

Chapter 15

System Reliability Concepts and Methods

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System Reliability Concepts and Methods

Chapter 15 Objectives

- Explain some important system reliability concepts like system structure; redundancy; nonrepairable and repairable systems; maintainability and availability.
- Describe some basic concepts of system reliability modeling.
- Give expressions for the distribution of system failure time as a function of individual component failure time distributions.
- Illustrate the analysis of failure-time data with two failure modes.
- Provide examples of the use of component test data to estimate system reliability.

Introduction

- **System:** a collection of components needed to realize a given task.
- **System structure:** a logic diagram illustrating the function of the system.

System Structures and System Failure Probability

System failure probability, $F_T(t; \theta)$: probability that a system fails before t .

The failure probability of the system is function of:

- Time in operation (or other measure of use)
- System structure.
- Reliability of system components, interconnections, and interfaces (including, for example, human operators).
- Environmental conditions.

Time Dependency of System Reliability

For the time to failure of a new system (all components starting a time 0)

- The cdf for component i is $F_i = F_i(t; \theta_i)$. The corresponding survival probability is $S_i = S_i(t; \theta_i) = 1 - F_i(t; \theta_i)$. The θ_i s may have some elements in common. Here θ denotes the unique elements in $(\theta_1, \dots, \theta_s)$.
- The cdf for the system is denoted by $F_T = F_T(t; \theta)$. This cdf is determined by the F_i 's and the system structure. Then

$$F_T(t; \theta) = g[F_1(t; \theta_1), \dots, F_s(t; \theta_s)]$$

or one of the simpler forms

$$F_T(\theta) = g[F_1(\theta_1), \dots, F_s(\theta_s)]$$

$$F_T = g(F_1, \dots, F_s).$$

To simplify the presentation, time-(and parameter)-dependency will usually be suppressed in this chapter.

A System with Components in Series



Examples of Systems with Components in Series

- Chain
- High-voltage multi-cell battery
- Inexpensive computer system
- Modern decorative tree lights

A System with Components in Series

A **series** structure with s components works iff all the components work. Then

- For two independent components

$$\begin{aligned}F_T(t) &= \Pr(T \leq t) = 1 - \Pr(T > t) \\ &= 1 - \Pr(T_1 > t \cap T_2 > t) \\ &= 1 - \Pr(T_1 > t) \Pr(T_2 > t) \\ &= 1 - (1 - F_1)(1 - F_2)\end{aligned}$$

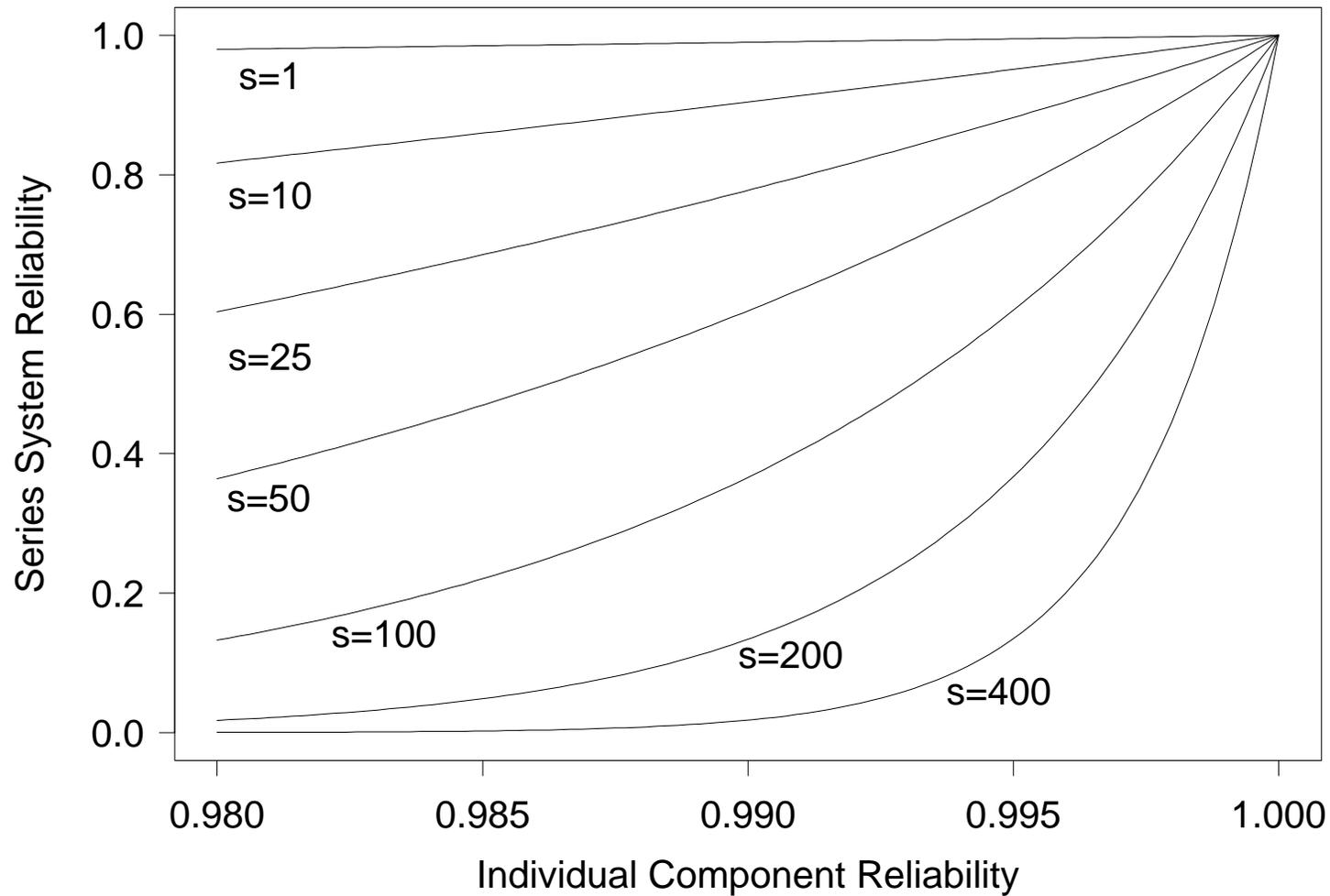
- For s independent components

$$F_T(t) = 1 - \prod_{i=1}^s (1 - F_i)$$

- For s iid components ($F = F_i, i = 1, \dots, s$)

$$F_T(t) = 1 - (1 - F)^s.$$

Reliability of a System with s Identical Independent Components in Series



Effect of Positive Dependency in a Two-Component Series System

- For a series system with two components and dependent failure times

$$F_T(t) = \Pr(T \leq t) = 1 - \Pr(T > t) = 1 - \Pr(T_1 > t \cap T_2 > t).$$

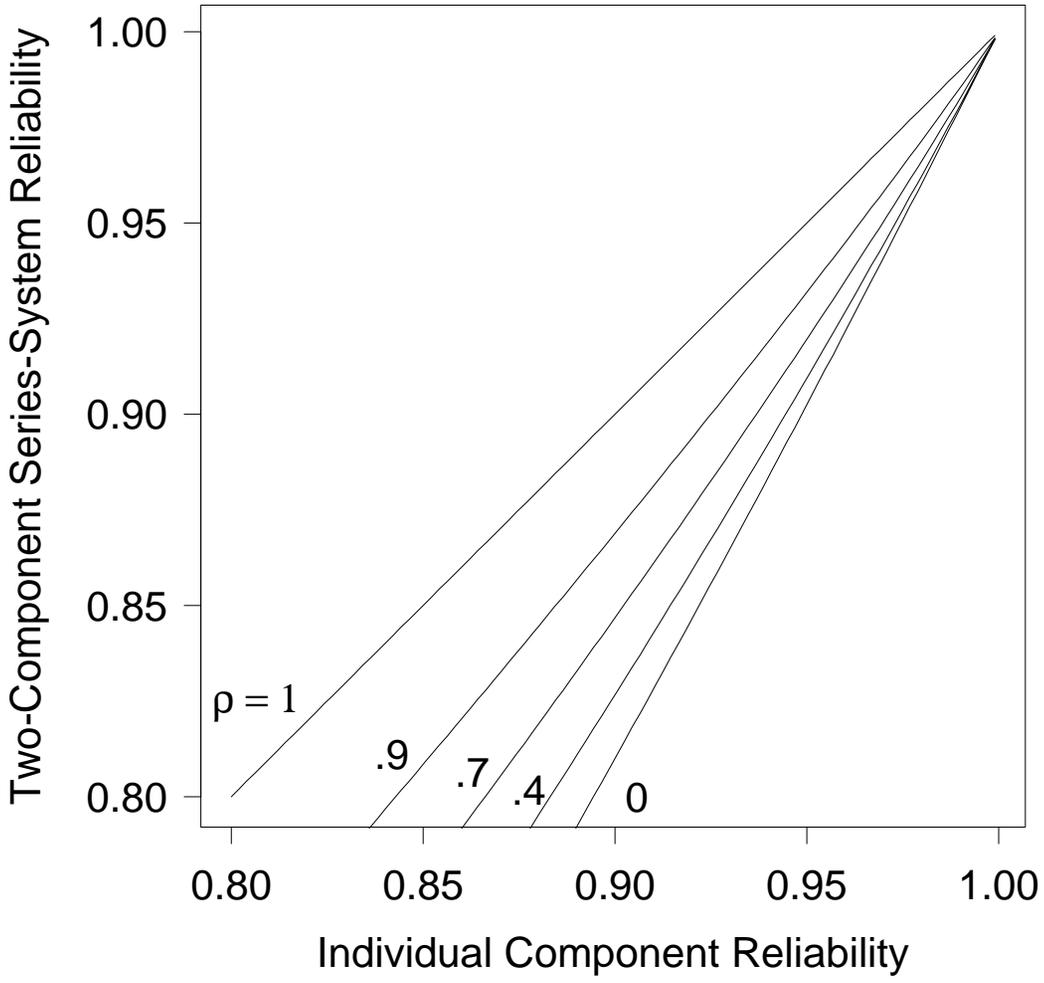
In this case, the evaluation has to be done with respect to the bivariate distribution of T_1 and T_2 .

- If the correlation between the two components is positive, then the assumption of independence is conservative in the sense that the actual $F_T(t)$ is smaller than that predicted by the independent-component model.
- These results extend to the s components in series, the system $F_T(t)$ would have to be computed with respect to the underlying s -variate distribution. Such computations are, in general, difficult.

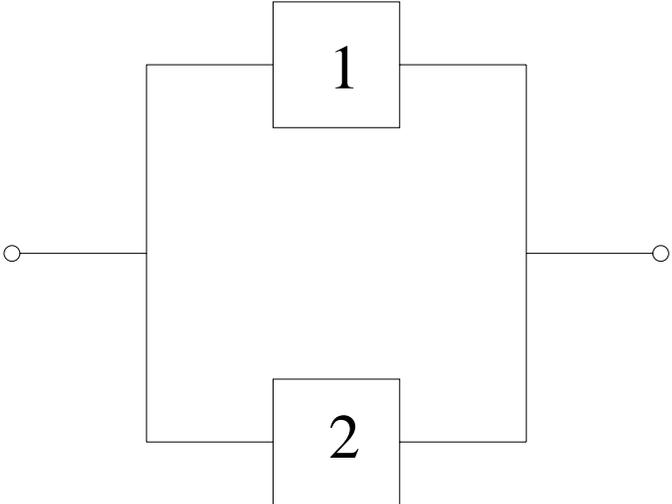
Effect of Positive Dependency in a Two-Component Series System with Lognormal Failure Times

- The distributions of log failure times for the individual components is bivariate normal with the same (arbitrary) mean and standard deviation for both components and correlation ρ .
- The reliability $1 - F_T(t)$ of the system can be expressed as a function of the individual reliability components $1 - F(t)$ and ρ .
- When $\rho = 1$ (so the two components are perfectly dependent and will fail at exactly the same time), the system reliability $1 - F_T(t)$ is the same as the reliability for a single component.
- When $\rho = 0$ (so the two components are independent), $1 - F_T(t)$ corresponds to the system reliability for a $s = 2$ independent series system.

Reliability of a System with 2 Dependent Components in Series



A System with Components in Parallel



Examples of Systems with Components in Parallel

- Automobile headlights
- RAID computer disk array systems
- Stairwells with emergency lighting
- Overhead projectors with backup bulb switch
- Multiple light banks in classroom

A System with Components in Parallel

A **parallel** structure with s components works if at least one of the components works. Then

- For two independent components

$$\begin{aligned}F_T(t) &= \Pr(T \leq t) \\ &= \Pr(T_1 \leq t \cap T_2 \leq t) \\ &= \Pr(T_1 \leq t) \Pr(T_2 \leq t) \\ &= F_1 F_2\end{aligned}$$

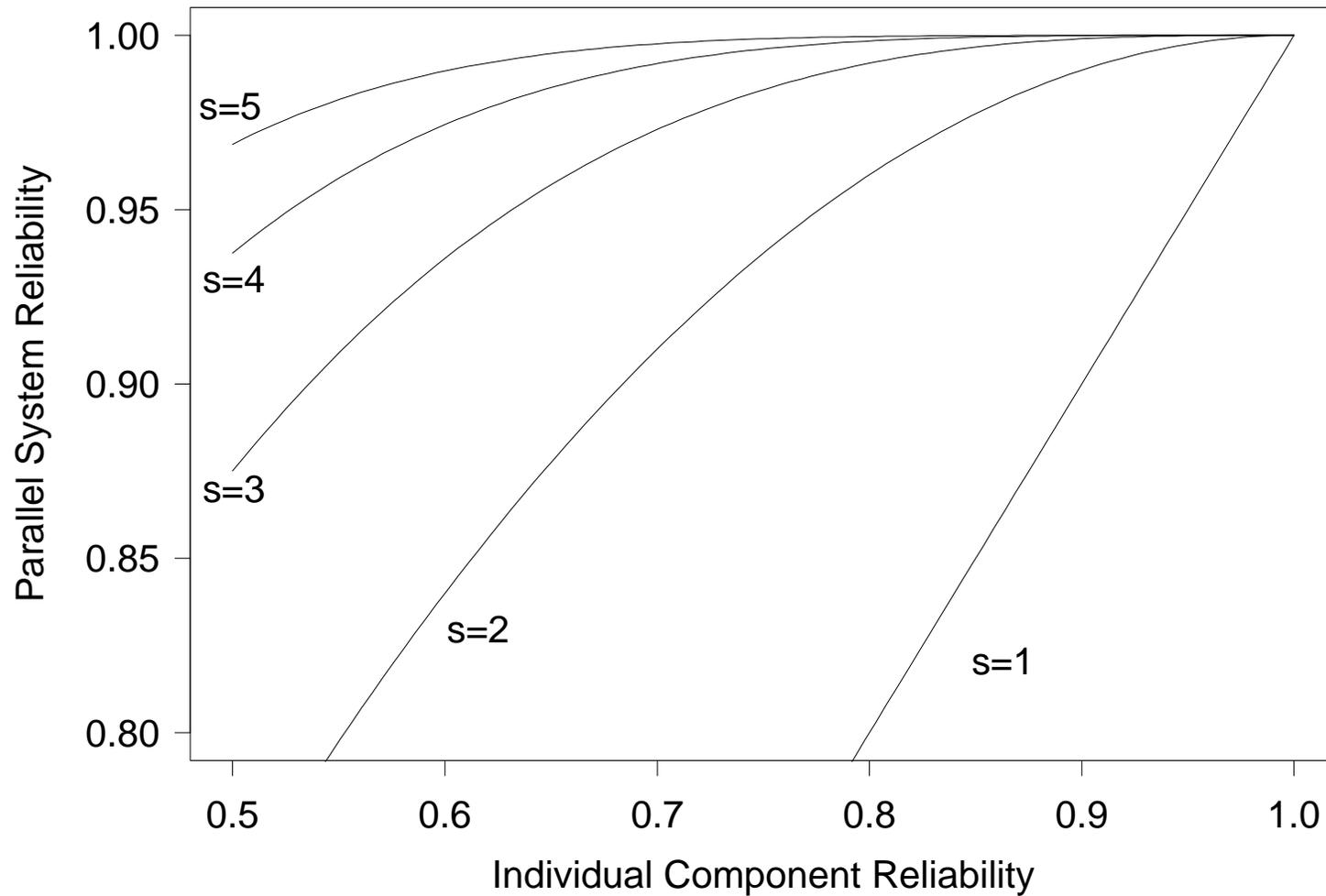
- For s independent components

$$F_T(t) = \prod_{i=1}^s F_i$$

- For s iid components ($F_i = F, i = 1, \dots, s$)

$$F_T(t) = F^s.$$

Reliability of a System with s Identical Independent Components in Parallel



Effect of Positive Dependency in a Two-Component Parallel System

- For a parallel system with two components and dependent failure times

$$F_T(t) = \Pr(T \leq t) = \Pr(T_1 \leq t \cap T_2 \leq t).$$

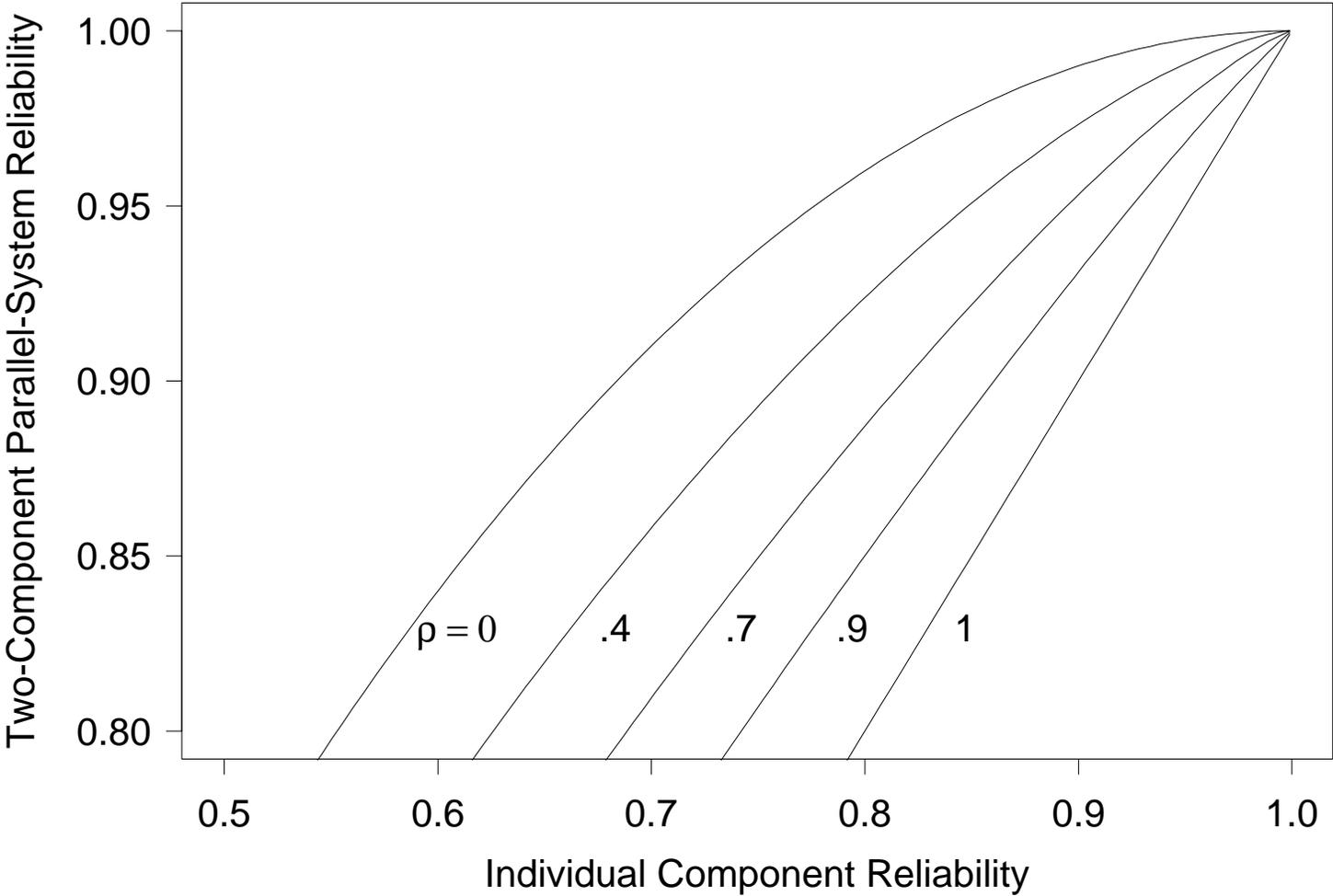
In this case, the evaluation has to be done with respect to the bivariate distribution of T_1 and T_2 .

- If the correlation between the two components is positive, then the assumption of independence is anti-conservative in the sense that the actual $F_T(t)$ is larger than that predicted by the independent-component model.
- These results extend to the s components in parallel, the system $F_T(t)$ would have to be computed with respect to the underlying s -variate distribution. Such computations are, in general, difficult.

Effect of Positive Dependency in a Two-Component Parallel System with Lognormal Failure Times

- The distributions of log failure times for the individual components is bivariate normal with the same (arbitrary) mean and standard deviation for both components and correlation ρ .
- The reliability $1 - F_T(t)$ of the system can be expressed as a function of the individual reliability components $1 - F(t)$ and ρ .
- When $\rho = 1$ (so the two components are perfectly dependent and will fail at exactly the same time), the system reliability $1 - F_T(t)$ is the same as the reliability for a single component.
- When $\rho = 0$ (so the two components are independent), $1 - F_T(t)$ corresponds to the system reliability for a $s = 2$ independent parallel system.
- The advantages of redundancy can be seriously degraded when the failure times of the individual components have positive dependence.

Reliability of a System With 2 Dependent Components in Parallel



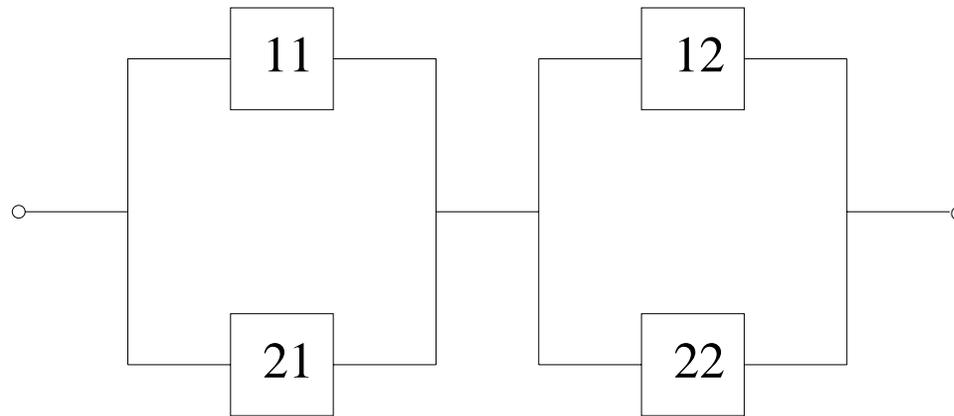
Some More Complicated System Structures

Series and parallel structures are the basis for building more complicated structures which use redundancy to increase system reliability.

Some examples are:

- Series-parallel with component-level redundancy.
- Series-parallel with system-level redundancy.
- Bridge structures.

A Series-Parallel System Structure with Component-Level Redundancy



Examples Series-Parallel System Structure with Component-Level Redundancy Parallel

- Dual repeaters in under-sea fiber-optic data transmission system
- Human body (lungs, kidneys)

Systems with Component-Level Redundancy

A $k \times r$ component-level redundant structure has k series structures each one made of r units in parallel.

- For 2×2 series-parallel with independent components

$$\begin{aligned} F_T(t) &= 1 - \Pr(T > t) \\ &= 1 - \Pr[\mathbf{parallel\ 1\ works} \cap \mathbf{parallel\ 2\ works}] \\ &= 1 - (1 - F_{11}F_{21})(1 - F_{12}F_{22}) \end{aligned}$$

where F_{ij} , $j = 1, 2$ are the cdfs for the parallel subsystem i .

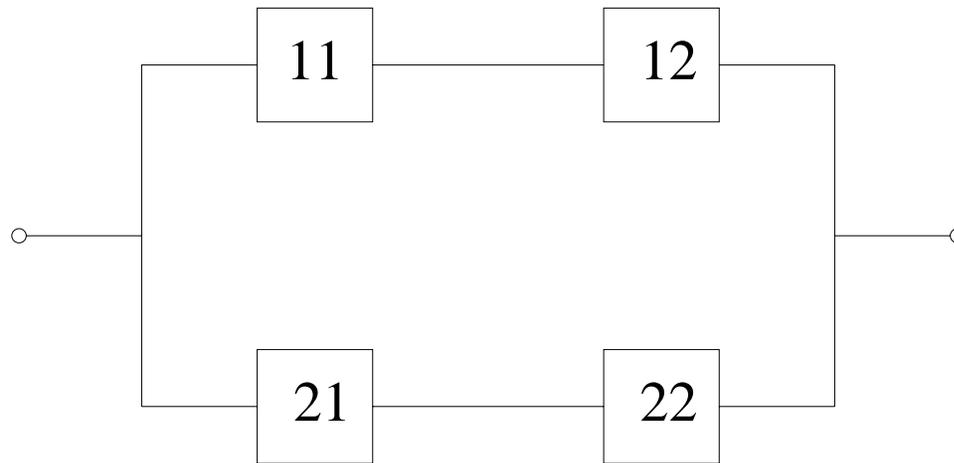
- For a $k \times r$ series-parallel with independent components

$$F_T(t) = 1 - \prod_{j=1}^k \left(1 - \prod_{i=1}^r F_{ij} \right)$$

- When all of the components are iid

$$F_T(t) = 1 - (1 - F^r)^k$$

A Series-Parallel System Structure with System-Level Redundancy



Examples Series-Parallel System Structure with System-Level Redundancy Parallel

- Dual central processors for a system-critical communications switching system
- Automobile break system (hydraulic and mechanical)
- Multiple trans-Atlantic transmission cables
- Fiber bundle or stranded wire

Series-Parallel Structure with System-Level Redundancy

A $r \times k$ series-parallel system-level redundancy structure has r parallel sets each of k units in series.

- For 2×2 structure with independent components

$$\begin{aligned}F_T(t) &= \Pr(T \leq t) \\ &= \Pr[\mathbf{series\ 1\ failed} \cap \mathbf{series\ 2\ failed}] \\ &= [1 - (1 - F_{11})(1 - F_{12})][1 - (1 - F_{21})(1 - F_{22})]\end{aligned}$$

where F_{ij} , $j = 1, 2$ are the cdf for the i series.

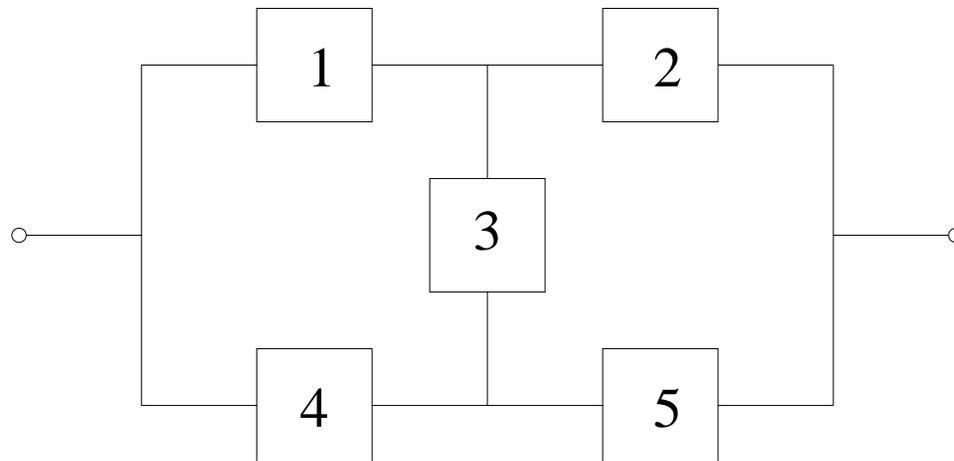
- For a $r \times k$ structure with independent components

$$F_T(t) = \prod_{i=1}^r \left[1 - \prod_{j=1}^k (1 - F_{ij}) \right]$$

- For a $r \times k$ parallel-series structure with iid components

$$F_T(t) = \left[1 - (1 - F)^k \right]^r$$

A Bridge System Structure

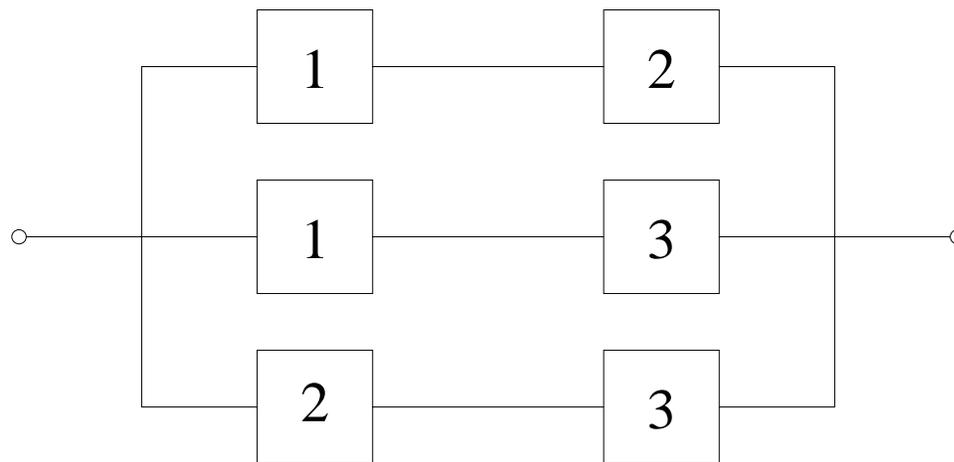


Bridge System Structure

Let A_i be the event that the i unit is working

$$\begin{aligned} F_T(t) &= \Pr(T \leq t \cap A_3) + \Pr(T \leq t \cap A_3^c) \\ &= \Pr(A_3) \Pr(T \leq t | A_3) + \Pr(A_3^c) \Pr(T \leq t | A_3^c) \\ &= \Pr(A_3) \Pr[(A_1^c \cap A_4^c) \cup (A_2^c \cap A_5^c) | A_3] + \\ &\quad \Pr(A_3^c) \Pr[(A_1^c \cup A_2^c) \cap (A_4^c \cup A_5^c) | A_3^c] \\ &= (1 - F_3) [F_1 F_4 + F_2 F_5 - F_1 F_2 F_4 F_5] + \\ &\quad F_3 [F_1 + F_2 - F_1 F_2] [F_4 + F_5 - F_4 F_5] \end{aligned}$$

A 2 out of 3 System Structure



Examples of k out of s System Structures

- Satellite battery system in which system will continue to operate as long as 6 of 10 batteries continue to operate correctly.
- Floppy disks which continue to provide service by blocking out bad sectors.

2 out of 3 System Structures

For a 2 out of 3 independent components

$$\begin{aligned}F_T(t) &= \Pr(T \leq t) \\&= \Pr(\mathbf{exactly\ two\ fail}) + \Pr(\mathbf{exactly\ three\ fail}) \\&= F_1F_2(1 - F_3) + F_1F_3(1 - F_2) + F_2F_3(1 - F_1) + F_1F_2F_3 \\&= F_1F_2 + F_1F_3 + F_2F_3 - 2F_1F_2F_3\end{aligned}$$

k out of s System Structures

- For k out of s independent components

$$F_T(t) = \sum_{j=s-k+1}^s \left\{ \sum_{\underline{\delta} \in A_j} \left[\prod_{i=1}^s F_i^{\delta_i} (1 - F_i)^{(1-\delta_i)} \right] \right\}$$

where $\underline{\delta}' = (\delta_1, \dots, \delta_s)$ with $\delta_i = 1$ indicating failure of unit i by time t and $\delta_i = 0$ otherwise and A_j is the set of all $\underline{\delta}$ such that $\underline{\delta}'\underline{\delta} = j$.

- For identically distributed components ($F = F_i, i = 1, \dots, n$)

$$F_T(t) = \sum_{j=s-k+1}^s \binom{s}{j} F^j (1 - F)^{s-j}.$$

Products With Two or More Causes of Failure (or Multiple Modes of Failure)

Many units, systems, subsystems, or components have more than one cause of failure. For example:

- A capacitor can fail open or as a short.
- Any of many solder joints in a circuit board can fail.
- A semiconductor device can fail at a junction or at a lead.
- A device can fail because a manufacturing defect (infant mortality) or because of mechanical wearout.
- For an automobile tire, tread can wearout or the tire may suffer a puncture.

Device-G Background

- Failure times and running times for a sample of devices from a field tracking study of a larger system.
- Thirty (30) units were installed in typical service environments.
- Cause of failure information was determined for each unit that failed.
- Mode S failures were caused by failures on an electronic component due to electrical surge. These failures predominated early in life.
- Mode W failures, caused by normal product wear, began to appear after 100 thousand cycles of use.

Device-G Data

Thousands of Cycles	Failure Mode	Thousands of Cycles	Failure Mode	Thousands of Cycles	Failure Mode
275	W	106	S	88	S
13	S	300	–	247	S
147	W	300	–	28	S
23	S	212	W	143	S
181	W	300	–	300	–
30	S	300	–	23	S
65	S	300	–	300	–
10	S	2	S	80	S
300	–	261	S	245	W
173	S	293	W	266	W

W indicates a wearout failure, S indicates an electrical surge failure, and – indicates a unit still operating after 300 thousand cycles.

Series-System Model

- Let T_S be the lifetime from the electrical surge failure mode (S) and T_W be the lifetime from the wearout failure mode (W).
- The failure-time of the product is

$$T = \min\{T_S, T_W\}$$

- When the failure modes are independent, the cdf for the failure-time is

$$\begin{aligned} F(t) &= \Pr(T \leq t) = 1 - \Pr(T > t) = 1 - \Pr(T_S > t \cap T_W > t) \\ &= 1 - \Pr(T_S > t) \Pr(T_W > t) \\ &= 1 - [1 - F_S(t)][1 - F_W(t)]. \end{aligned}$$

- The same concepts apply to the failure-time of a product with more than two failure modes.

General Data Analysis Strategies

- Analyze the failure modes separately.
 - ▶ Of interest to managers or engineers wanting to improve product reliability.
 - ▶ Requires that failure modes act independently of each other or information about the joint distribution of (T_S, T_W) (see discussion later).
- Ignore cause of failure information.
 - ▶ Of interest to a consumer concerned with product life.
 - ▶ Sometimes adequate within the range of the data; can be seriously incorrect when extrapolating.

Analysis of Device-G Data Independent Failure Modes

When the failure modes S and W act independently, one can:

- Analyze the mode S failures only. In this case mode W failures are treated as right censored observations.

This is the estimate of the failure-time distribution if mode W could be completely eliminated.

- Analysis of the mode W failures only. In this case mode S failures are treated as right censored observations.

This is the estimate of the failure-time distribution if mode S could be completely eliminated.

- A combined analysis using the series system model and independence between mode S and mode W .

Weibull Distribution Models for the Device-G Data

- Failure times for each of the two failure modes modeled with a separate Weibull distribution:

$$F_i(t) = \Phi_{\text{sev}} \left(\frac{\log(t) - \mu_i}{\sigma_i} \right), \quad i = S, W.$$

- The **series model** for two independent failure modes acting together is

$$F(t) = 1 - [1 - F_S(t)] \times [1 - F_W(t)].$$

- Ignoring the cause of failure information:

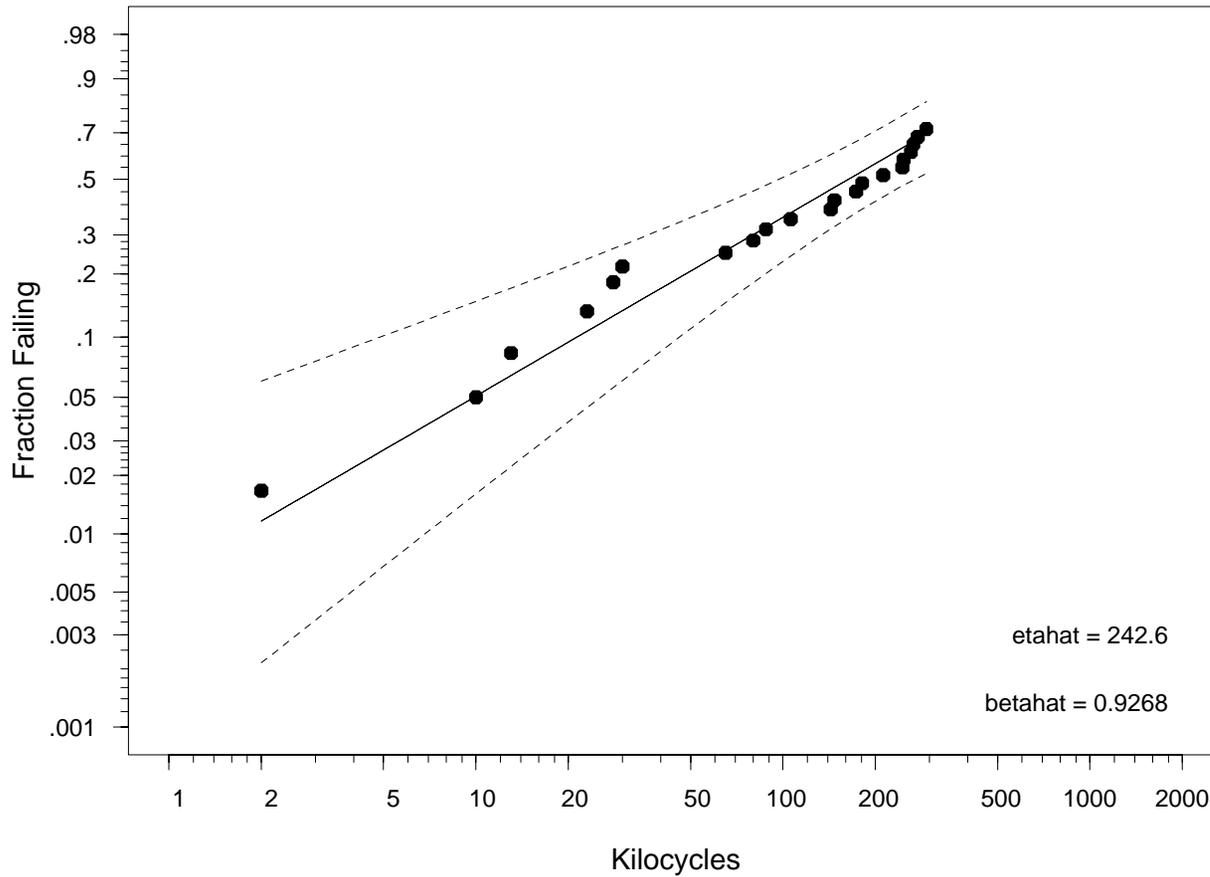
$$F(t) = \Phi_{\text{sev}} \left(\frac{\log(t) - \mu_{SW}}{\sigma_{SW}} \right).$$

Device-G Field-Tracking Data
ML Weibull Distribution Estimation Results for the
Electric Surge (S) and Wearout (W) Failure Modes

Mode	Parameter	ML Estimate	Standard Error	95% Approximate Confidence Interval	
				Lower	Upper
S	μ_S	6.11	.427	5.27	6.95
	σ_S	1.49	.35	.94	2.36
W	μ_W	5.83	0.11	5.62	6.04
	σ_W	.23	.08	.12	.44
S & W	μ_{SW}	5.49	0.23	5.04	5.94
	σ_{SW}	1.08	.21	.74	1.57

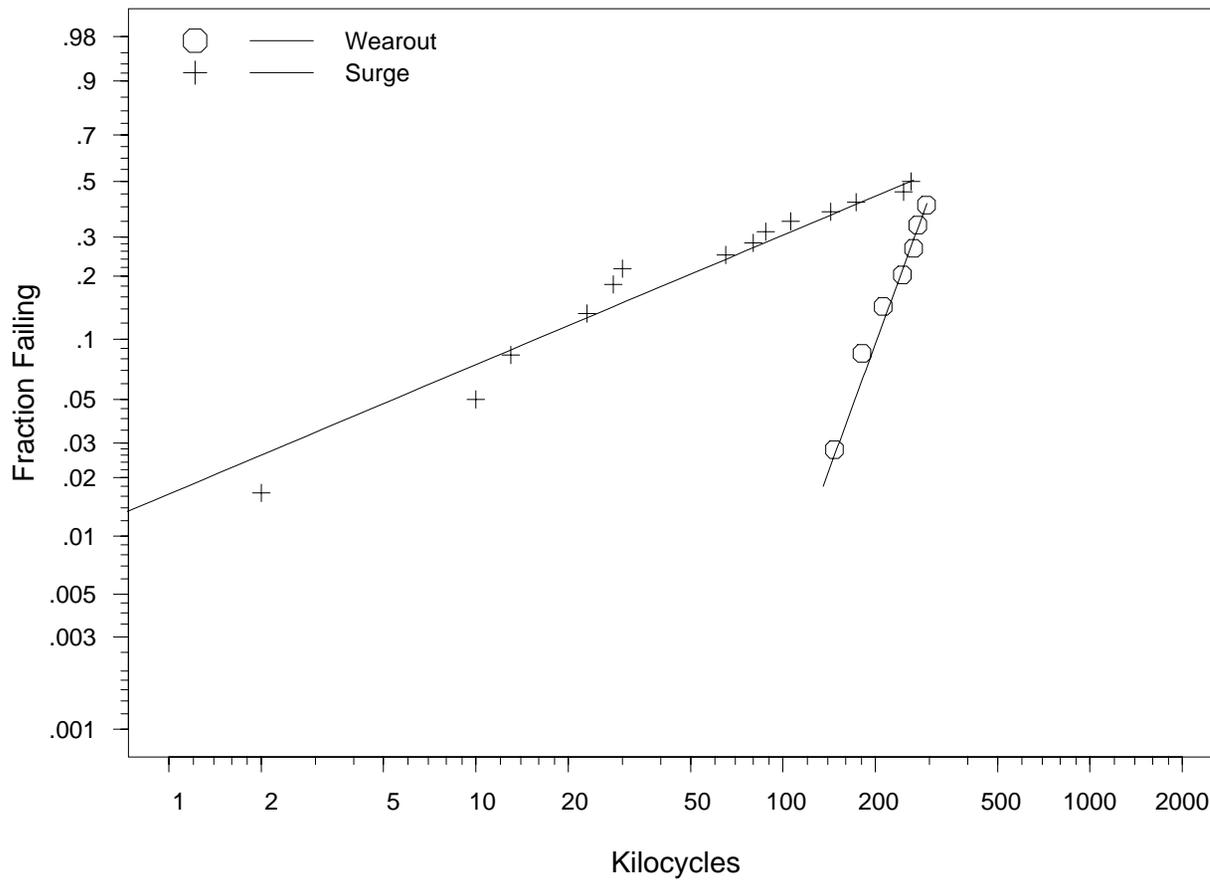
For Mode S alone, $\mathcal{L}_S = -101.36$ for Mode W alone, $\mathcal{L}_W = -47.16$, and for both modes together, $\mathcal{L}_{SW} = -142.62$.

Weibull Analyses of Device-G Data Estimating Time to Failure Ignoring the Cause of Failure

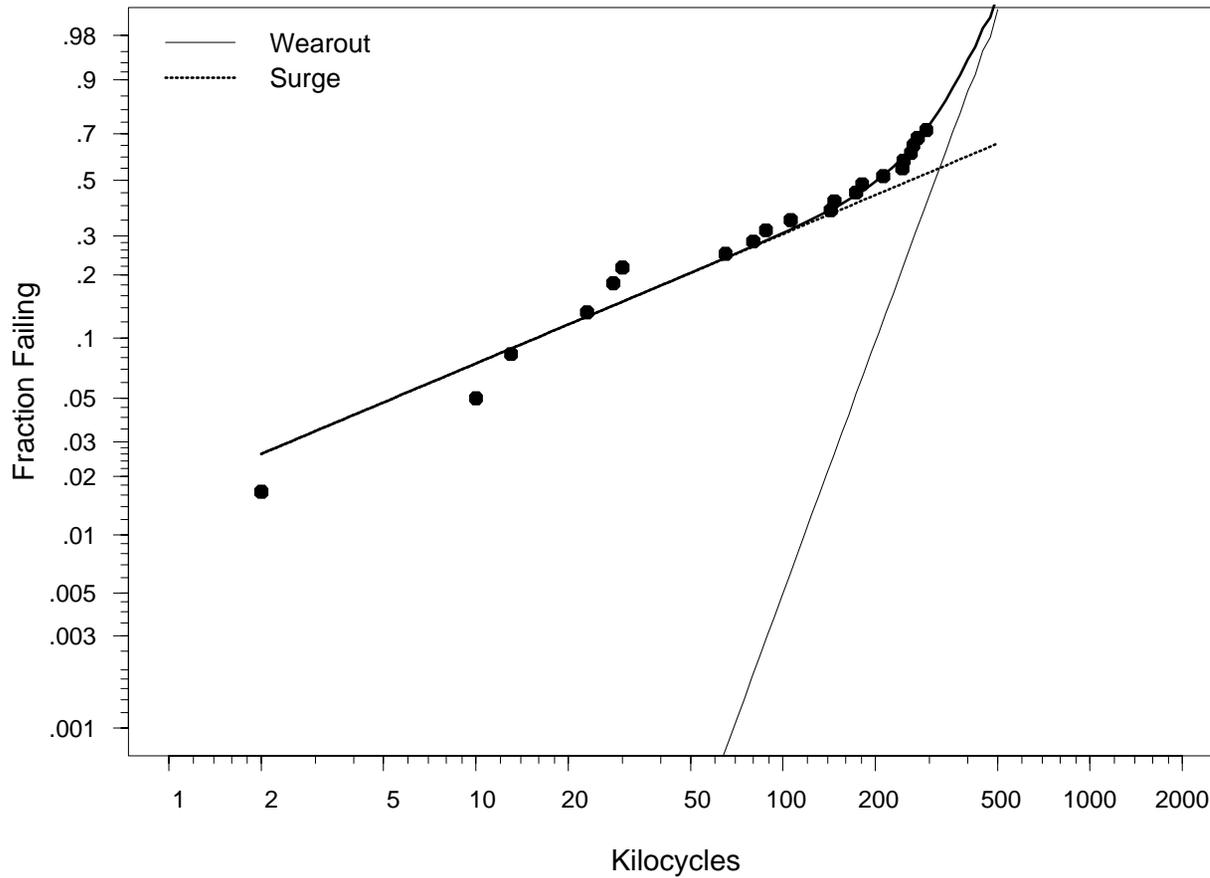


Weibull Analyses of Device-G Data

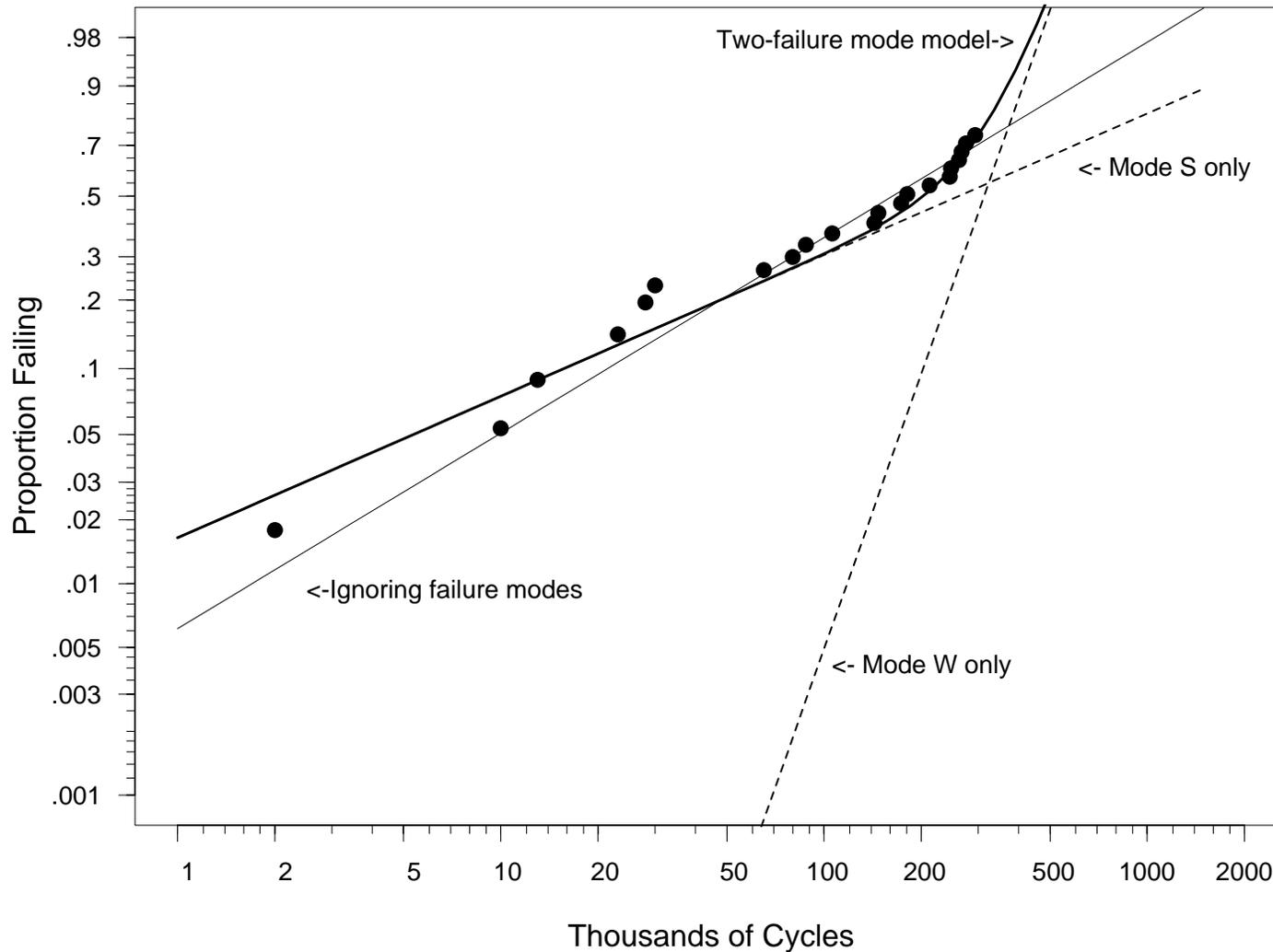
Individual Failure Modes



Weibull Analyses of Device-G Data Estimating Time to Failure Using Series System Model



Weibull Analyses of Device-G Data Estimating Time to Failure Mode S Only, Failure Mode W Only, Ignoring the Cause of Failure, and Series System Model



Some Comments on the Weibull Analyses of Device-G Data

- The Weibull distribution provides a good fit to both data the S failure mode and the W failure mode.
- Weibull analysis ignoring the cause of failure information shows evidence of a change in the slope of the plotted points, indicating a gradual shift from one failure mode to another.
- The Weibull cdf estimate obtained from ignoring the cause of failure and the series-system cdf estimate for the two failure modes acting together diverge rapidly after 200 thousand cycles.
- Estimates of the mean time to failure computed from $\widehat{MTTF} = \int_0^{\infty} [1 - \widehat{F}_T(t)] dt$ were 251.3 and 196.0 thousands of cycles, respectively, for the models ignoring and using the failure mode information.

Connection Strength Data (King 1971 and Nelson 1982)

- Wires are bonded at one end to a semiconductor wafer and at the other end to a terminal post.
- The wire or the bond can fail.
- The engineers wanted to know if the manufacturing process meets the specification that no more than 1% of the strengths be below 500mg.

Connection Strength Data (King 1971 and Nelson 1982)

Strength mg	Failure Mode	Strength mg	Failure Mode
550	B	1250	B
750	W	1350	W
950	B	1450	B
950	W	1450	B
1150	W	1450	W
1150	B	1550	B
1150	B	1550	W
1150	W	1550	W
1150	W	1850	W
1250	B	2050	B

B indicates that the bond failed and
W indicates that the wire failed

Normal Distribution Series Model for Connection Strength Data

- Each failure mode is modeled separately with a normal model

$$F_i(t) = \Phi_{\text{nor}} \left(\frac{t - \mu_i}{\sigma_i} \right), \quad i = B, W.$$

- The series model for the two failure modes acting together is

$$F(t) = 1 - [1 - F_B(t)] \times [1 - F_W(t)].$$

- When the cause of failure information is ignored

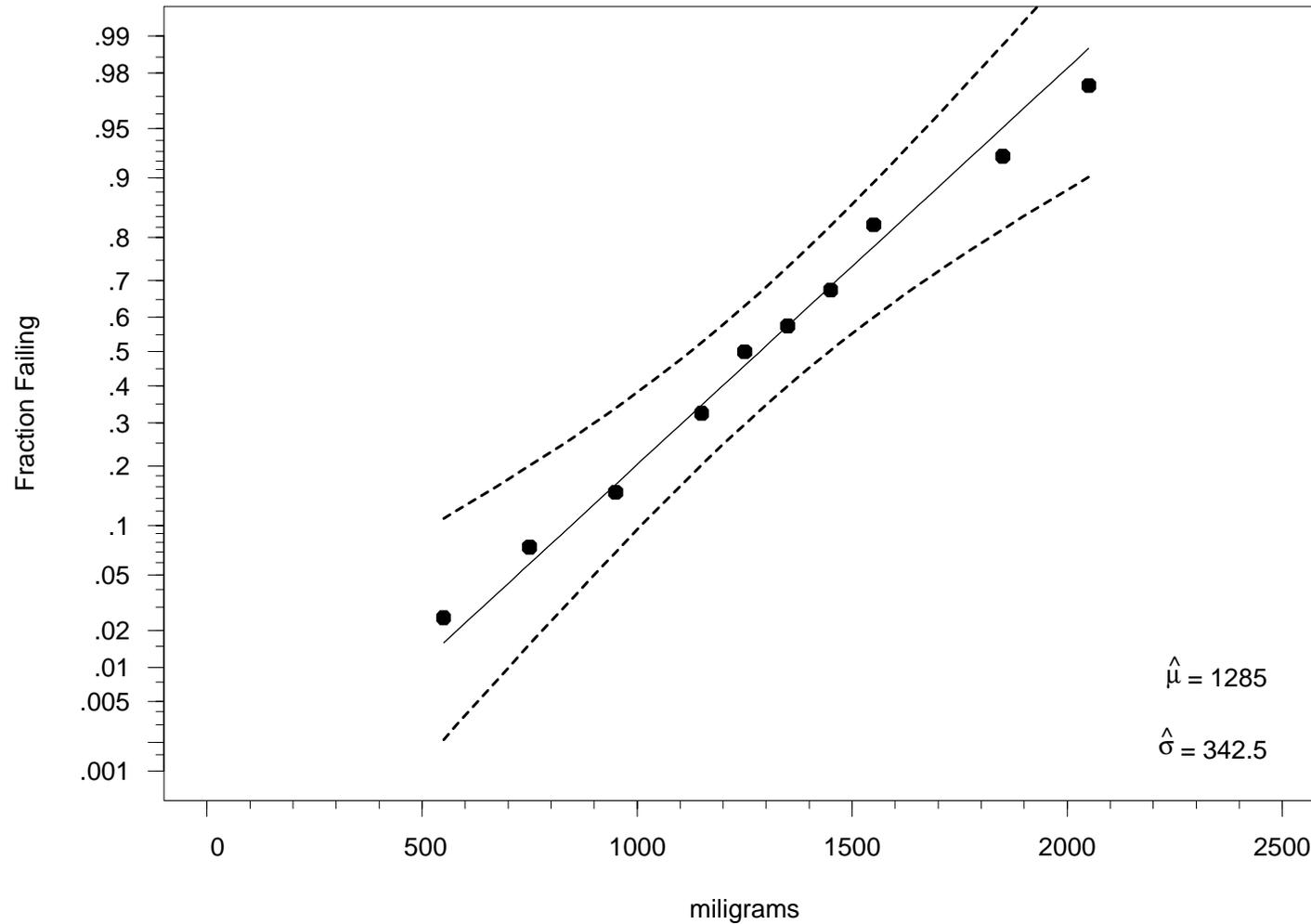
$$F(t) = \Phi_{\text{nor}} \left(\frac{t - \mu_{SW}}{\sigma_{SW}} \right).$$

Connection Strength Data Normal ML Estimation for Bond (B) and Wire (W) Failure Modes

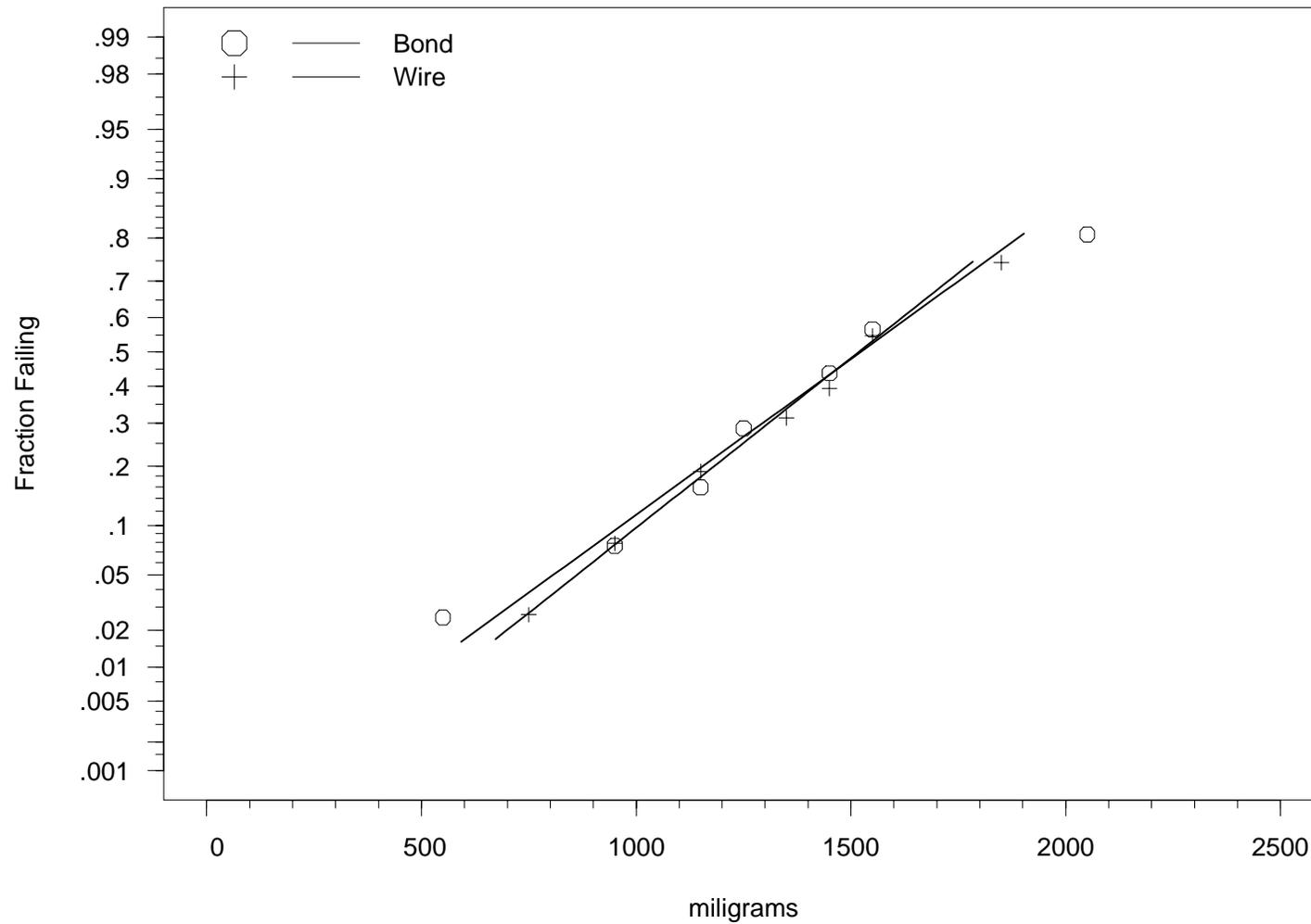
Mode	Parameter	ML Estimate	Standard Error	95% Approximate Confidence Interval	
				Lower	Upper
B	μ_B	1522.32	121.61	1304.20	1831.80
	σ_B	434.97	97.96	295.71	728.46
W	μ_W	1517.36	111.43	1316.00	1799.46
	σ_W	398.70	89.86	270.97	667.58
B&W	μ_{BW}	1285.00	76.58	1127.41	1442.59
	σ_{BW}	342.45	54.14	258.58	483.78

For mode S alone, $\mathcal{L}_B = -79.96$ for mode W alone, $\mathcal{L}_W = -79.02$, and for both modes together, $\mathcal{L}_{BW} = -145.10$.

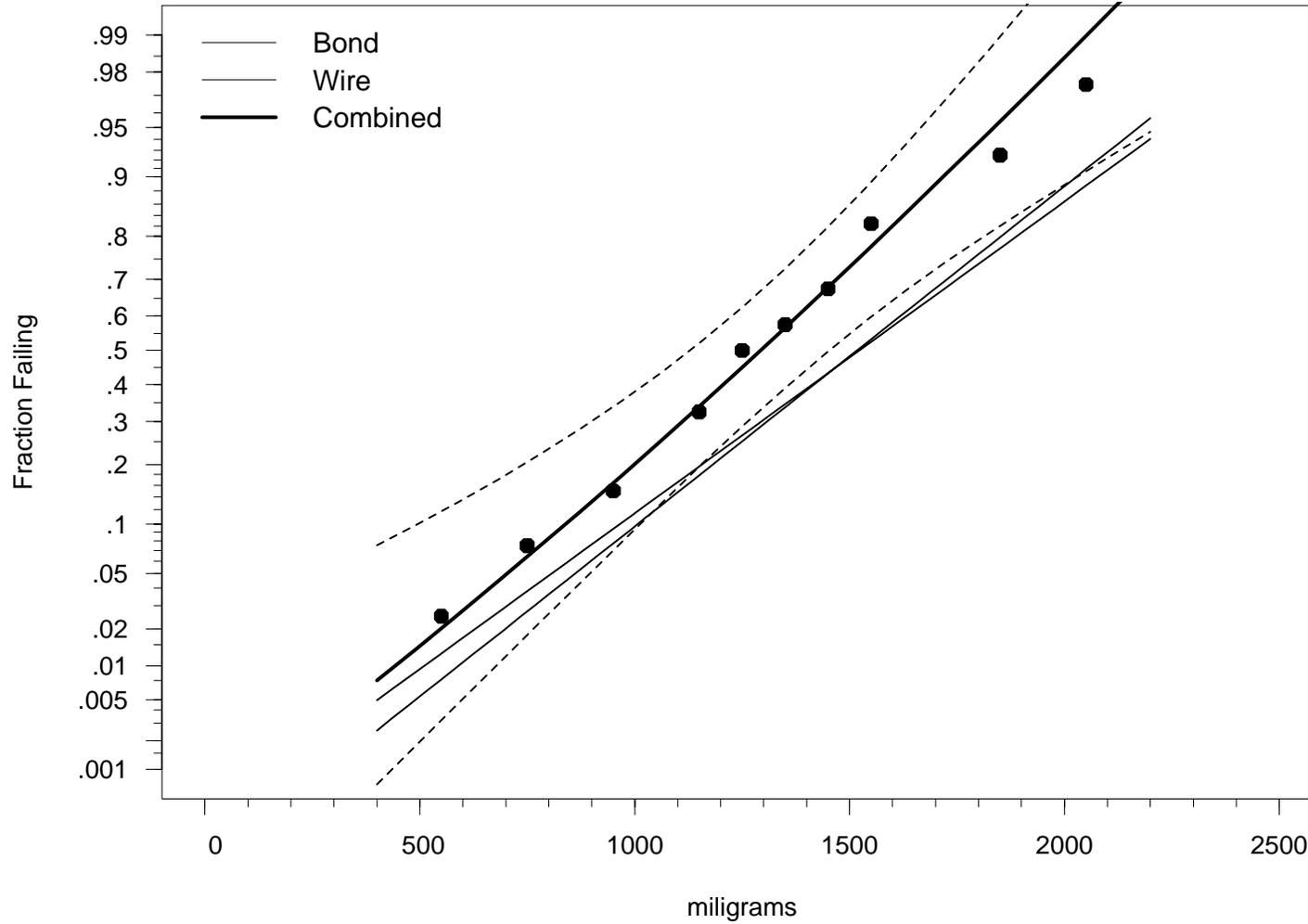
Normal Probability Plots for Connecting Strength Data Ignoring Cause of Failure



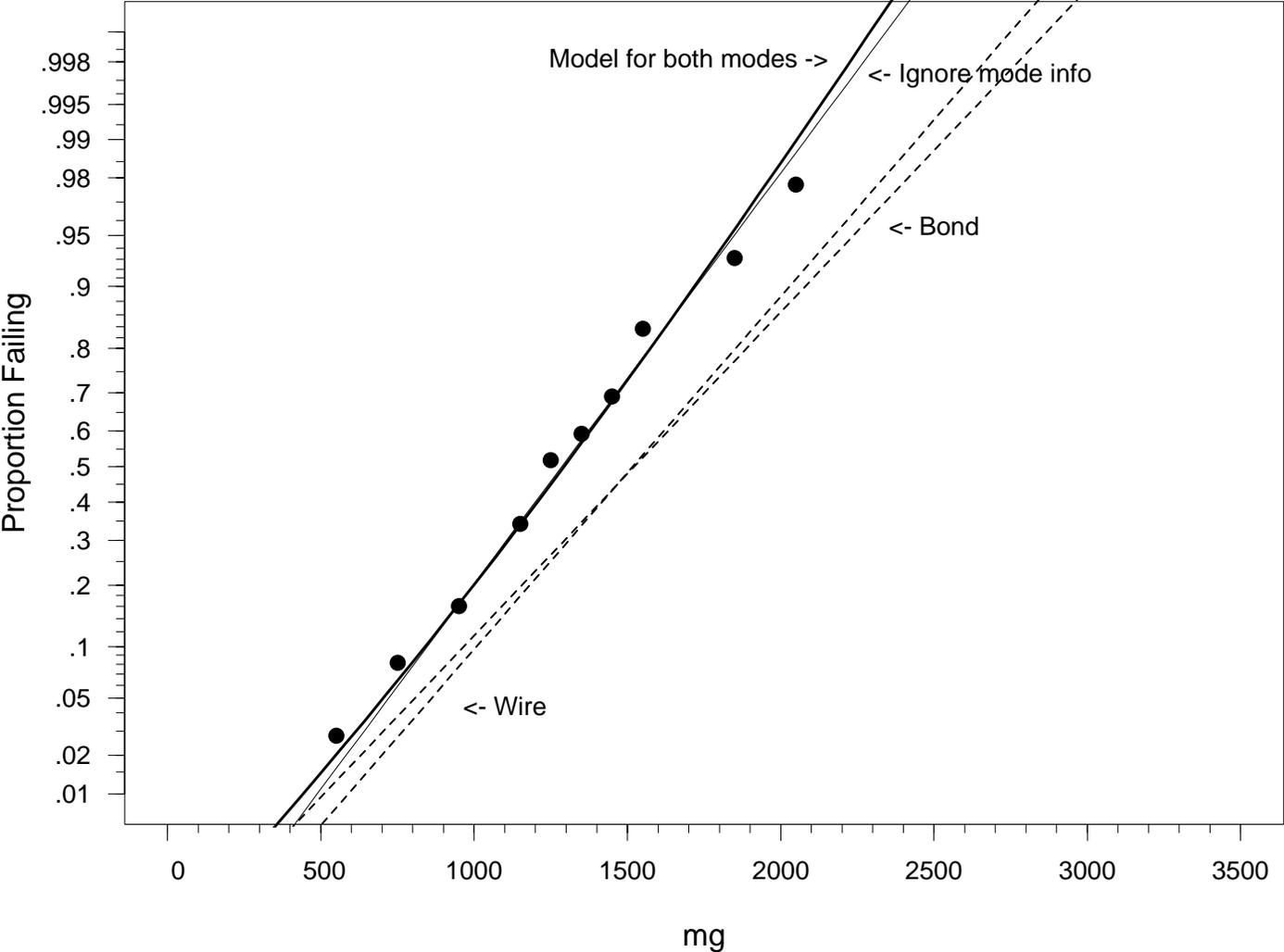
Normal Probability Plots for Connecting Strength Data Individual Failure Modes



Normal Distribution Series Model for Connection Strength Data



Normal Probability Plots for Bond, Wire, Ignoring Mode of Failure and Series System Model



Some Computations for the Connection Strength Data

- The estimate of the failure probability at 500mg are:
- For bonds

$$\hat{F}_B(500) = \Phi \left[\frac{500 - 1522.32}{434.98} \right] = .0094$$

- For wires

$$\hat{F}_W(500) = \Phi \left[\frac{500 - 1517.36}{398.70} \right] = .0054$$

- When both modes act together (independent failures modes)

$$\hat{F}(500) = 1 - (1 - .0094)(1 - .0054) = .0147$$

- Ignoring cause of failure

$$\hat{F}(500) = \Phi \left[\frac{500 - 1285.0}{342.45} \right] = .0109$$

Estimation When Failure Mode is Identified for Only Some Failures

When failure modes are not identified or are only partially identified for some units, it is still possible to estimate the individual $F_i(t)$ distributions by using maximum likelihood.

- Known as **masking** of failure modes.
- Difficult because the analysis are not separable.
- Parameter estimates for the distribution for one mode will be correlated with those of the other modes.
- In practice, one is likely to analyze the data as if there were only a single mode. Potentially misleading if extrapolating outside the range of the data.

Effect of Dependency Among Failure Modes

- The common assumption of independent failure modes is sometimes unrealistic.

When there is dependence, still one can use the relationship

$$F(t) = \Pr(T \leq t) = 1 - \Pr(T_1 > t \cap T_2 > t)$$

but the evaluation has to be done with respect to the bivariate joint distribution of T_1 and T_2 .

- Usually, when there is dependence, the dependence is positive. Then long (short) failure times of one mode tend to go with long (short) failure times of another.

In this case, eliminating one of the failure modes has little effect in increasing the reliability of the system because the other mode assumes the role of the eliminated mode.

- If the failure modes are positively dependent, then attempting to predict the effect of eliminating one of the failure modes, using the independent failure mode model, can give seriously incorrect and overly optimistic predictions.

Computing System cdf from Component Information

When there are component data available, one can estimate the θ_i 's from the component data. This yields the estimates F_1, \dots, F_s . Evaluate as a function of time.

To compute the system cdf, one can

- Use $F_T = g(F_1, \dots, F_s)$ when g is known.
- If g cannot be expressed in closed form or is otherwise difficult to compute, one can use a computer simulation of the system based on the F_i and the system structure.
- When the F_i are unknown, an estimate of the system F_T can be obtained by evaluating F_T at the ML estimates of the needed F_i values.

Sources of Reliability Data and Other Information

- Laboratory tests.
- Field data.
- Books and data banks.
- Expert knowledge.

Maximum Likelihood Estimation of System Reliability

The system cdf and other related functions can be estimated using ML estimates for the components.

- Let $\hat{\theta}$ be the ML estimate of θ , and $\hat{\Sigma}_{\hat{\theta}}$ the ML estimate of $\Sigma_{\hat{\theta}}$ obtained from the component data. Then using same methods as in previous chapters

$$\hat{F}_T = F_T(\hat{\theta}) = g[F_1(\hat{\theta}), \dots, F_s(\hat{\theta}_s)]$$

$$\widehat{\text{Var}}(\hat{F}_T) = \left(\frac{\partial F_T}{\partial \theta} \right)' \hat{\Sigma}_{\hat{\theta}} \left(\frac{\partial F_T}{\partial \theta} \right)$$

where the derivatives are evaluated at $\hat{\theta}$.

Example of Maximum Likelihood Estimation for a Simple System

For a parallel structure with s iid components

$$\hat{F}_T = [\hat{F}]^s = [F(\hat{\theta})]^s$$

$$\begin{aligned}\widehat{\text{Var}}(\hat{F}_T) &= \left(\frac{\partial F_T}{\partial \theta}\right)' \hat{\Sigma}_{\hat{\theta}} \left(\frac{\partial F_T}{\partial \theta}\right) \\ &= \left(\frac{\partial F_T}{\partial F} \frac{\partial F}{\partial \theta}\right)' \hat{\Sigma}_{\hat{\theta}} \left(\frac{\partial F_T}{\partial F} \frac{\partial F}{\partial \theta}\right) \\ &= \left(s\hat{F}^{s-1} \frac{\partial F}{\partial \theta}\right)' \hat{\Sigma}_{\hat{\theta}} \left(s\hat{F}^{s-1} \frac{\partial F}{\partial \theta}\right)\end{aligned}$$

$$\widehat{\text{se}}(\hat{F}_T) = \sqrt{\widehat{\text{Var}}(\hat{F}_T)}$$

Normal-Approximation Confidence Intervals for System Reliability

An approximate $100(1 - \alpha)\%$ confidence intervals can be based on $Z_{\text{logit}(\hat{F}_T)} \sim \text{NOR}(0, 1)$

$$[\underset{\sim}{F}_T, \tilde{F}_T] = \left[\frac{\hat{F}_T}{\hat{F}_T + (1 - \hat{F}_T) \times w}, \frac{\hat{F}_T}{\hat{F}_T + (1 - \hat{F}_T)/w} \right]$$

where $w = \exp\{z_{(1-\alpha/2)} \hat{\text{se}}_{\hat{F}} / [\hat{F}(1 - \hat{F})]\}$.

Bootstrap Approximate Confidence Intervals for System Reliability

- Needed bootstrap samples consist of
 - ▶ Bootstrap estimates \hat{F}_i^* , $i = 1, \dots, s$ (as in Chapter 13).
 - ▶ Bootstrap estimate $\hat{F}_T^* = g(\hat{F}_1^*, \dots, \hat{F}_s^*)$
- A $100(1 - \alpha)\%$ approximate confidence interval based on $Z_{\text{logit}(\hat{F}_T)} \sim Z_{\text{logit}(\hat{F}_T^*)}$ and B bootstrap samples is

$$[\underset{\sim}{F}_T, \tilde{F}_T] = \left[\frac{\hat{F}_T}{\hat{F}_T + (1 - \hat{F}_T) \times \underset{\sim}{w}}, \frac{\hat{F}_T}{\hat{F}_T + (1 - \hat{F}_T) \times \tilde{w}} \right]$$

where $\underset{\sim}{w} = \exp\{z_{\text{logit}(\hat{F}_T^*)_{(1-\alpha/2)}} \widehat{\text{se}}_{\hat{F}_T} / [\hat{F}_T(1 - \hat{F}_T)]\}$ and $\tilde{w} = \exp\{z_{\text{logit}(\hat{F}_T^*)_{(\alpha/2)}} \widehat{\text{se}}_{\hat{F}_T} / [\hat{F}_T(1 - \hat{F}_T)]\}$ are obtained from the quantiles of the B bootstrap estimates \hat{F}_T^* .

Other System Structures

Standby or passive redundancy: a redundant unit is activated only when another unit fails and the redundant unit is need to keep the system working.

There are many variations of this:

- Cold standby.
- Partially loaded redundancy.

Need to consider the reliability of the switching mechanism that activates the standby units.

Other Topics in System Reliability

- Other systems structures.
- System repair, maintainability, and availability.
- Dependent failures: the common assumption of components with independent failures is sometimes unrealistic. It is possible that a component failure improves or degrades the reliability of other system components.
- Markov models for handling dependencies and common-cause failures.

Repairable and Nonrepairable Systems

- **Nonrepairable** system: a system (component) that is discarded the first time that it fails.
- **Repairable** system: a system (component) that can be repaired or replaced after failure.

Modeling the reliability of a system containing many components of different ages is complicated (components not iid).

Repairable System Reliability Metrics

- For repairable systems with failures at T_1, T_2, \dots and negligible repair time define $\tau_i = T_i - T_{i-1}$, where $T_0 = 0$,
 - ▶ Mean time between failures (MTBF): average time elapsed between failures, $\text{MTBF}_i = E(\tau_i)$. In general, MTBF_i depends on i and $\tau_1, \dots, \tau_{i-1}$.
 - ▶ The failure occurrence rate (or intensity) is defined as:

$$v(t) = \frac{dE[N(t)]}{dt}$$

where $N(t)$ is the number of failures in the interval $(0, t]$.

Markov and Other More General Models

Markov models allow the modeling of repairable models allowing for dependence among components and common-cause failures.

- Markov models are, however, only suitable for small systems.
- The Markov models are also limited by the life and repair distributions that can be employed.
- Non-Markovian generalizations possible, but lead to computational difficulties. Analysis of such models generally done with simulation.