Question 5:

(a) To prove: $P(A/B,C) = \frac{P(B|A,C)\,P(A|C)}{P(B|C)}$

we know that,

$$P(X|Y) = \frac{P(XY)}{P(Y)}$$

$$\Rightarrow \frac{P(B/A,C)\ P(A|C)}{P(B/C)}$$

$$= \frac{P(B,A,C)}{\cancel{P(A,C)}} \cdot \frac{\cancel{P(A,C)}}{\cancel{P(C)}} \cdot \frac{\cancel{P(C)}}{P(B,C)}$$

$$= \frac{P(B,A,C)}{P(B,C)} \qquad (\because P(B,A,C) = P(A,B,C))$$

$$= \frac{P(A,B,C)}{P(B,C)}$$

$= \underline{\underline{LHS}}$ Proved.

(b)

The probability of the coin being fair $\frac{F}{1+F}$.

$\therefore$ probability of the coin being double headed $\frac{1}{1+F}$.

hence, the mixture coefficient is

$$\pi = \left\{ \frac{F}{1+F}, \frac{1}{1+F} \right\} \text{ with prob of head } \{0.5, 1\}.$$

Suppose, the coin is tossed n times and for the coin to have a more chance to be ~~fair~~ double head.

$$\frac{1}{1+F}(1)^n > \left(\frac{1}{2}\right)^n \left(\frac{F}{1+F}\right)$$

$$2^n \cdot \frac{1}{\cancel{(1+F)}} > \frac{F}{\cancel{(1+F)}}$$

$$\therefore \quad 2^n > F$$

i.e. $n \log 2 > \log F$

i.e. $n > \dfrac{\log F}{\log 2}$.