Open Data for Open Cities: Re-use and discovery level applied to the spatial point analysis process on linear networks
point pattern session

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Outline

Traffic accidents in an area of Medellin, Colombia

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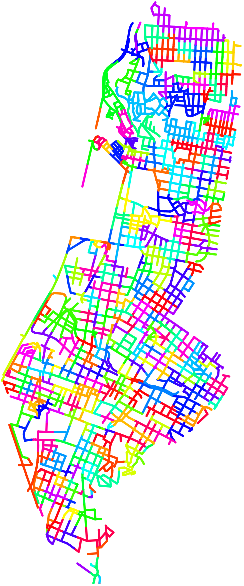

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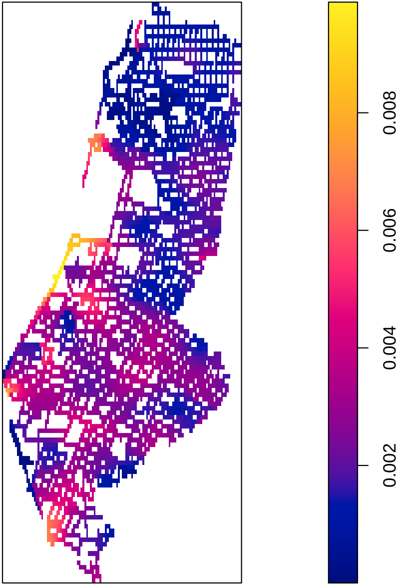

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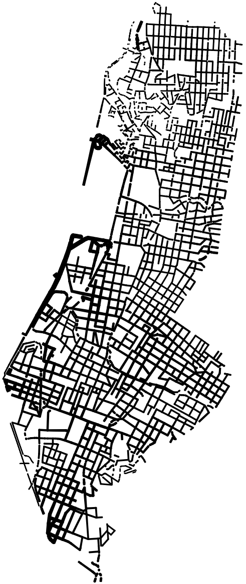

Mathematical structure of linear networks

A line segment with endpoints $u \in \mathbb{R}^2$ and $v \in \mathbb{R}^2 \setminus \{u\}$ is given by $l = [u, v] = \{tu + (1-t)v : 0 \leq t \leq 1\} \subseteq \mathbb{R}^2$, $u \neq v$ and a *linear network* L is defined as the union

$$L = \bigcup_{i=1}^{k} l_i$$

of $k \geq 1$ line segments $l_i = [u_i, v_i]$, the *components* of L, which we assume satisfy $l_i \nsubseteq l_j$, $i \neq j$, and boundedness, i.e. $|l_i| = |u_i - v_i| < \infty$.

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of $k \geq 1$ line segments $l_i = [u_i, v_i]$, the *components* of L, which we assume satisfy $l_i \not\subseteq l_j$, $i \neq j$, and boundedness, i.e. $|l_i| = |u_i - v_i| < \infty$.

The *total network length* is given by the sum of the component lengths, i.e.

$$|L| = \sum_{i=1}^{k} |l_i|,$$

whereby $|L| < \infty$ if $k < \infty$. On the other hand, if $k = \infty$, since each component has positive length, we have that $|L| = \infty$.

Mathematical structure of linear networks

A *path* from a starting point $u \in L$ to some end point $v \in L$ is a sequence of points $u_0, \ldots, u_n \in L$, $n > 1$, such that $u_0 = u$, $u_n = v$ and $[u_i, u_{i+1}] \subseteq L$ for all $i = 0, \ldots, n-1$.
Denoting by $p_{u,v}$ the collection of all paths between $u \in L$ and $v \in L$, the shortest-path (geodesic) distance between u and v is given by

$$\begin{aligned} d_L(u,v) &= \min_{(u_0,\ldots,u_n)\in p_{u,v}} \sum_{i=0}^{n-1} |u_i - u_{i+1}| \\ &= \min_{(u_0,\ldots,u_n)\in p_{u,v}} |u - u_1| + \sum_{i=1}^{n-2} |u_{i+1} - u_i| + |v - u_{n-1}|. \end{aligned}$$

Network example

Example

The first very simple example is any $L = \bigcup_{i=1}^{\infty} l_i$, $|L| = \infty$, *where each node has degree 2. Since the 1-dimensional Euclidean space* $\mathbb{R}$ *may be expressed as a countable union of compacts, we note that* L *is isometric to* $\mathbb{R}$ *and may be viewed as a bending of the real line at* $k = \infty$ *places. For the particular case where* $L = \mathbb{R}$ *it additionally follows that* $d_L(\cdot, \cdot)$ *coincides with the Euclidean metric.*

Example

The second example is the graph $L = \mathbb{Z}^2$, *which has its nodes at all integer pairs* $(i, j) \in \mathbb{R}^2$, $i, j \in \mathbb{Z}$. *The edges* $\mathcal{E}_L$ *are formed by the line segments joining the nodes horizontally and vertically. Note that we may scale everything by an arbitrary positive constant* $\alpha > 0$ *so that all edges have length* α.

Linear network point processes

Assume X is a point process on linear network L with intensity measure μ,

$$\mu(A) = E(N(A)) = \int_A \rho(u) \mathrm{d}_1 u, \qquad A \subseteq L,$$

where $N(A)$ is the number of points belong to subnetwork A, we then say X has *intensity function* $\rho(u)$, $u \in L$.

Linear network point processes

Example

A Poisson process X on L with intensity function $\rho(\cdot)$ is a LNPP such that $X(A_1), \ldots, X(A_n)$ are mutually independent for any disjoint line segments $A_1, \ldots, A_n \subseteq L$, $n \geq 1$, and $X(A)$ is Poisson distributed with mean $\mu(A) = \int_A \rho(u) \mathrm{d}_1 u$. Conditioning on $X(L) = n$, we obtain a classical random sample with density $f(u) = \rho(u)/\mu(L) = \rho(u)/n$, a so-called Binomial point process (Møller and Waagepetersen; 2004).

Voronoi tessellations/estimator

In the context of a linear network L, the *Voronoi cell/subnetwork* of $x \in X$ is given by (Okabe and Sugihara; 2012)

$$\mathcal{V}_x = \mathcal{V}_x(X) = \{u \in L : d_L(x,u) \leq d_L(y,u) \text{ for all } y \in X \setminus \{x\}\}.$$

Definition

Given a LNPP X on a linear (sub)network L with intensity function $\rho(u)$, $u \in L$, the *Voronoi intensity estimator* is given by

$$\hat{\rho}^V(u) = \hat{\rho}^V(u; X, L) = \sum_{x \in X \cap L} \frac{\mathbf{1}\{u \in \mathcal{V}_x(X) \cap L\}}{|\mathcal{V}_x(X) \cap L|}, \qquad u \in L,$$

if $X \cap L \neq \emptyset$; set $\hat{\rho}^V(u) = 0$ otherwise.

Kernel smoothing

The Kernel-based intensity estimator (Diggle corrected) of a point pattern on a linear network L is as

$$\hat{\lambda}_{\epsilon}^{(\mathrm{D})}(\mathbf{u}) = \sum_{i=1}^{n} \frac{\kappa_{\epsilon}(d_L(\mathbf{u}, \mathbf{v}_i))}{C_{L,\epsilon}(\mathbf{v}_i)}, \qquad \mathbf{u} \in L, \tag{1}$$

where κ_ϵ is a one-dimensional kernel function with bandwidth ϵ, and

$$C_{L,\epsilon}(\mathbf{v}) = \int_L \kappa_{\epsilon}(d_L(\mathbf{v}, \mathbf{u}))\mathrm{d}_1 u, \qquad \mathbf{v} \in X \cap L, \tag{2}$$

is an edge-correction factor (Moradi et al.; 2016).

Kernel smoothing

The diffusion estimator of intensity function of a point pattern on a linear network L with bandwidth $\epsilon = \sqrt{t}$ is as

$$\hat{\lambda}_\epsilon(\mathbf{u}) = \sum_{i=1}^{n} \kappa_{\epsilon^2}(\mathbf{u}|x_i), \qquad \mathbf{u} \in L \tag{3}$$

where κ_t is the heat kernel (McSwiggan et al.; 2016).

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$$\sum_{i=1}^{n} log(\hat{\lambda}_{-i,\epsilon}(u_i)) - \int_L \hat{\lambda}_{\epsilon}(u_i)\mathrm{d}_1 u \tag{4}$$

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- use the smoothing bandwidth parameter ϵ which minimizes (Cronie and van Lieshout (2016))

$$\left| \sum_{i=1}^{n} \frac{1}{\hat{\lambda}_{\epsilon}^{(\mathrm{D})}(u_i)} - |L| \right|.$$

Acknowledgements

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Reference

Cronie, O. and van Lieshout, M. (2016). Bandwidth selection for kernel estimators of the spatial intensity function, *arXiv preprint arXiv:1611.10221* .

Loader, C. (1999). *Local Regression and Likelihood*, Springer, New York.

McSwiggan, G., Baddeley, A. and Nair, G. (2016). Kernel density estimation on a linear network, *Scandinavian Journal of Statistics* . doi: 10.1111/sjos.12255.

Møller, J. and Waagepetersen, R. P. (2004). *Statistical inference and simulation for spatial point processes*, CRC Press.

Moradi, M. M., Rodriguez-Cortes, F. and Mateu, J. (2016). On the intensity estimator of spatial point patterns on linear networks, *submitted* .

Okabe, A. and Sugihara, K. (2012). *Spatial analysis along networks: statistical and computational methods*, John Wiley & Sons.