
Epoch: 98 w= 2.00 cost= 0.00
Epoch: 99 w= 2.00 cost= 0.00
Predict (after training) 4 8.00
```



Cost in each epoch

# Stochastic Gradient Descent

## Gradient Descent

$$\omega = \omega - \alpha \frac{\partial cost}{\partial \omega}$$



## Stochastic Gradient Descent

$$\omega = \omega - \alpha \frac{\partial loss}{\partial \omega}$$

## Derivative of Cost Function

$$\frac{\partial cost}{\partial \omega} = \frac{1}{N} \sum_{n=1}^N 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$



## Derivative of Loss Function

$$\frac{\partial loss_n}{\partial \omega} = 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$

# Implementation of SGD

```
x_data = [1.0, 2.0, 3.0]
y_data = [2.0, 4.0, 6.0]

w = 1.0

def forward(x):
    return x * w

def loss(x, y):
    y_pred = forward(x)
    return (y_pred - y) ** 2

def gradient(x, y):
    return 2 * x * (x * w - y)

print('Predict (before training)', 4, forward(4))

for epoch in range(100):
    for x, y in zip(x_data, y_data):
        grad = gradient(x, y)
        w = w - 0.01 * grad
        print("\tgrad: ", x, y, grad)
        l = loss(x, y)

    print("progress:", epoch, "w=", w, "loss=", l)

print('Predict (after training)', 4, forward(4))
```

```
def loss(x, y):
    y_pred = forward(x)
    return (y_pred - y) ** 2
```

Calculate loss function:

Loss Function

$$loss = (\hat{y} - y)^2 = (x * \omega - y)^2$$

# Implementation of SGD

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def gradient(x, y):
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Calculate loss function:

Derivative of Loss Function

$$\frac{\partial loss_n}{\partial \omega} = 2 \cdot x_n \cdot (x_n \cdot \omega - y_n)$$



# Implementation of SGD

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        print("\tgrad: ", x, y, grad)
        l = loss(x, y)
```

Update weight by every grad of sample of train set.



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