Chapter 21

Analysis of Accelerated Degradation Data

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Based on the authors' text *Statistical Methods for Reliability Data*, John Wiley & Sons Inc. 1998.

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12h 27min

Chapter 21
Analysis of Accelerated Degradation Data
Objectives

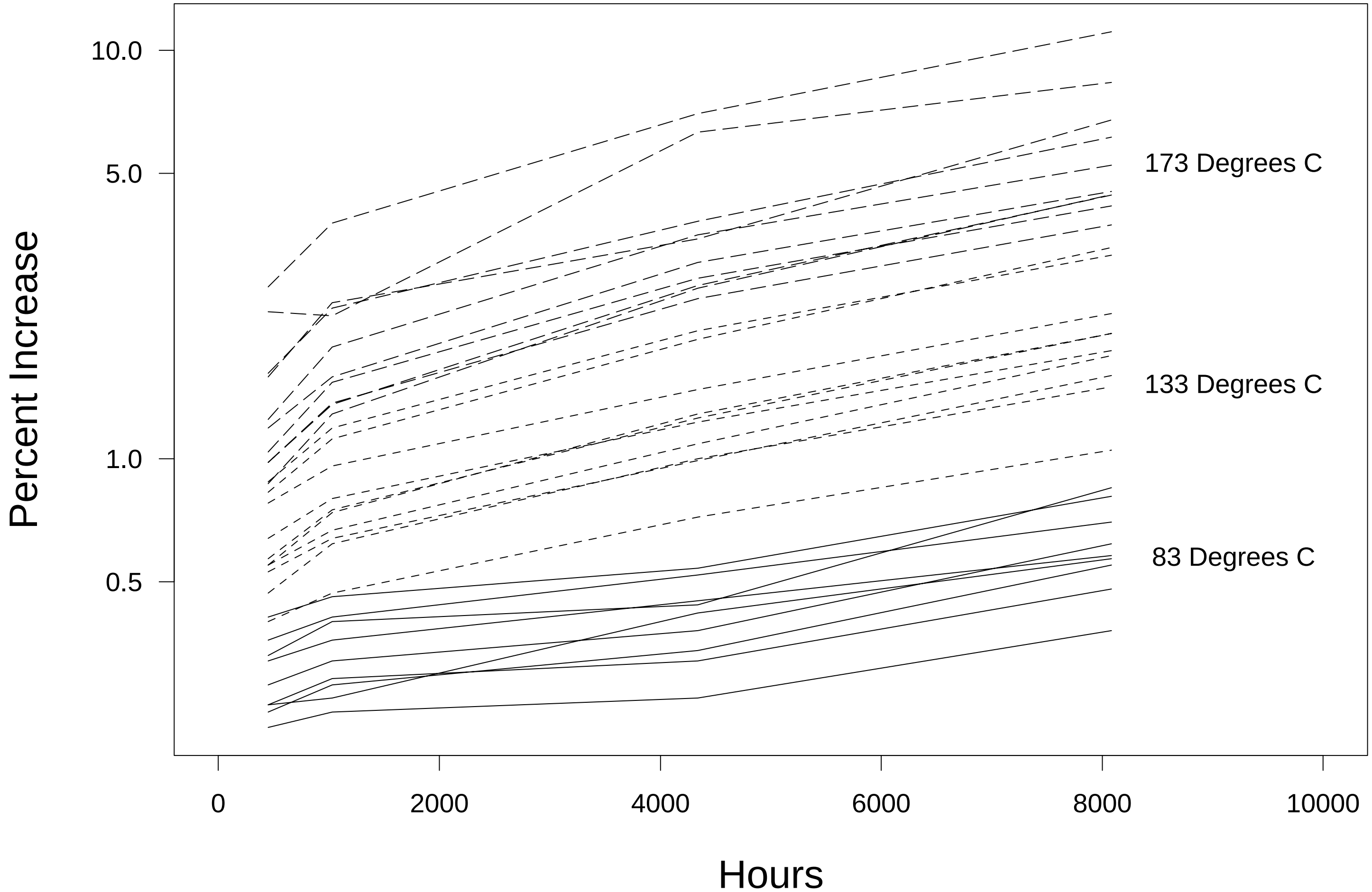
- Show how accelerated degradation tests can be used to assess and improve product reliability.
- Present models, methods of analysis, and methods of inference for accelerated degradation tests.
- Show how to analyze data from accelerated degradation tests.
- Compare accelerated degradation test methods with traditional accelerated life test methods using failure-time data.

Background

Today's manufacturers face strong pressure to:

- Develop newer, higher technology products in record time.
- Improve productivity, product field reliability, and overall quality.
- Increased the need for **up-front** testing of materials, components and systems.
- Accelerated degradation tests can be useful for such **up-front** testing.

Percent Increase in Resistance Over Time for Carbon-Film Resistors (Shiomi and Yanagisawa 1979)



Advantages of Using Degradation Data Over Failure-Time Data

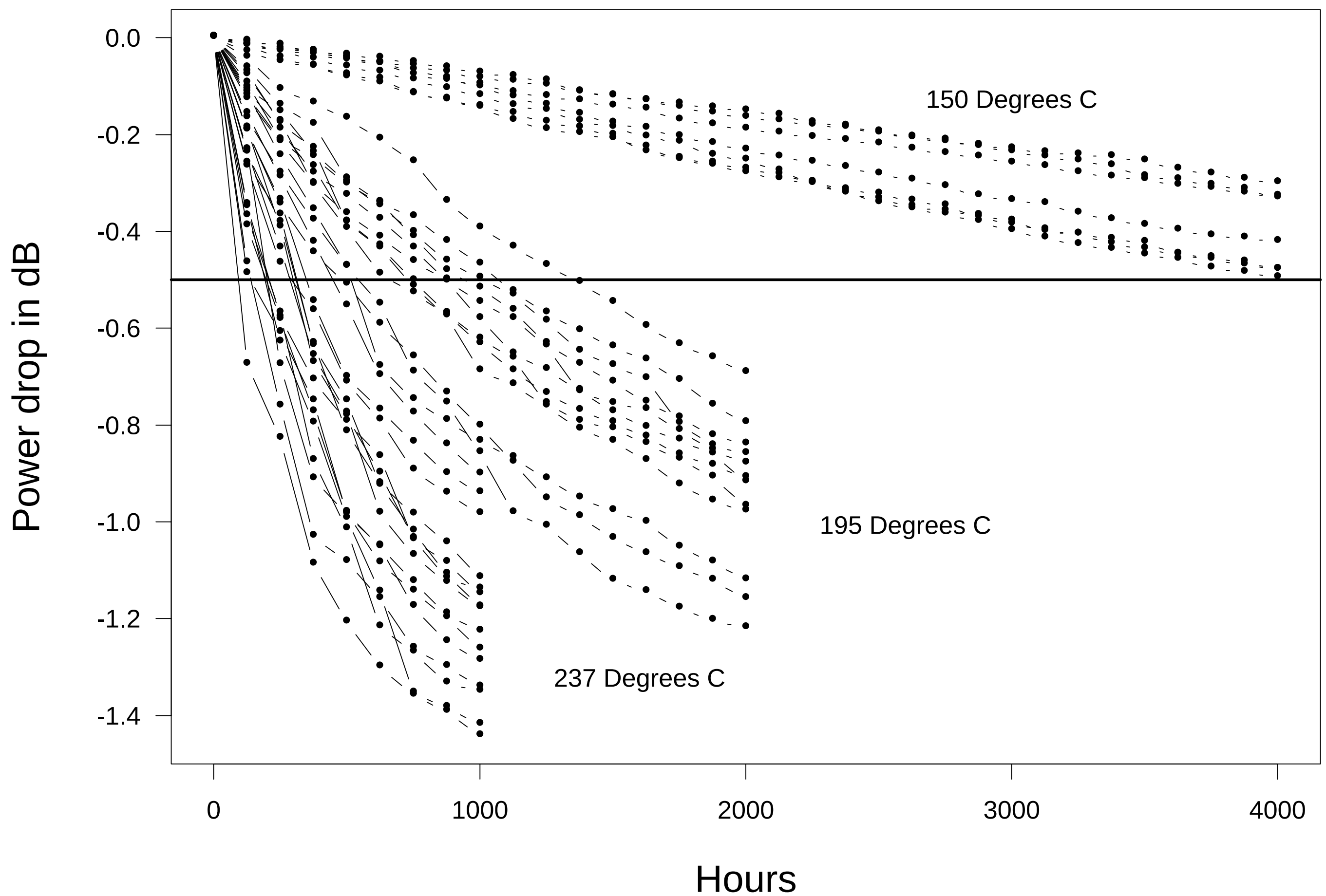
- Degradation is natural response for some tests.

- Useful reliability inferences even with 0 failures.

- More justification and credibility for extrapolative acceleration models.
(Modeling closer to physics-of-failure)

- Can be more informative than failure-time data.
(Reduction to failure-time data loses information)

Device-B Power Drop
Accelerated Degradation Test Results
at 150°C, 195°C, and 237°C
(Use Conditions 80°C)



Device-B Power Drop Simple One-Step Chemical Reaction Leading to Failure

- $A_1(t)$ is the amount of harmful material available for reaction at time t
- $A_2(t)$ is proportional to the amount of failure-causing compounds at time t.
- Chemical reaction:

$$A_1 \xrightarrow{k_1} A_2$$

- Power drop proportional to $A_2(t)$
- The rate equations for this reaction are

$$\frac{dA_1}{dt} = -k_1 A_1 \quad \text{and} \quad \frac{dA_2}{dt} = k_1 A_1$$

Device-B Power Drop Simple One-Step Chemical Reaction Leading to Failure (continued)

- Solution to differential equations:

$$
\begin{aligned}
A_1(t) &= A_1(0)\exp(-k_1 t) \\
A_2(t) &= A_2(0) + A_1(0)[1 - \exp(-k_1 t)]
\end{aligned}
$$

  where $A_1(0)$ and $A_2(0)$ are initial conditions.

- If $A_2(0) = 0$, then $\mathcal{D}_\infty = \lim_{t\to\infty} A_2(t) = A_1(0)$ and the solution for $A_2(t)$ (the function of primary interest) can be reexpressed as

$$
\begin{aligned}
A_2(t) &= A_1(t)[1 - \exp(-k_1 t)] \\
\mathcal{D}(t) &= \mathcal{D}_\infty\,[1 - \exp\,(-\mathcal{R}\,t)]
\end{aligned}
$$

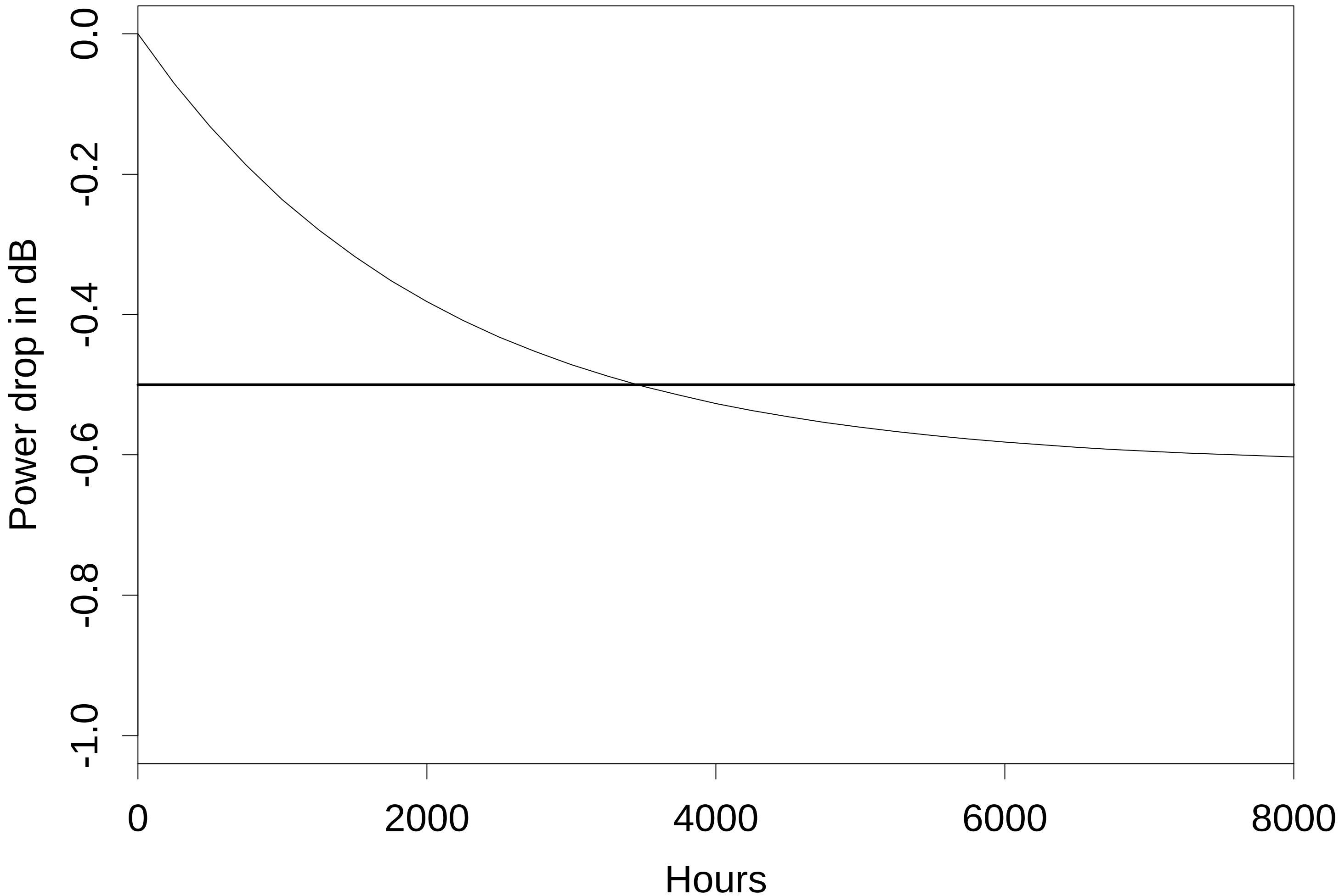
  where $\mathcal{D}(t) = A_2(t)$ is the degradation at time t and $\mathcal{R} = k_1$ is the reaction rate.

- A simple 1-step diffusion process has the same solution.

Device-B Power Drop

$$\mathcal{D}(t) = \mathcal{D}_{\infty}[1 - \exp(-\mathcal{R}\, t)]$$

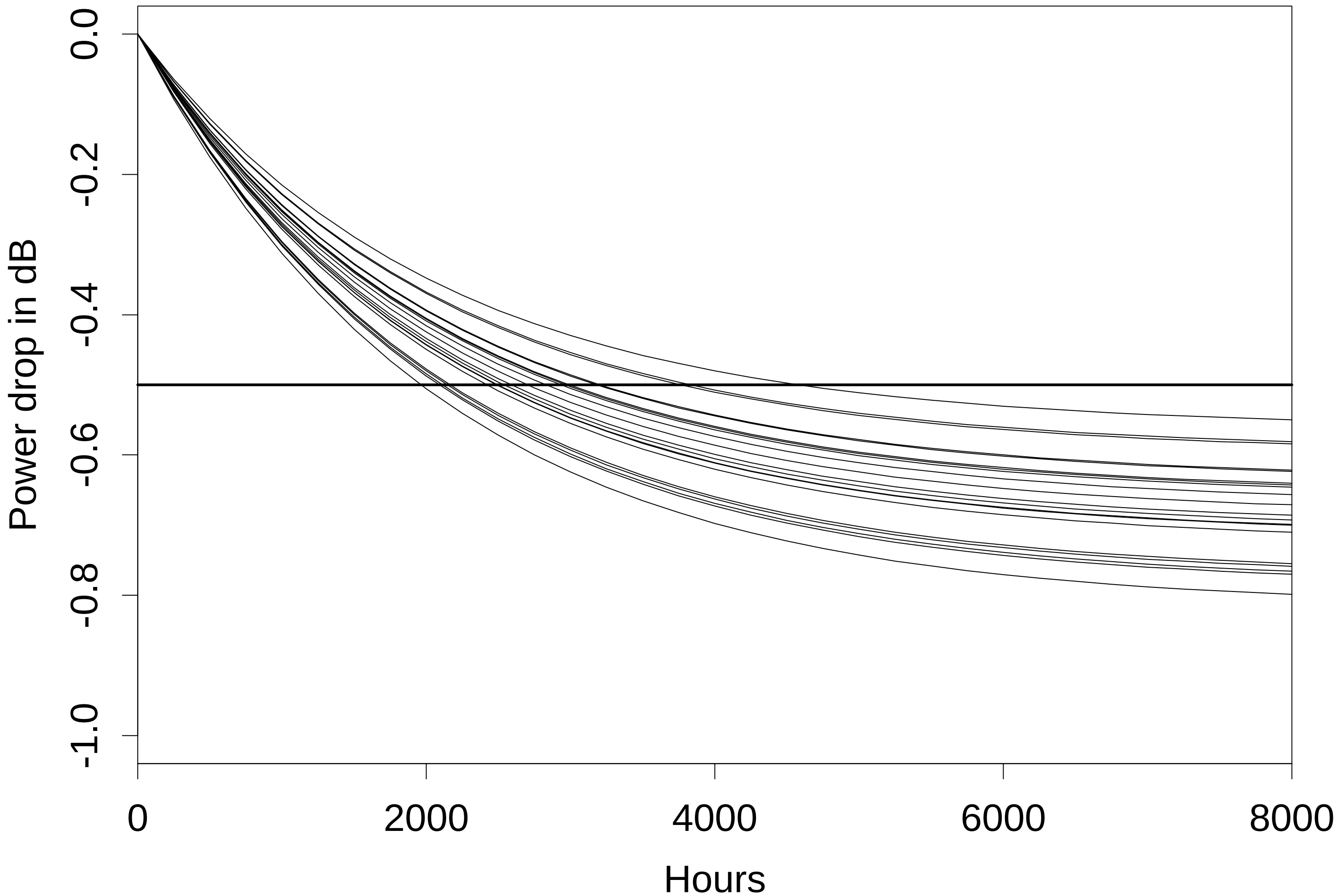
Fixed $\mathcal{D}_{\infty}$ and Rate $\mathcal{R}$



Device-B Power Drop

$$\mathcal{D}(t) = \mathcal{D}_{\infty}[1 - \exp(-\mathcal{R}\,t)]$$

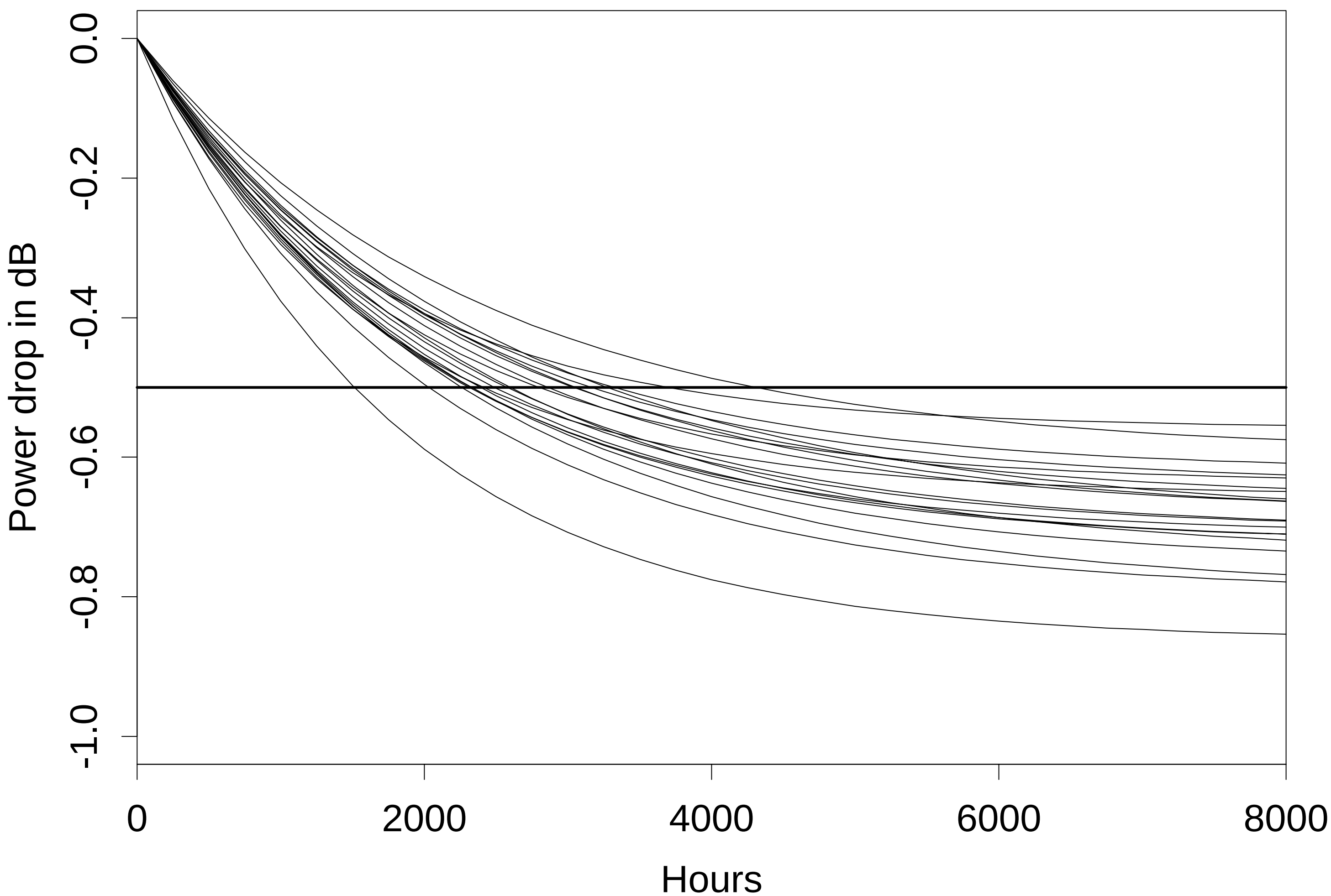
Variability in Asymptote $\mathcal{D}_{\infty}$



Device-B Power Drop

$$\mathcal{D}(t) = \mathcal{D}_{\infty}[1 - \exp(-\mathcal{R}\, t)]$$

Variability in Asymptote $\mathcal{D}_{\infty}$ and Rate $\mathcal{R}$



Model for Degradation Data

- **Actual degradation path model:** Actual path of unit ith at time t_{ij} is

$$\mathcal{D}_{ij} = \mathcal{D}(t_{ij}, \beta_{1i}, \ldots, \beta_{ki})$$

- **Path parameters:** $\beta_{1i}, \ldots, \beta_{ki}$ may be random from unit-to-unit or fixed in the population/process.

- **Sample path model:** Sample degradation path of unit ith at t_{ij} (the jth inspection time for unit i) is

$$y_{ij} = \mathcal{D}_{ij} + \epsilon_{ij}, \quad \epsilon_{ij} \sim \text{NID}(0, \sigma_\epsilon^2), \quad i = 1, \ldots, n, \quad j = 1, \ldots, m_i.$$

- Can use transformations on the response, time, or random parameters, as suggested by physical/chemical theory, past experience, or the data.

Acceleration of Degradation

- The **Arrhenius** model describing the effect that temperature has on the **rate** of a simple one-step chemical reaction is

$$\mathcal{R}(\texttt{temp}) = \gamma_0 \exp\left(\frac{-E_a}{k_{\text{B}}(\texttt{temp} + 273.15)}\right)$$

  where $\texttt{temp}$ is temperature in $^\circ$C and $k_{\text{B}} = 8.6 \times 10^{-5}$ is Boltzmann's constant in units of electron volts per $^\circ$C.

- The pre-exponential factor γ_0 and the reaction activation energy E_a are characteristics of the product or material.

- The **Acceleration Factor** between $\texttt{temp}$ and $\texttt{temp}_U$ is

$$\mathcal{AF}(\texttt{temp}) = \mathcal{AF}(\texttt{temp}, \texttt{temp}_U, E_a) = \frac{\mathcal{R}(\texttt{temp})}{\mathcal{R}(\texttt{temp}_U)}$$

  When $\texttt{temp} > \texttt{temp}_U$, $\mathcal{AF}(\texttt{temp}, \texttt{temp}_U, E_a) > 1$.

Arrhenius Model Temperature Effect on Time to an Event

- Re-expressing the single-step chemical reaction degradation path model to allow for acceleration:

$$\mathcal{D}(t;\texttt{temp}) = \mathcal{D}_{\infty} \times \{1 - \exp\left[-\{\mathcal{R}_U \times \mathcal{AF}(\texttt{temp})\} \times t\right]\}$$

  where $\mathcal{R}_U$ is the rate reaction at $\texttt{temp}_U$.

- Failure defined by $\mathcal{D}(t) < \mathcal{D}_{\mathsf{f}}$.

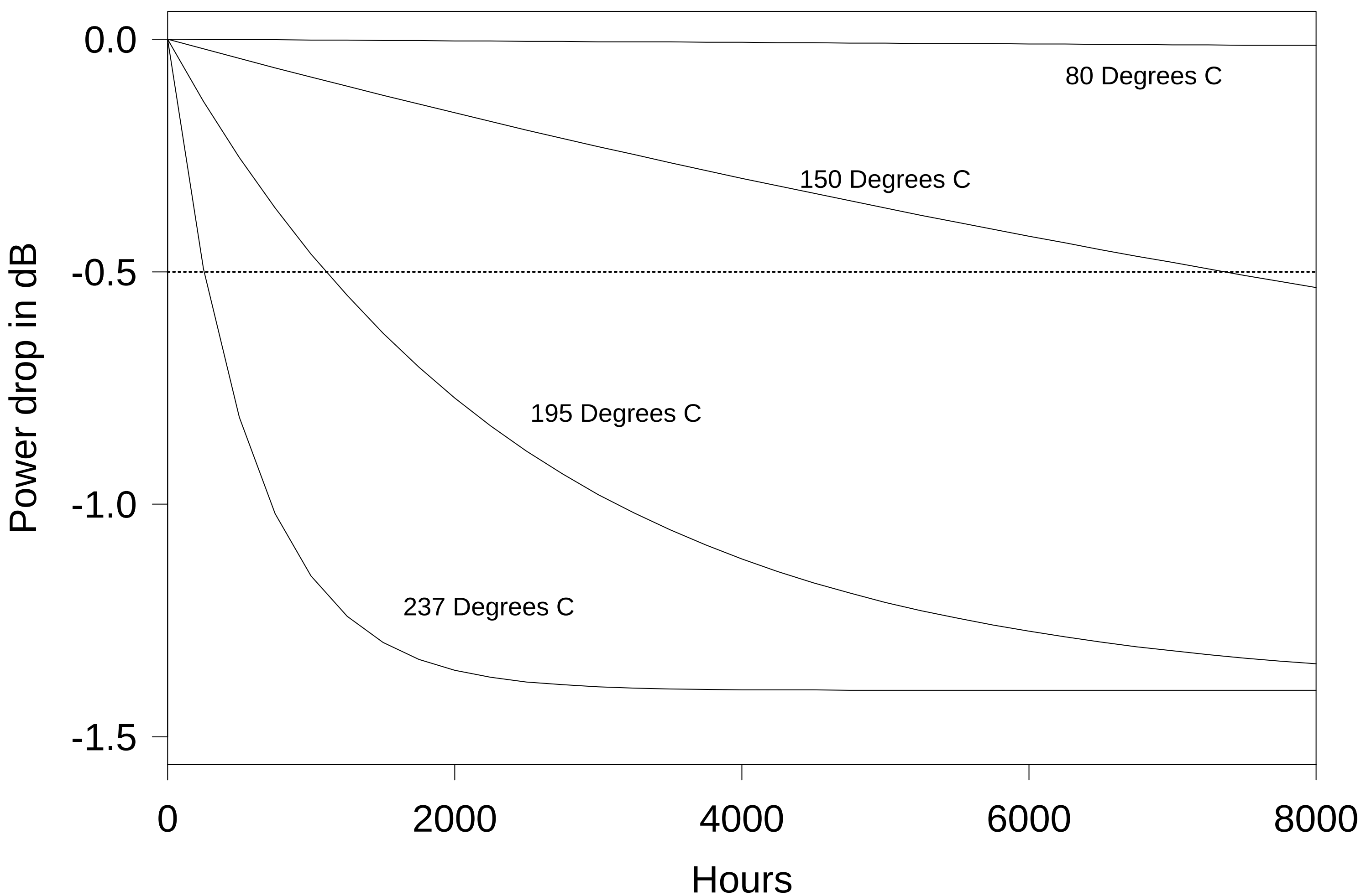
- Equating $\mathcal{D}(T;\texttt{temp})$ to $\mathcal{D}_{\mathsf{f}}$ and solving for T gives the failure time at temperature $\texttt{temp}$ as

$$T(\texttt{temp}) \;=\; \frac{\left[-\frac{1}{\mathcal{R}_U}\log\left(1-\frac{\mathcal{D}_{\mathsf{f}}}{\mathcal{D}_{\infty}}\right)\right]}{\mathcal{AF}(\texttt{temp})} = \frac{T(\texttt{temp}_U)}{\mathcal{AF}(\texttt{temp})}$$

- Thus the simple degradation process induces a Scale Accelerated Failure Time (SAFT) model.

Illustration of the Effect of Arrhenius Temperature Dependence on the Degradation Caused by a Single-Step Chemical Reaction

$$\mathcal{D}(t;\texttt{temp}) = \mathcal{D}_\infty \times \{1 - \exp\left[-\{\mathcal{R}_U \times \mathcal{AF}(\texttt{temp})\} \times t\right]\}$$



Device-B Power Drop
Degradation Model and Parameters

- **Basic parameters:** $\mathcal{R}_U = \mathcal{R}(80)$, $\mathcal{D}_\infty$, E_a.

- **Estimation parameters:**
  $\beta_1 = \log[\mathcal{R}(195)]$, $\beta_2 = \log(-\mathcal{D}_\infty)$, and $\beta_3 = E_a$.

- Assume that (β_1, β_2) follow a bivariate normal distribution.

- Assume that activation energy $\beta_3 = E_a$ is a fixed (but unknown) characteristic of Device-B.

- **Variability in path model parameters:** $(\beta_1, \beta_2, \beta_3) \sim \mathrm{MVN}(\boldsymbol{\mu}_{\boldsymbol{\beta}}, \Sigma_{\boldsymbol{\beta}})$ [but $\mathrm{Var}(\beta_3)=0$].

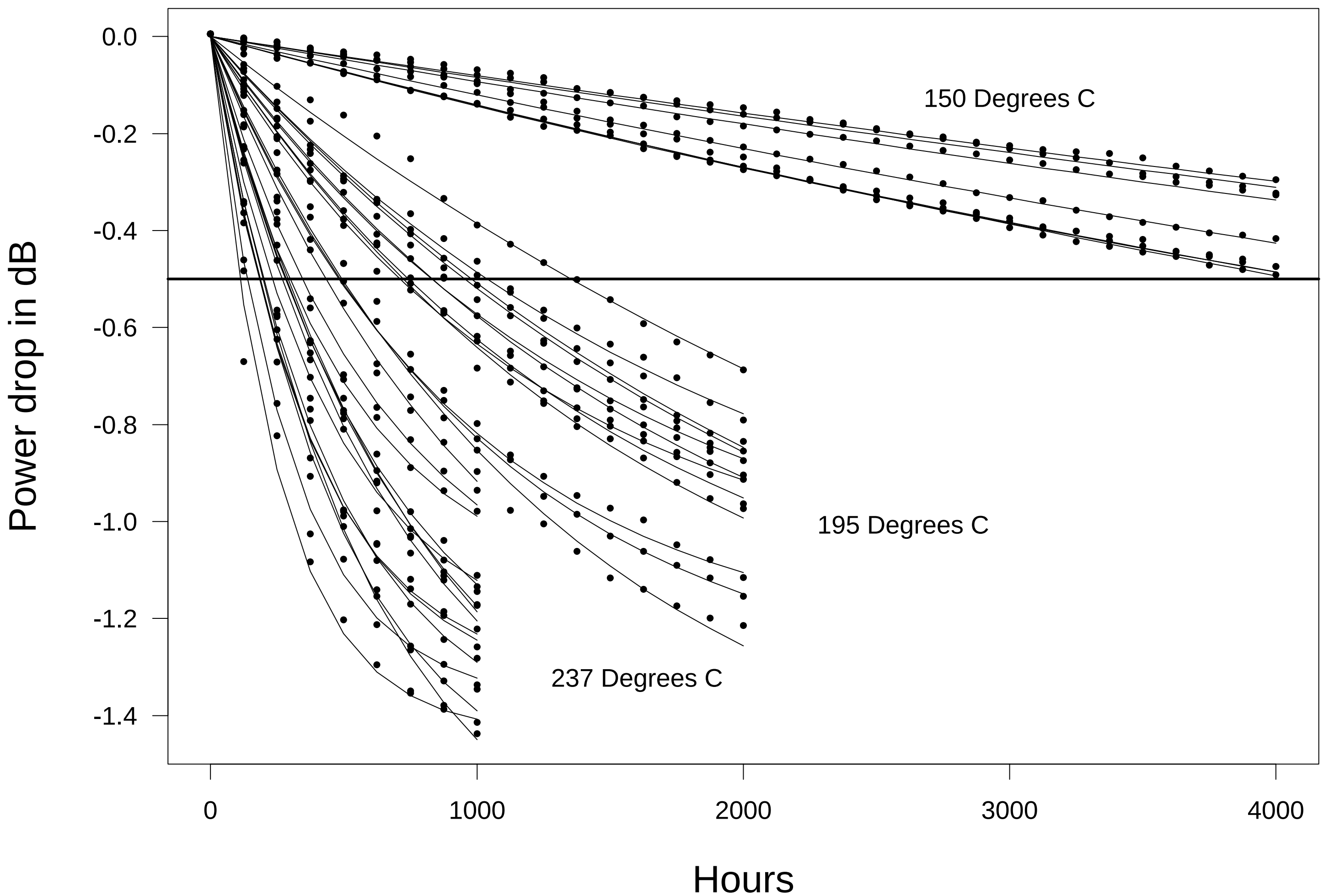
Device-B Power Drop Data
Approximate ML Estimates
(Computed with Program of Pinheiro and Bates 1995)

$$\widehat{\boldsymbol{\mu}}_{\boldsymbol{\beta}} = \begin{pmatrix} -7.572 \\ .3510 \\ .6670 \end{pmatrix}, \qquad \widehat{\Sigma}_{\boldsymbol{\beta}} = \begin{pmatrix} .15021 & -.02918 & 0 \\ -.02918 & .01809 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$\widehat{\sigma}_{\epsilon} = .0233$,

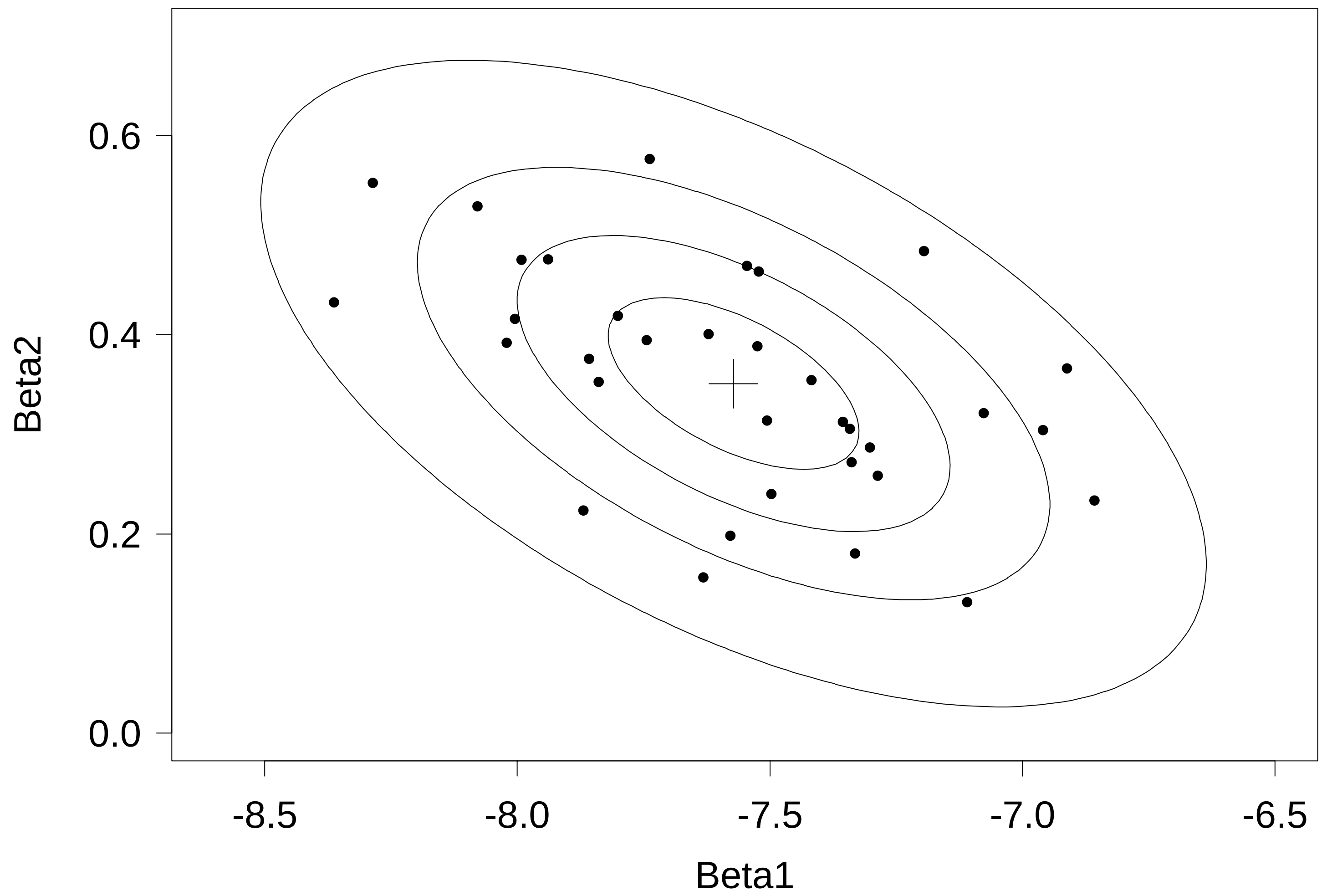
Loglikelihood = 1201.8.

Device-B Power Drop Observations and Fitted Degradation Model for the $i = 1, \ldots, 34$ Sample Paths



Plot of $\beta_1 = \log[\mathcal{R}(195)]$ Versus $\beta_2 = \log(-\mathcal{D}_\infty)$ for the $i = 1, \ldots, 34$ Sample Paths from Device-B

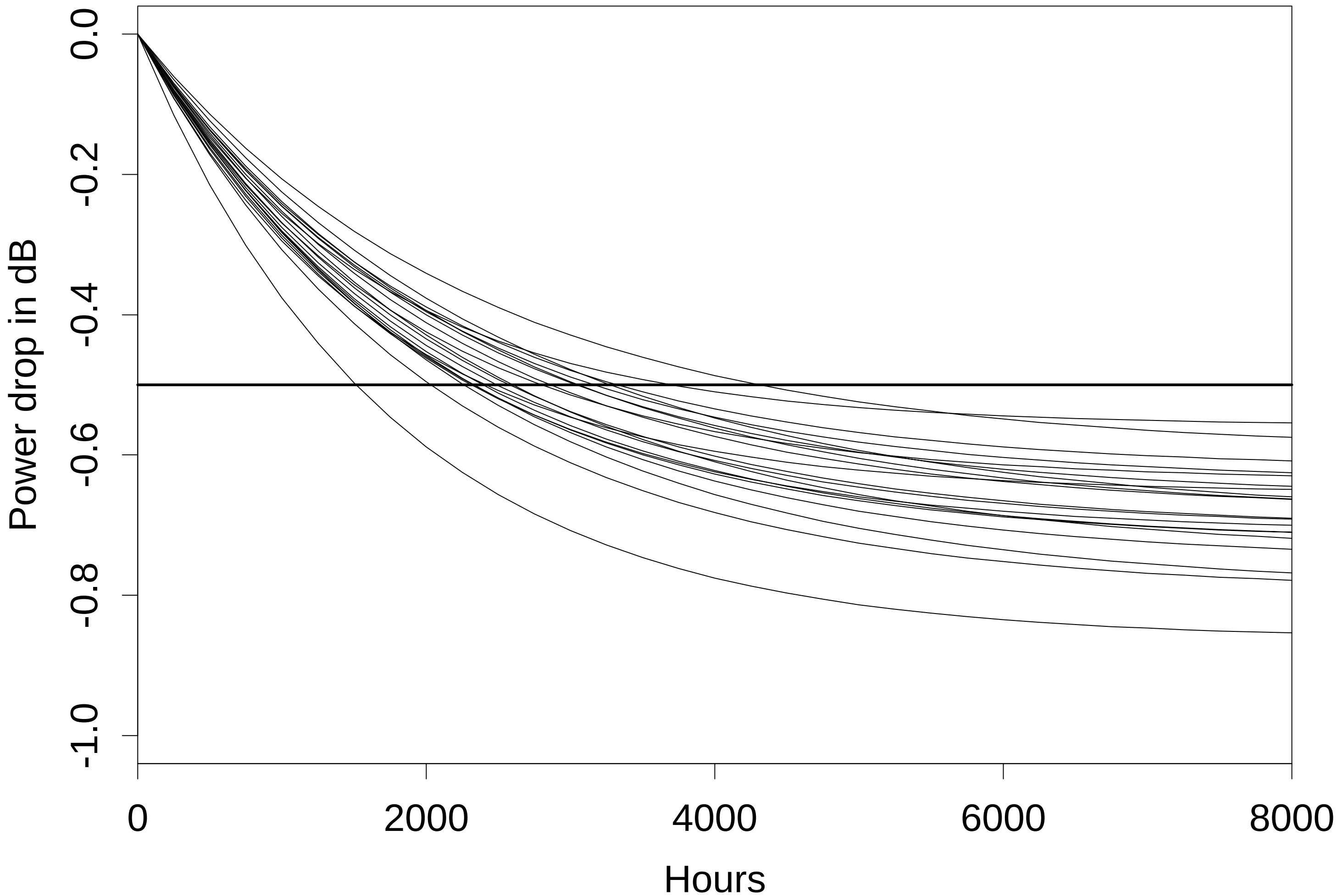
$$\widehat{\rho}_{\beta_1\beta_2} = -.56$$



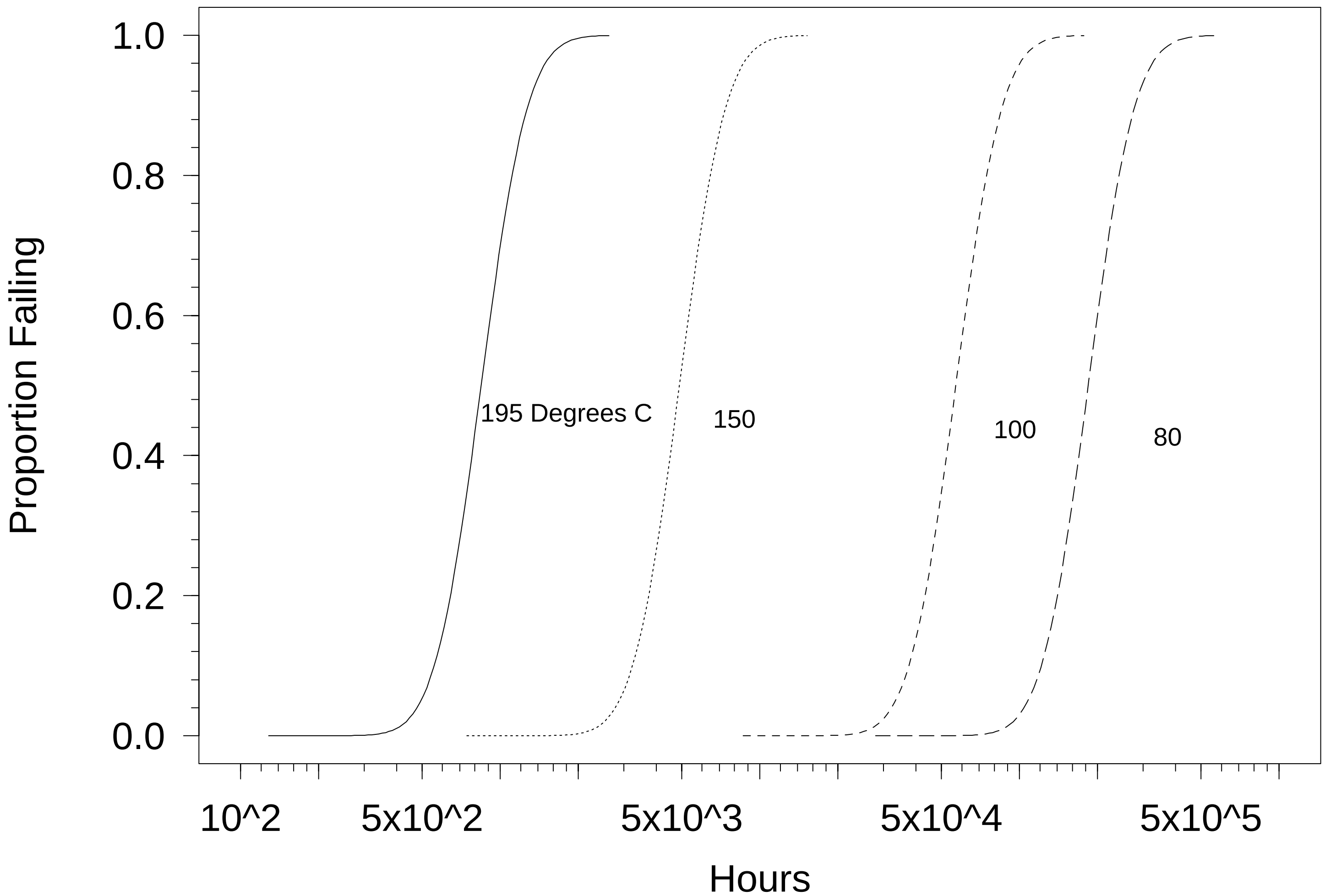
Device-B Power Drop

$$\mathcal{D}(t) = \mathcal{D}_\infty[1 - \exp(-\mathcal{R}\,t)]$$

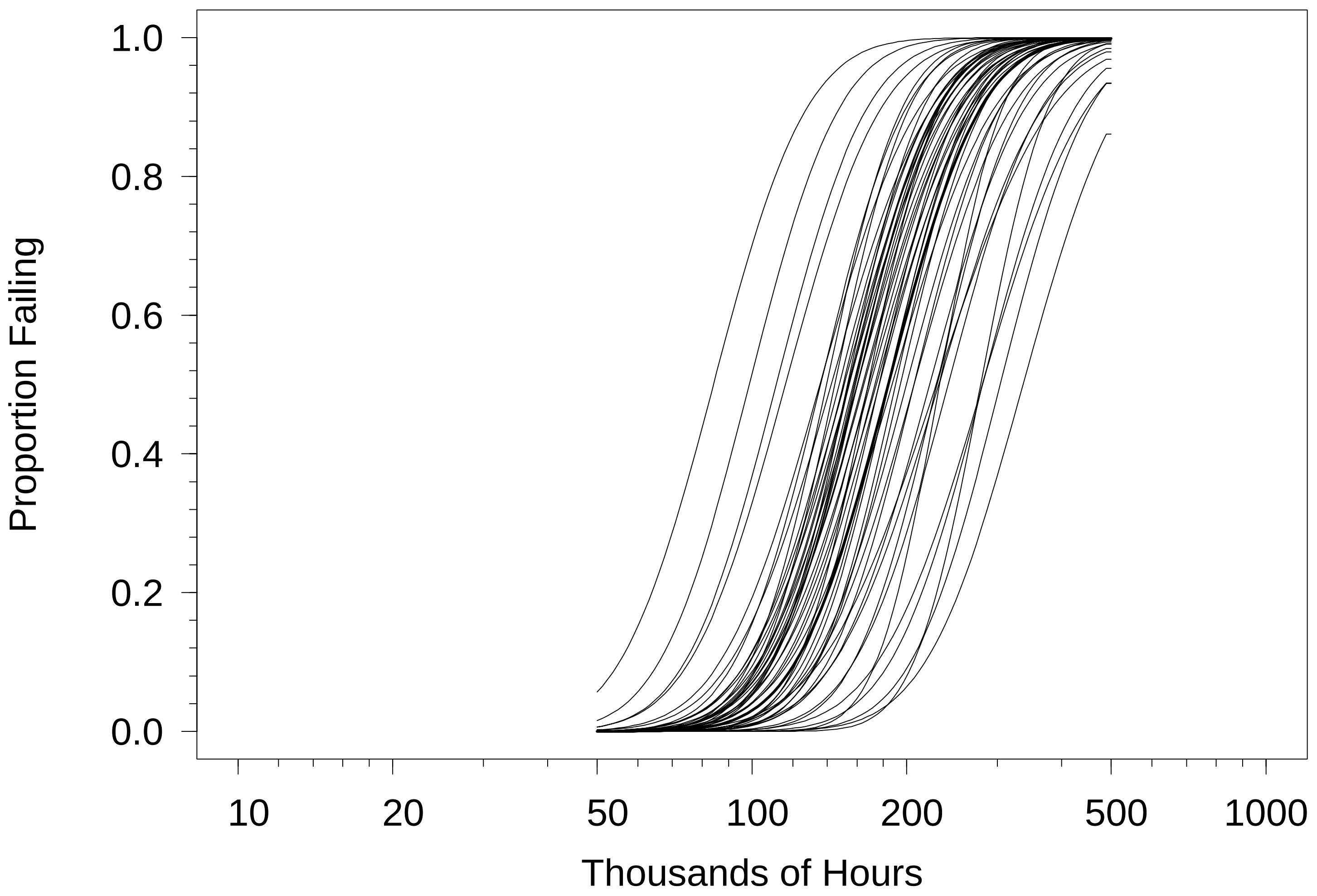
Variability in Asymptote $\mathcal{D}_\infty$ and rate $\mathcal{R}$



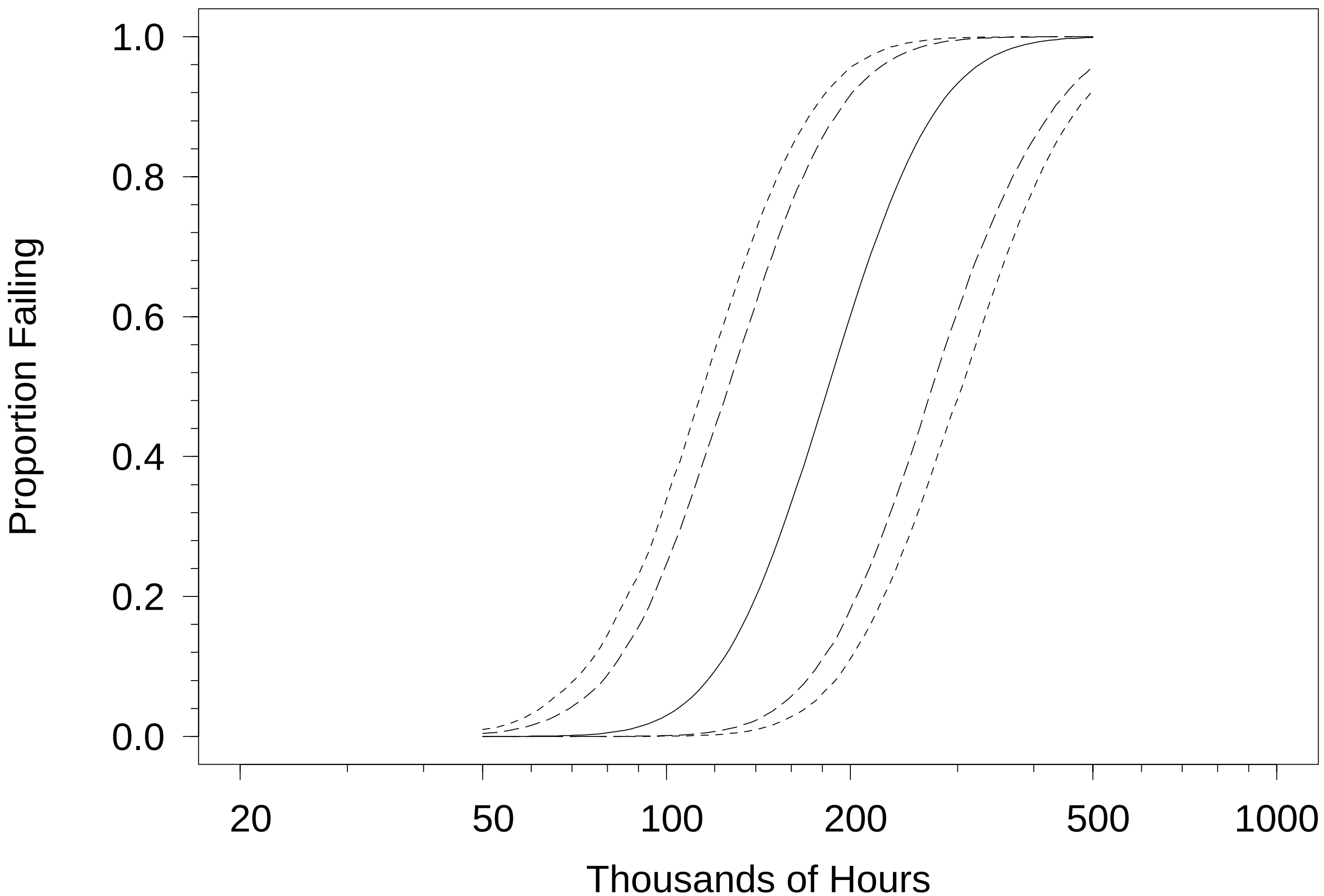
Estimates of the Device-B Life Distributions at 80, 100, 150, and 190°C, Based on the Degradation Data



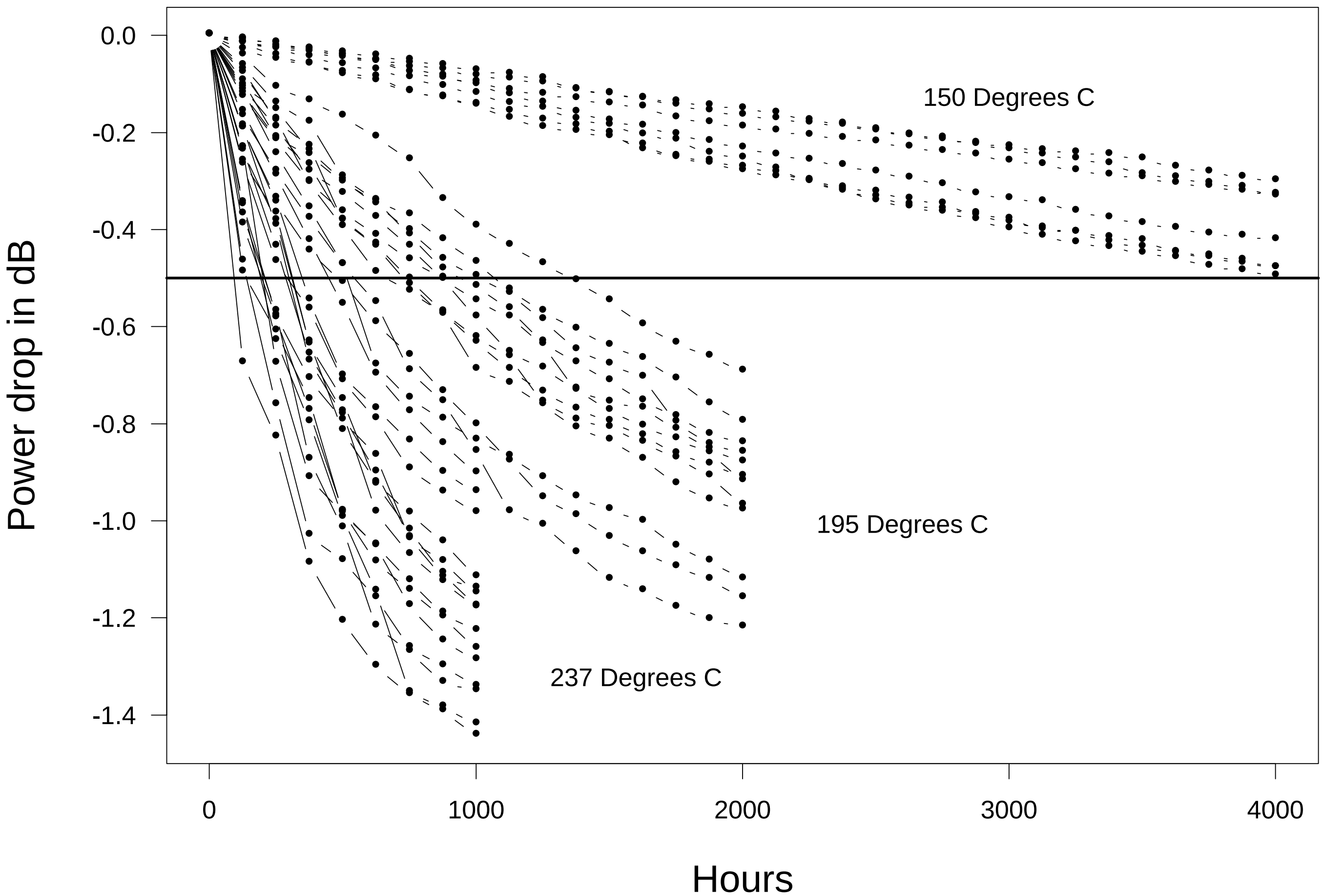
Bootstrap Sample Estimates of $F(t)$ at $80°\mathrm{C}$



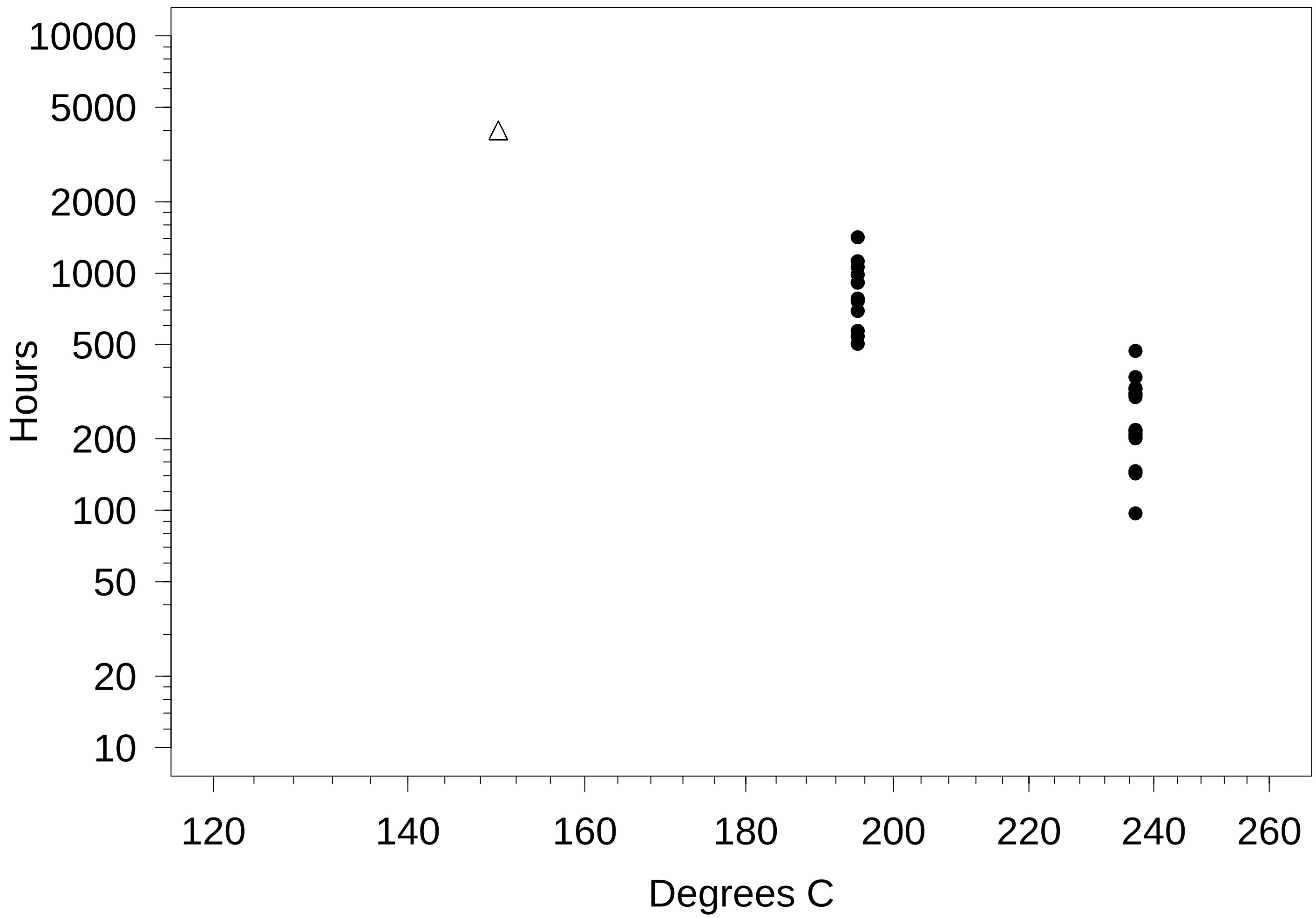
80% and 90% Bias-Corrected Percentile Bootstrap Confidence Intervals for $F(t)$ at 80°C



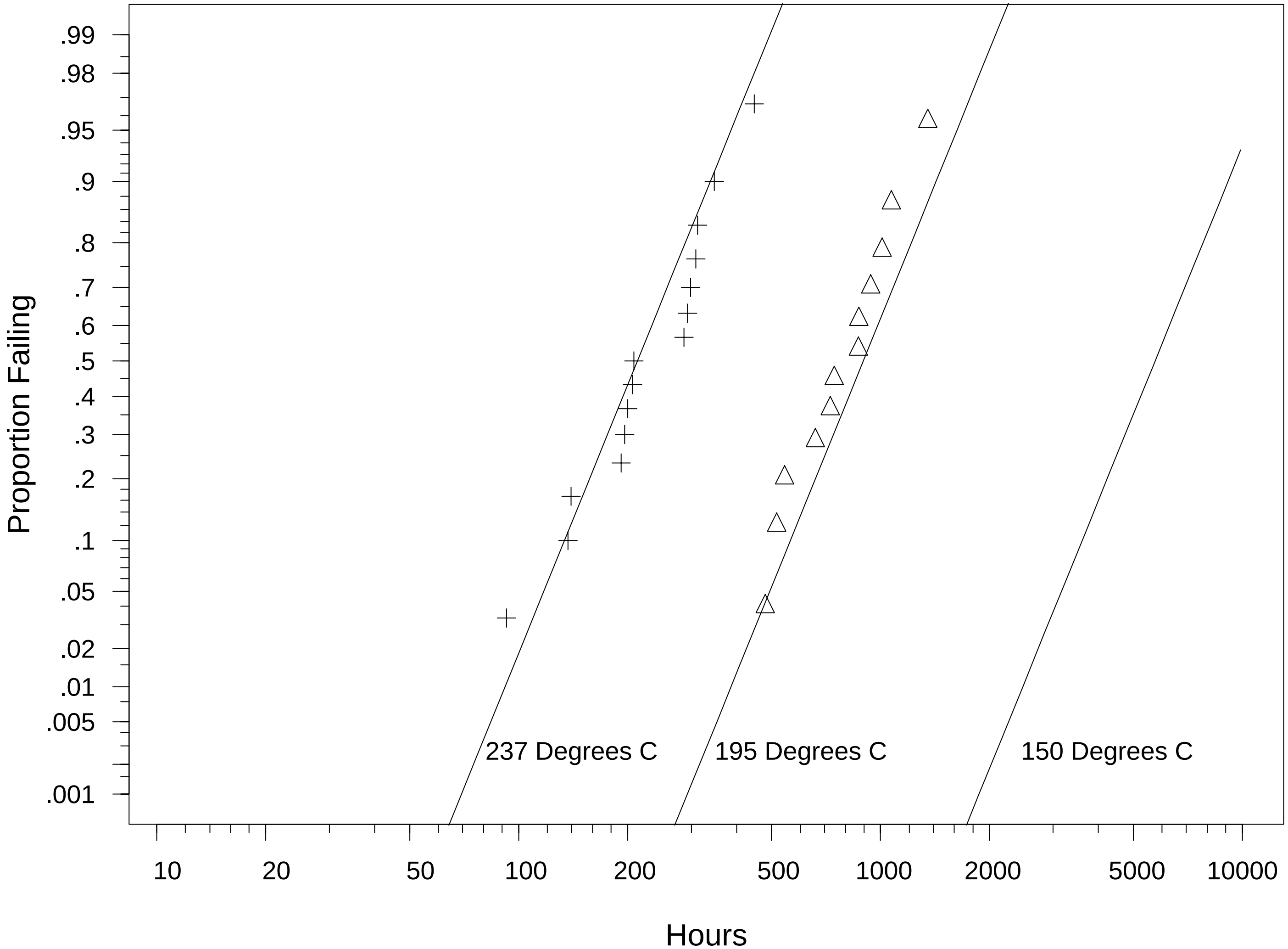
Device-B Power Drop
Accelerated Degradation Test Results
at 150°C, 195°C, and 237°C
(Use Conditions 80°C)



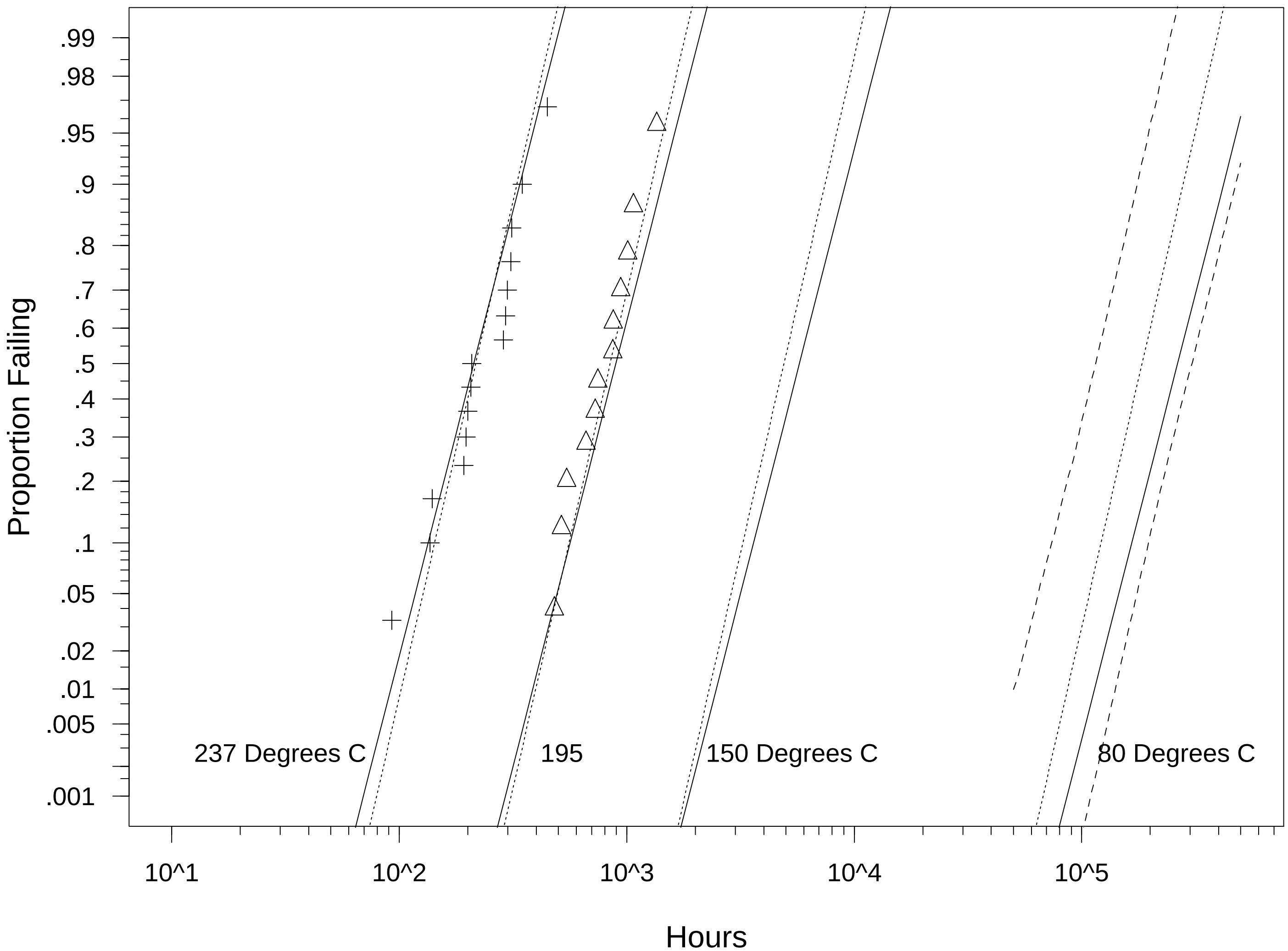
Scatterplot of Device-B Failure-Time Data with Failure Defined as Power Drop Below $-.5$ dB



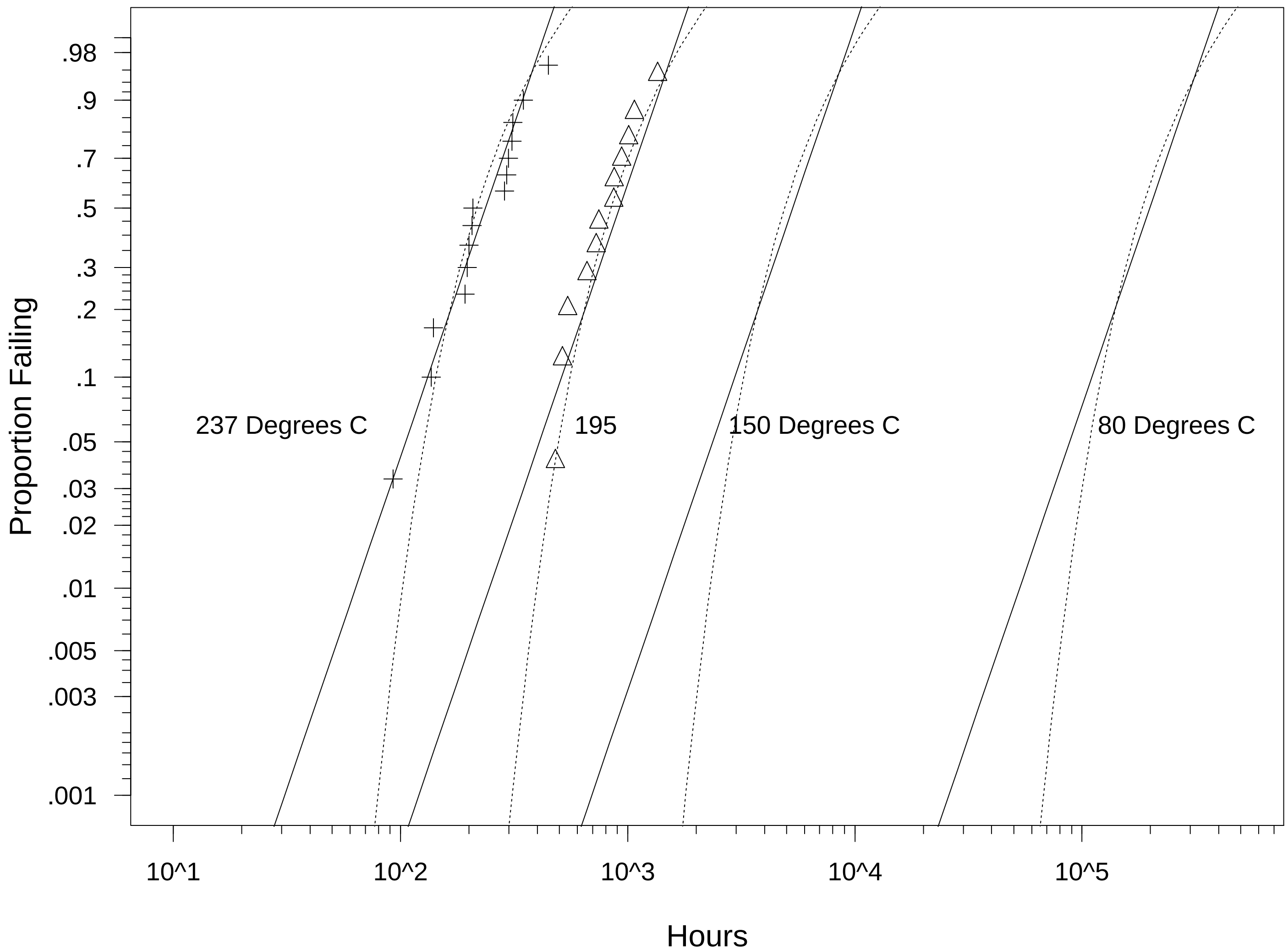
Lognormal-Arrhenius Model Fit to the Device-B Failure-Time Data



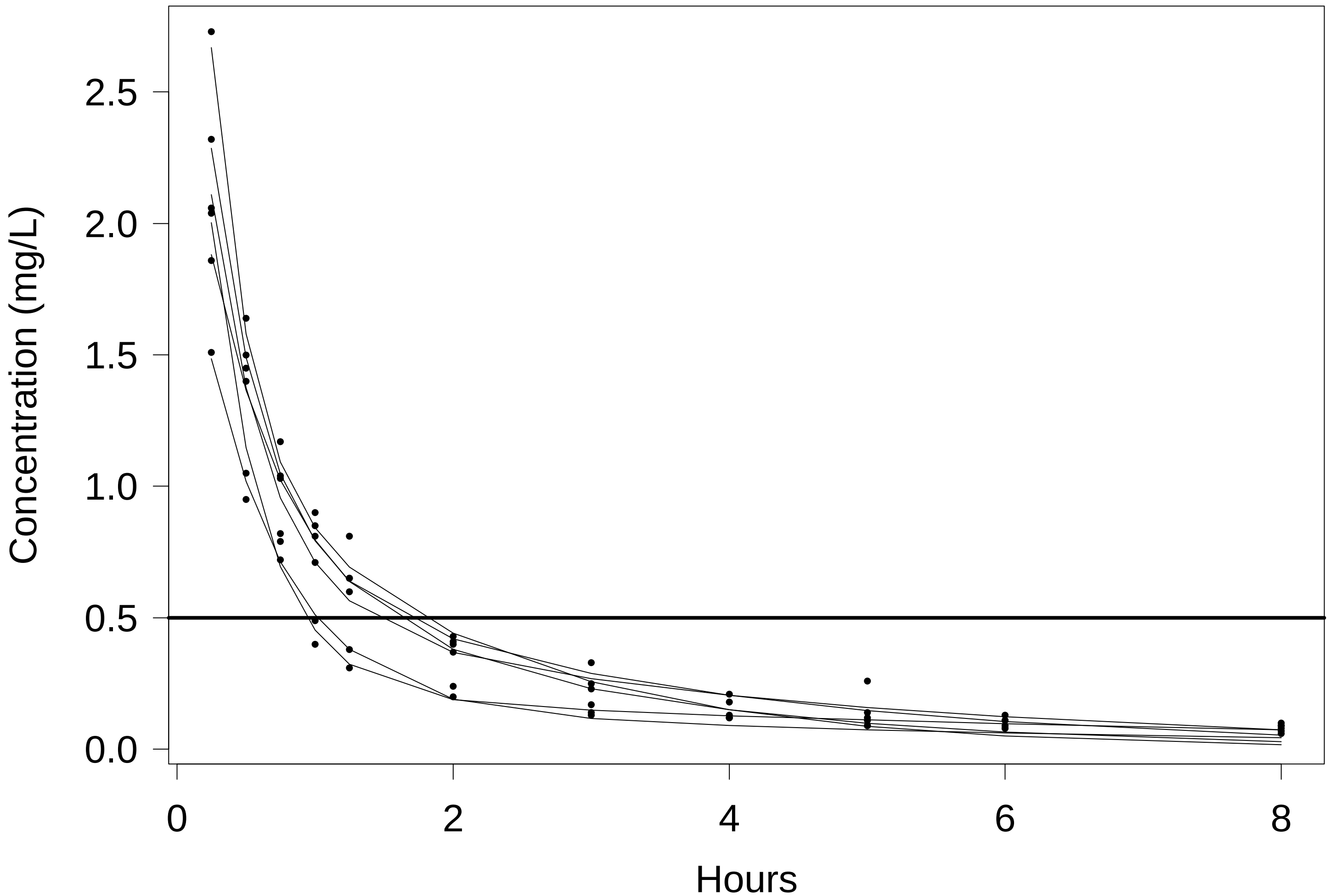
Lognormal-Arrhenius Model Fit to the Device-B Failure-Time Data with Degradation Model Estimates



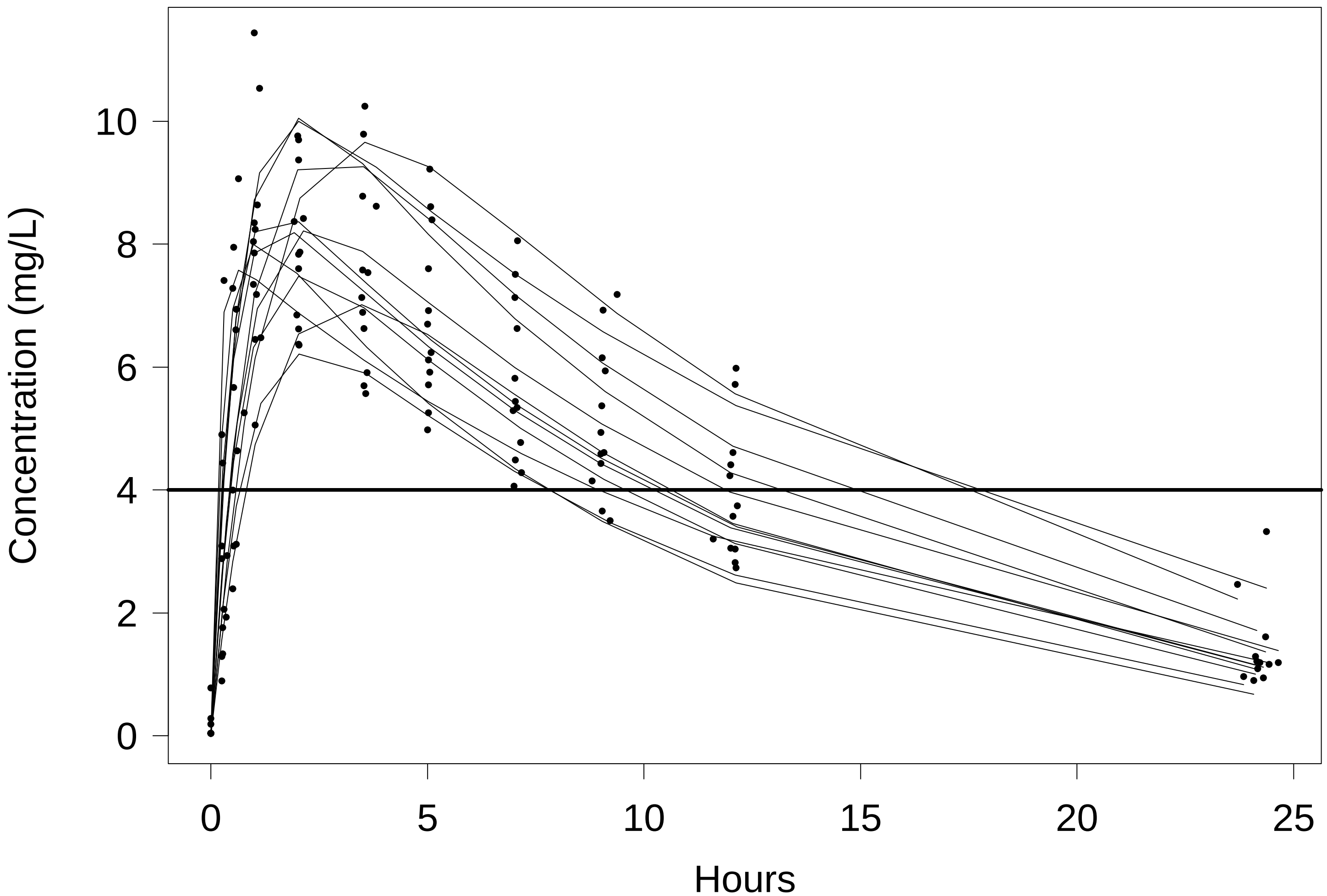
Weibull-Arrhenius Model Fit to the Device-B Failure-Time Data with Degradation Model Estimates



Plasma Concentrations of Indomethicin Following Intravenous Injection
Fitted Biexponential Model



Theophylline Serum Concentrations
Fitted Curves for a First-Order Compartment Model



Approximate Accelerated Degradation Analysis

The simple method for degradation data analysis extends directly to accelerated degradation analysis.

- For each sample path one uses the algorithm described to predict the failure times.
- These data can be analyzed using the methods to analyze ALT data.
- It is important to remember, however, that such an analysis has the same limitations described in for the simple analysis of degradation data.

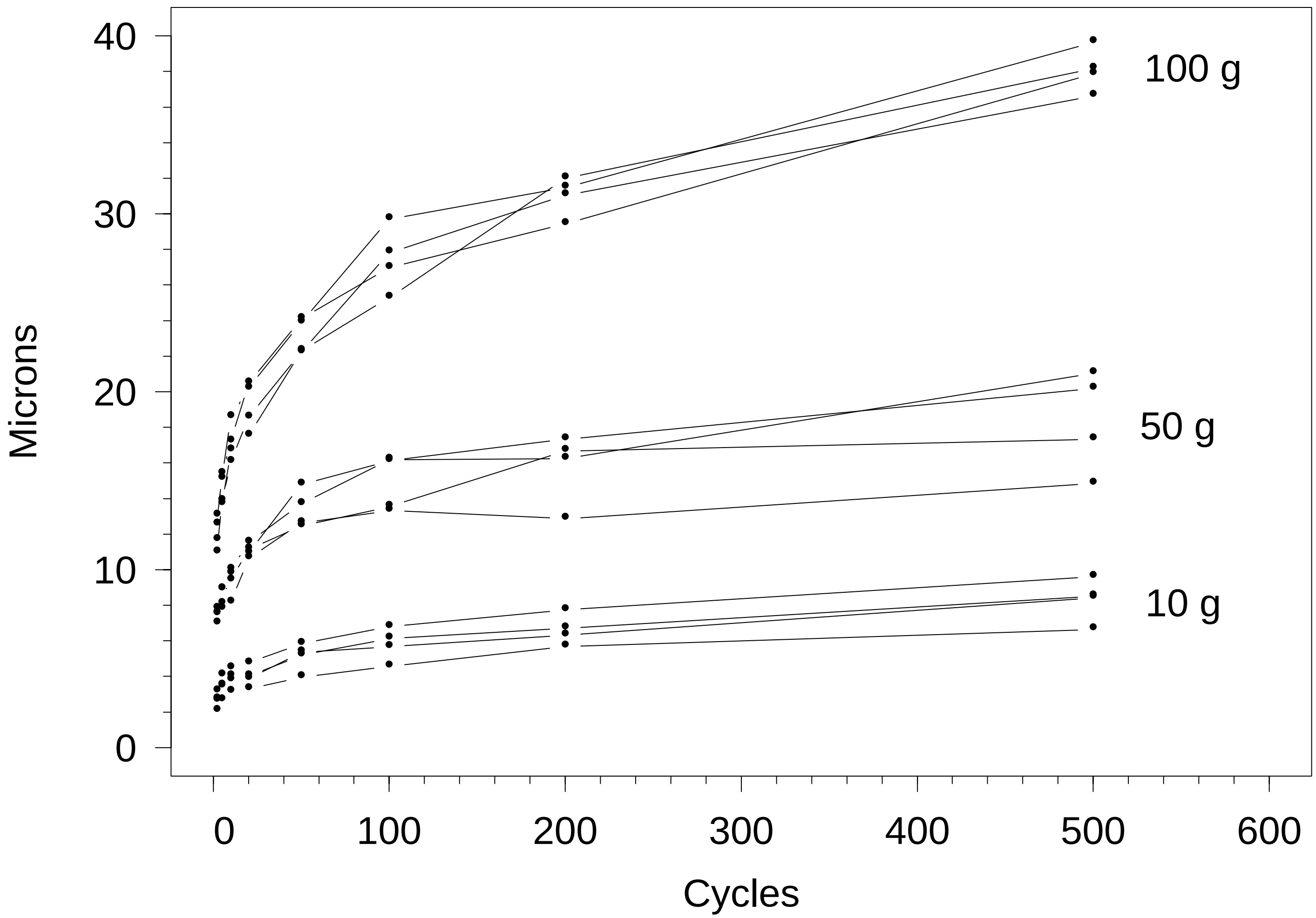
Sliding Metal Wear Data Analysis

- An experiment was conducted to test the wear resistance of a particular metal alloy.

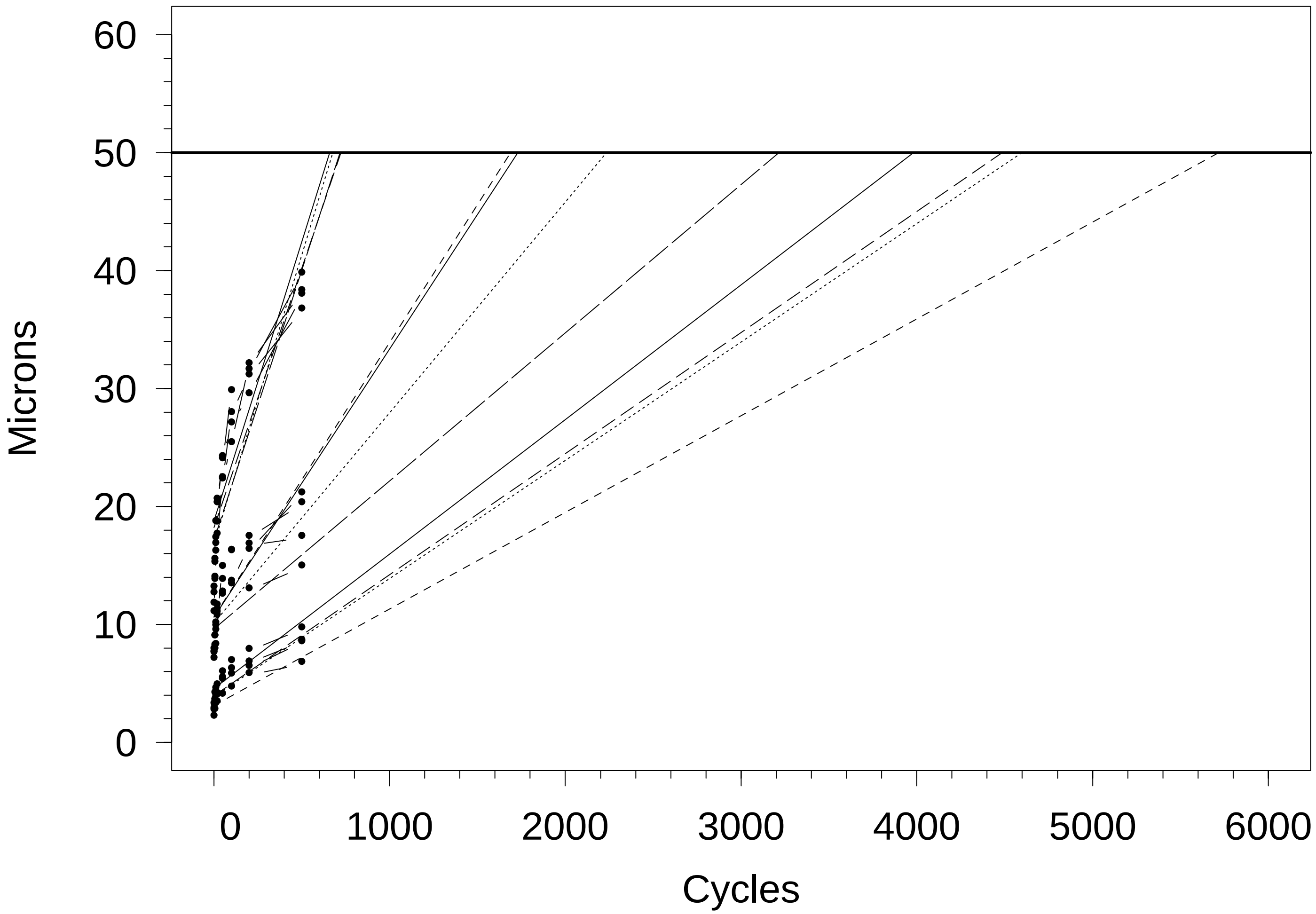
- The sliding test was conducted over a range of different applied weights in order to study the effect of weight and to gain a better understanding of the wear mechanism.

- The predicted pseudo failure times were obtained by using ordinary least squares to fit a line through each sample path on the log-log scale and extrapolating to the time at which the scar width would be 50 microns.

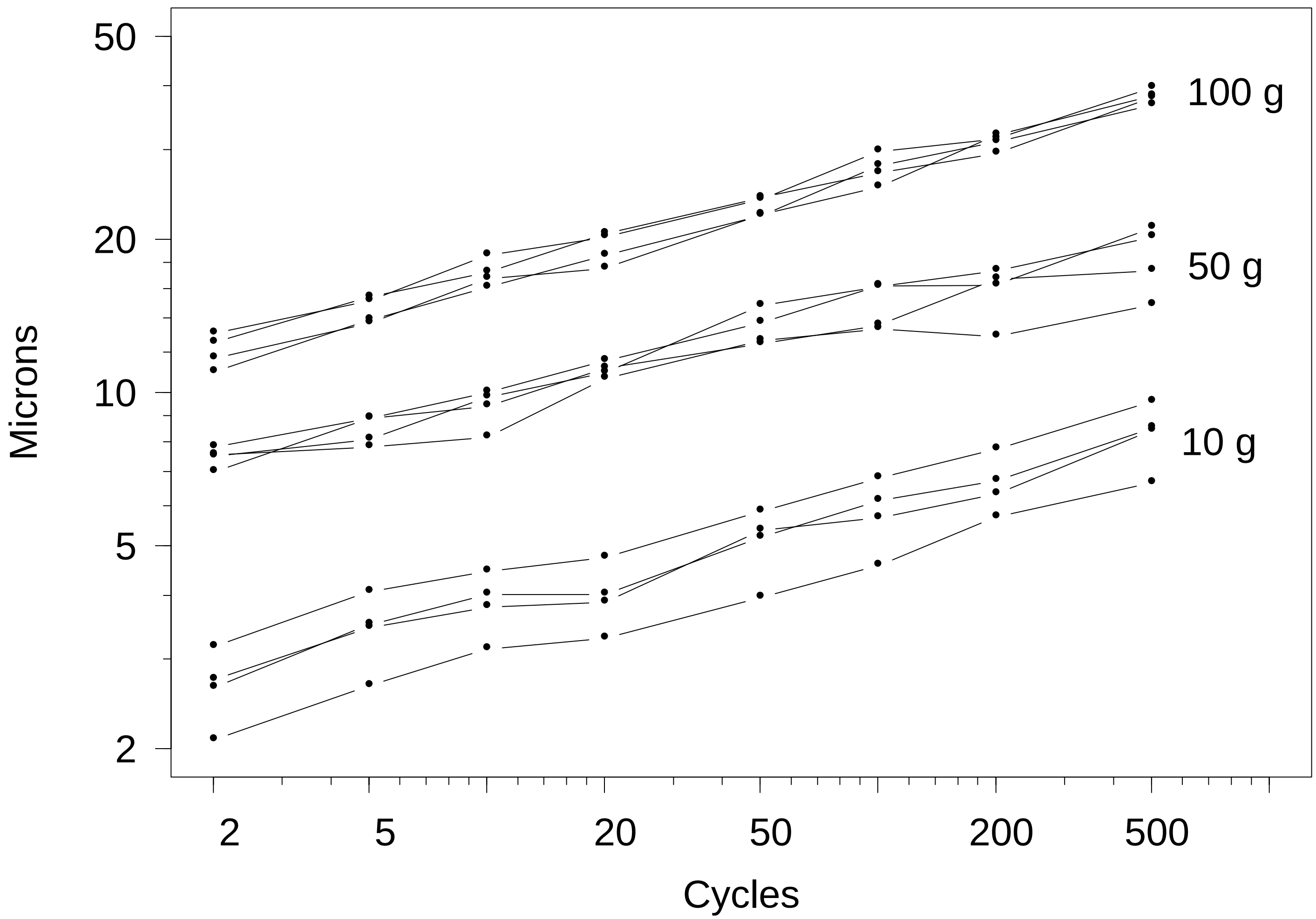
Scar Width Resulting from a Metal-to-Metal Sliding Test for Different Applied Weights



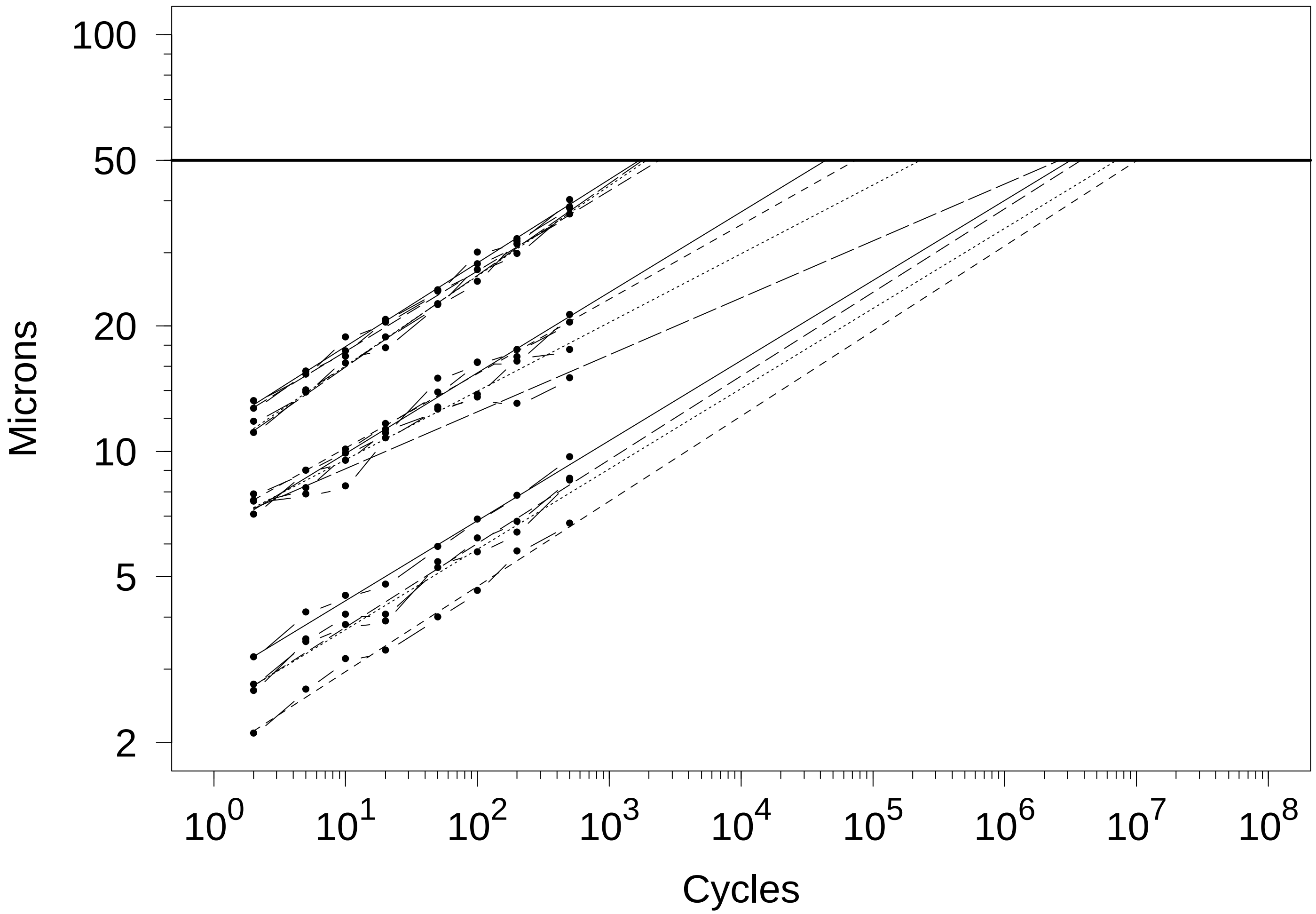
Metal-to-Metal Sliding Test for Different Applied Weights
Extrapolation to Failure Definition
(Using linear regression on linear axes)



Scar Width Resulting from a Metal-to-Metal Sliding Test for Different Applied Weights (Using log-log Axes)



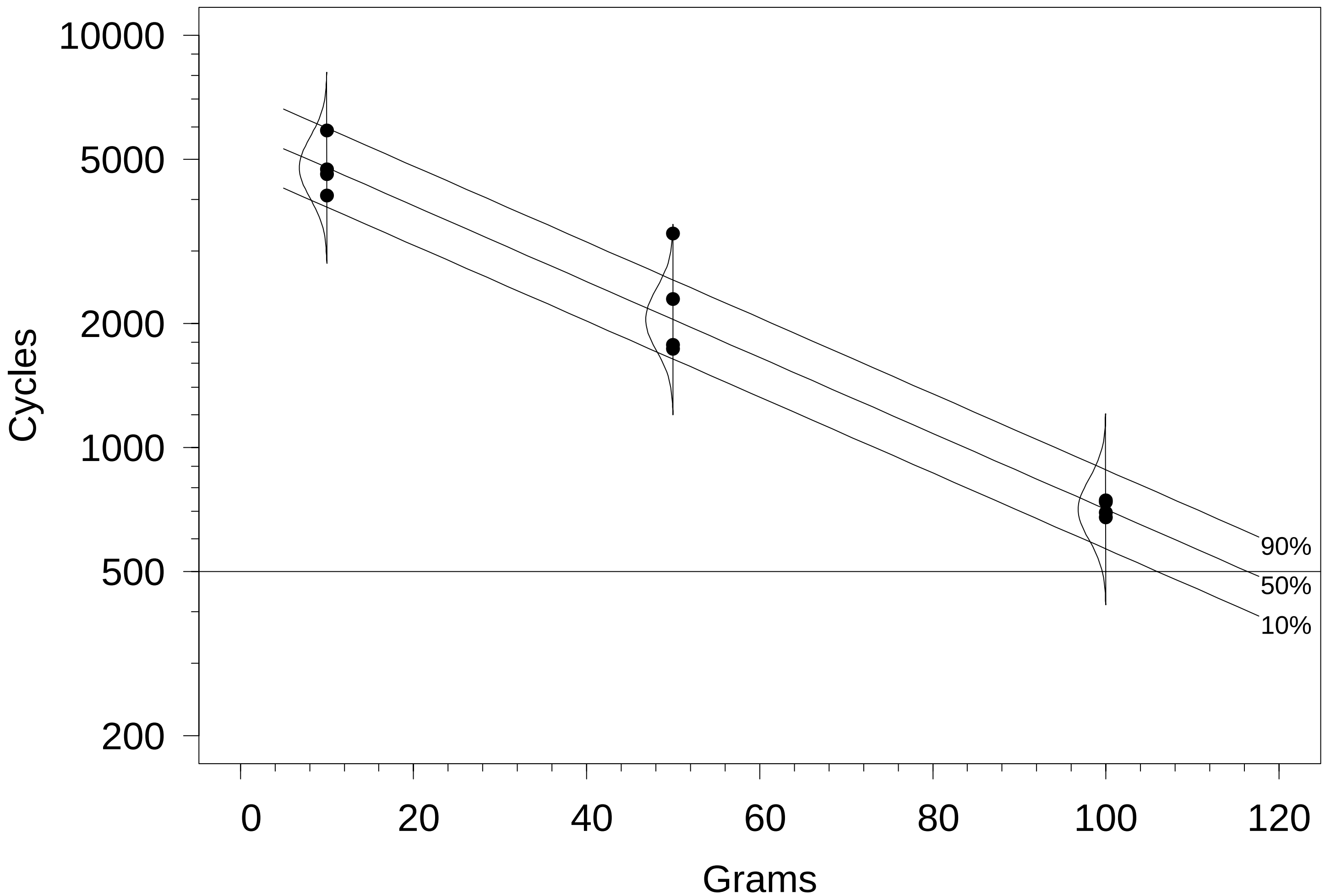
Metal-to-Metal Sliding Test for Different Applied Weights
Extrapolation to Failure Definition
(Using linear regression on log-log axes)



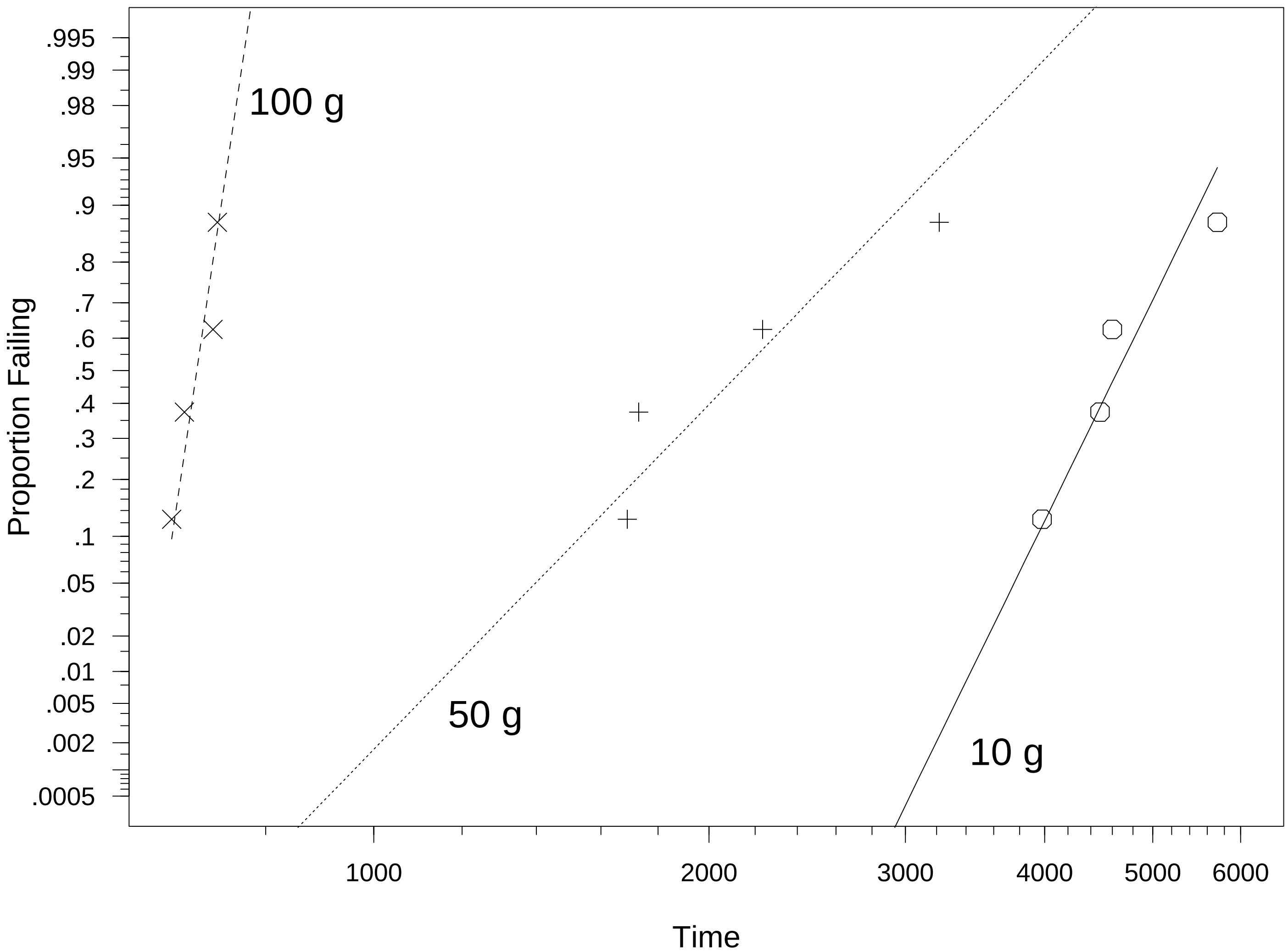
Metal-Wear Failure Times in Hours

Grams	Pseudo Failure Times			
100	724	718	659	677
50	3216	1729	2234	1689
10	3981	4600	5718	4487

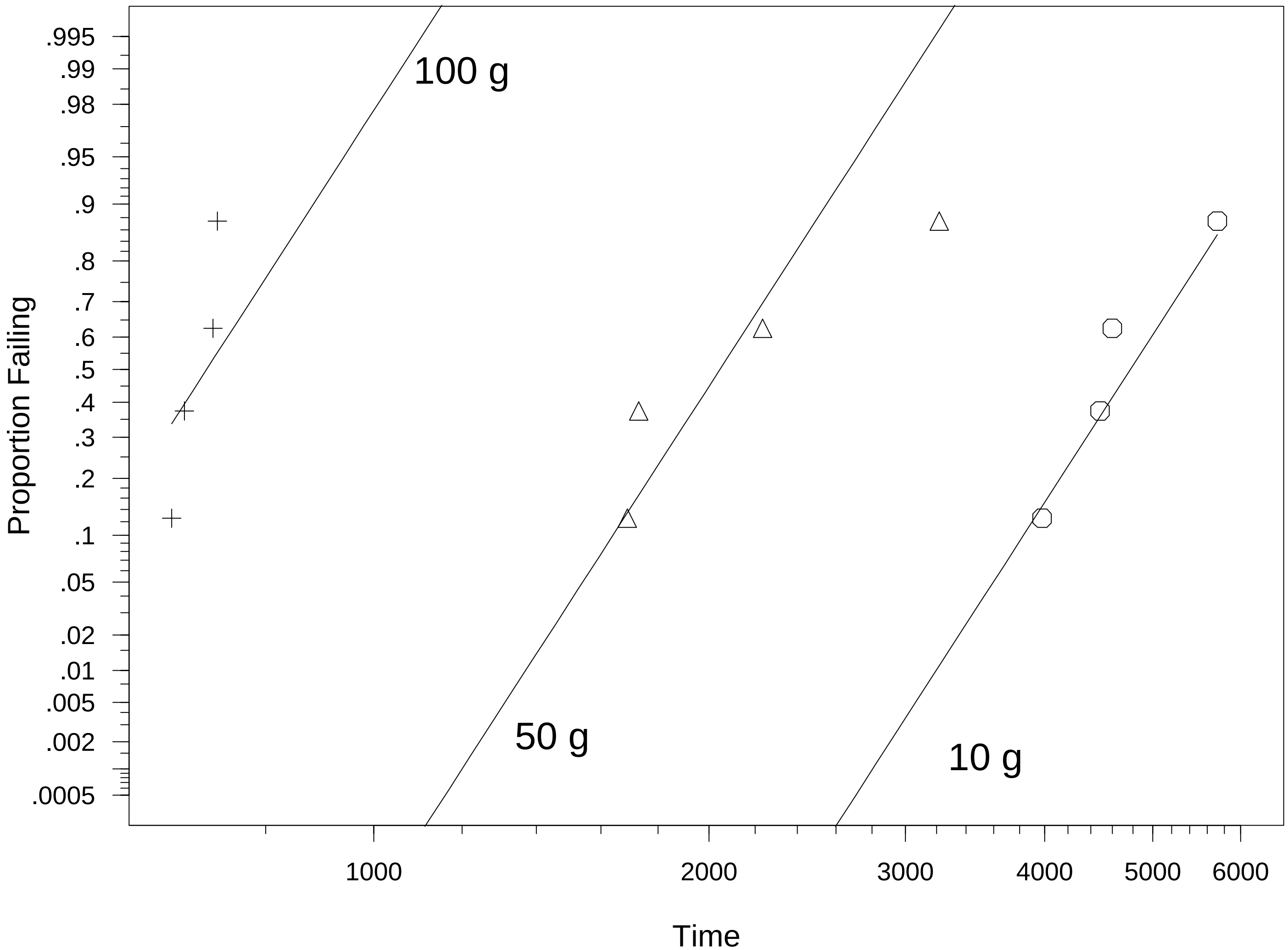
Pseudo Failure Time to 50 Microns Scar Width Versus Applied Weight for the Metal-to-Metal Sliding Test



Lognormal Probability Plot Showing the ML Estimates of Time to 50 Microns Width for Each Weight



Lognormal Probability Plot Showing the Lognormal Regression Model ML Estimates of Time to 50 Microns Width for Each Weight



Other Topics in Chapter 21

- Choice of parameter transformation in the estimation/bootstrap procedure.

- Stochastic process degradation models.

Test planning case study in Chapter 22.