Monotone Grid Drawings of Planar Graphs

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Bangladesh University of Engineering and Technology
Dhaka, Bangladesh

International Frontiers of Algorithmics Workshop (FAW) 2014

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LAKSHMIPUR
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NOAKHALI
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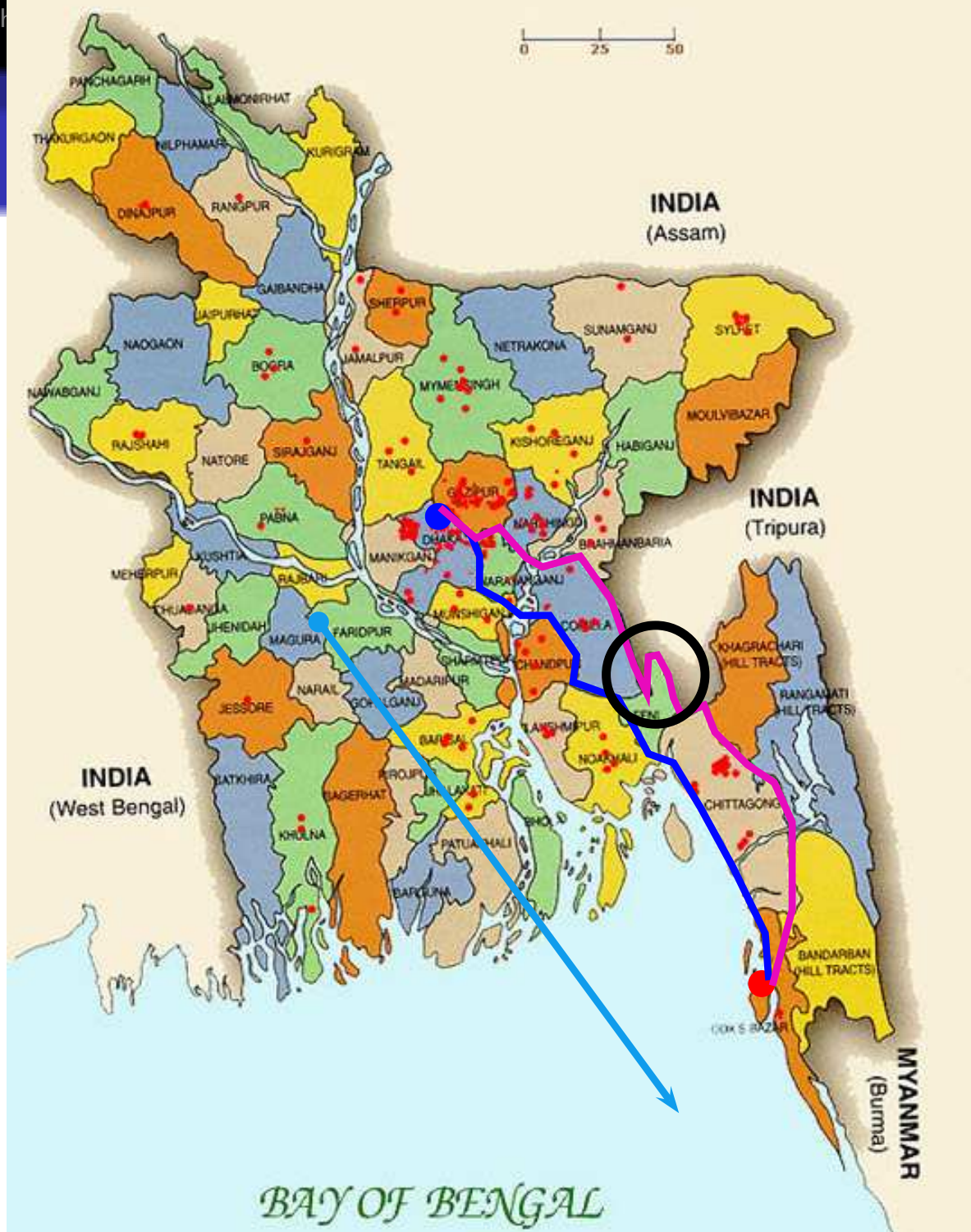
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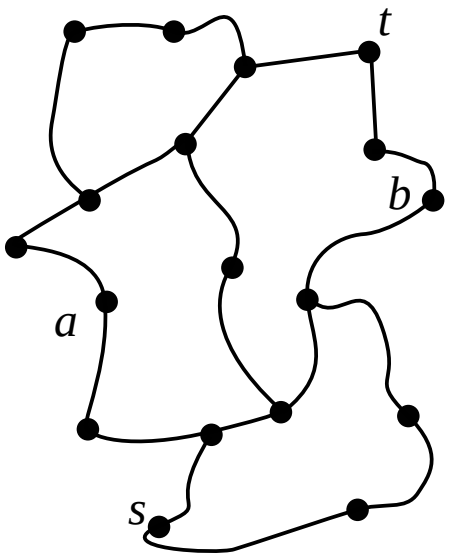
Bangladesh

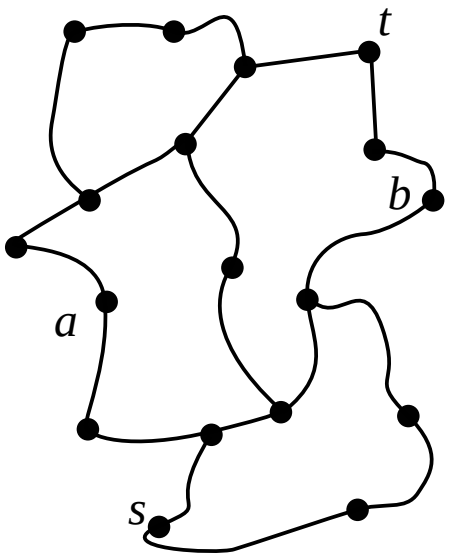
Does a good route exist between every two cities?

Monotone Grid Drawings



Input
Planar Graph G

Monotone Grid Drawings



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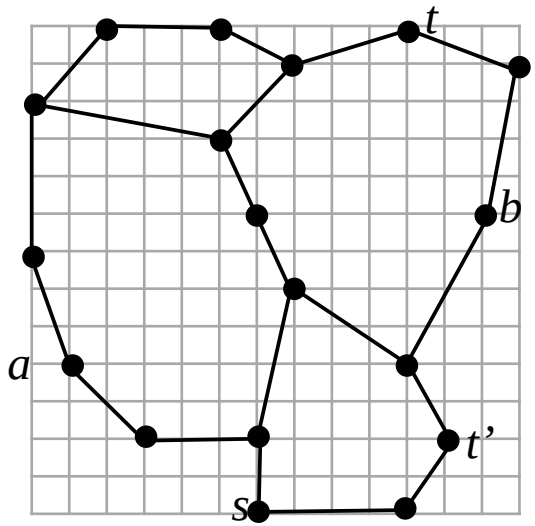

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Monotone Grid Drawing of G

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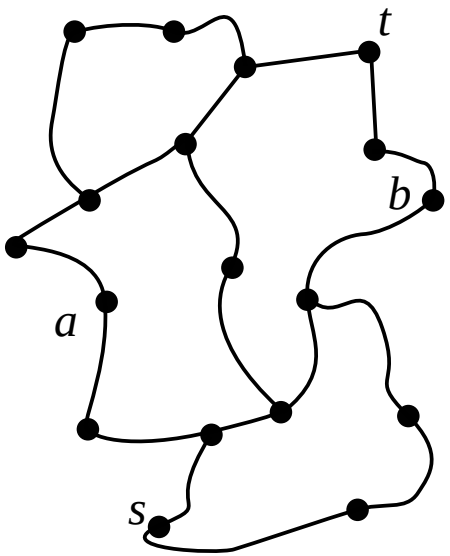

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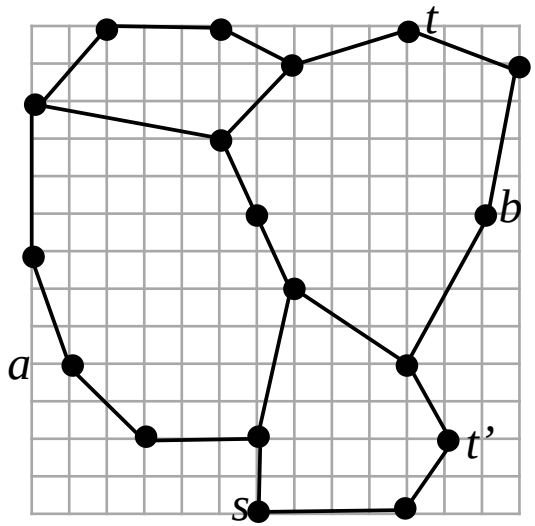

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- Each vertex is drawn at a grid point on an integer grid.
- Each edge is drawn as a straight-line segment without edge crossings.
- At least a monotone path exists between every pair of vertices.

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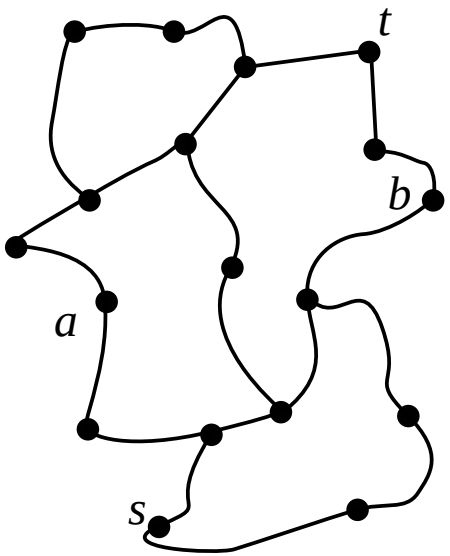

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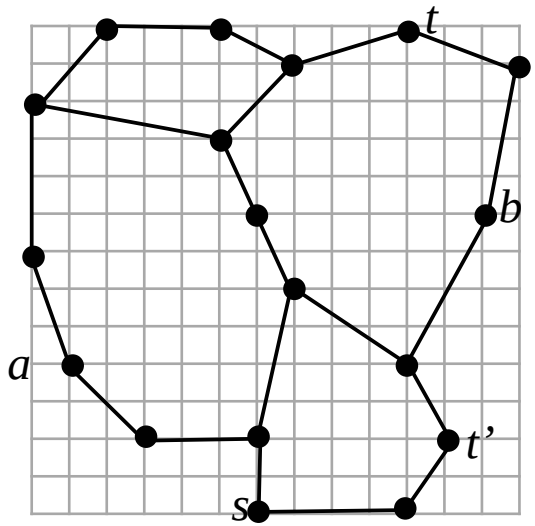

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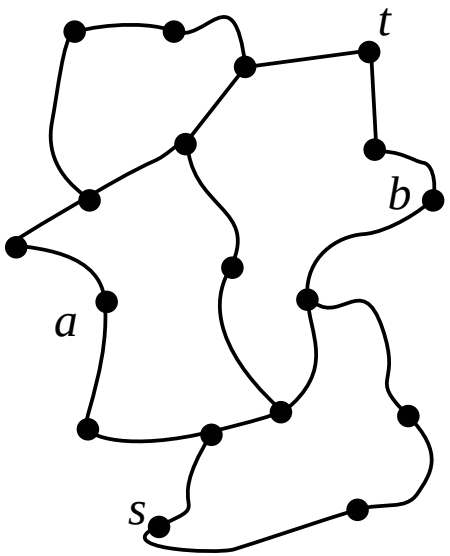

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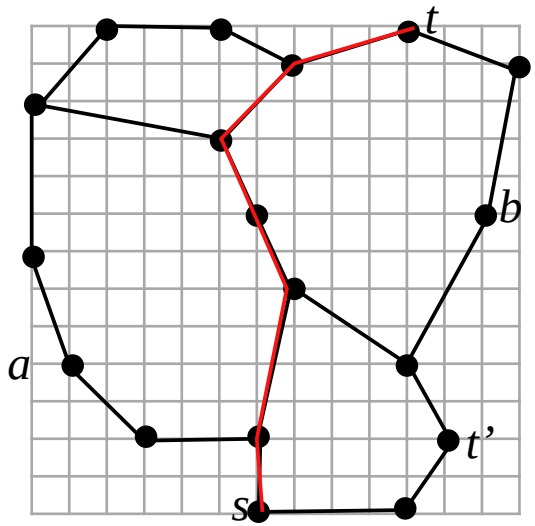

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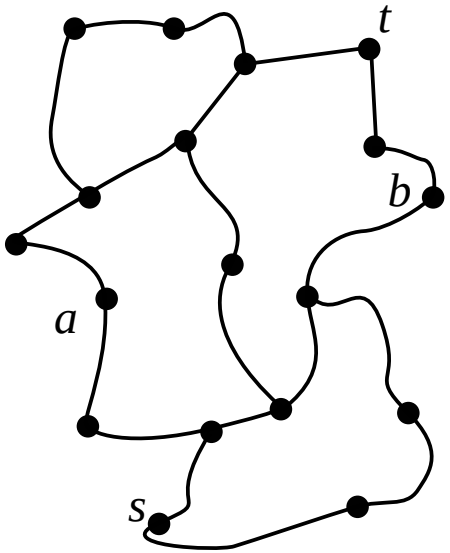

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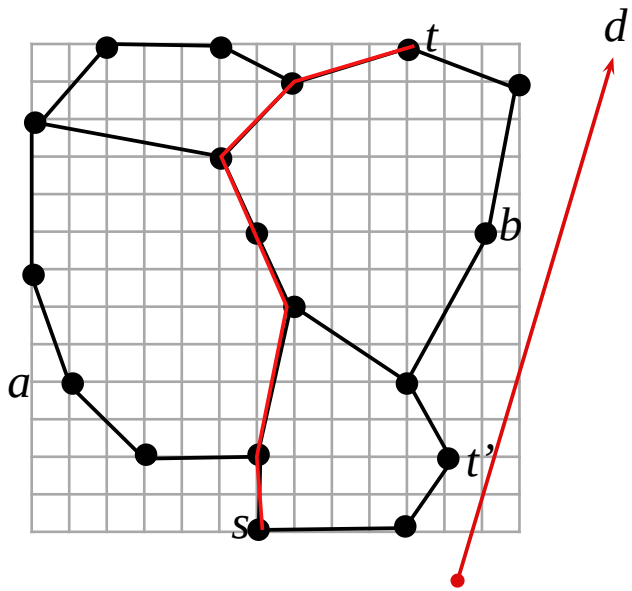

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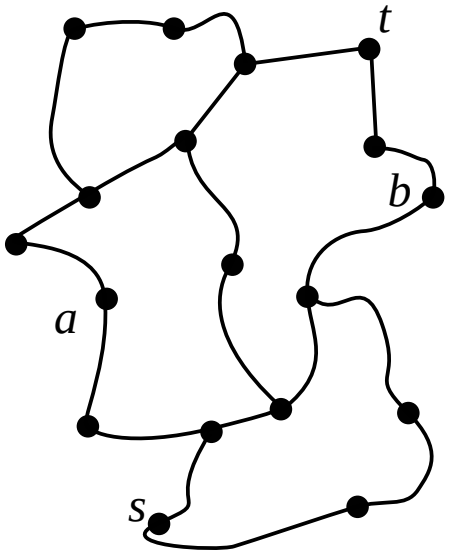

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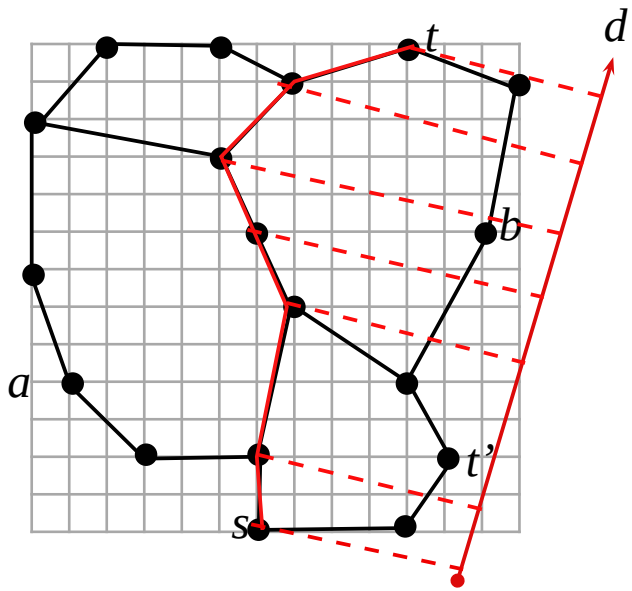

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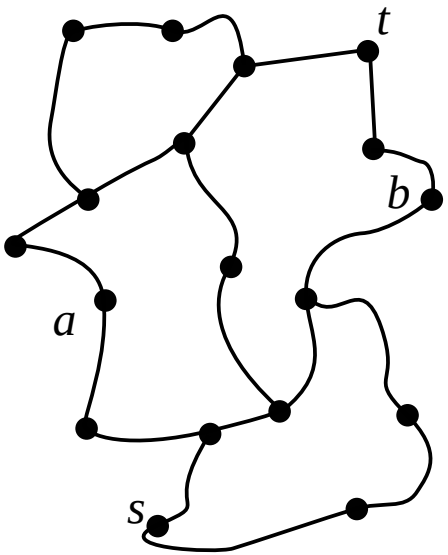

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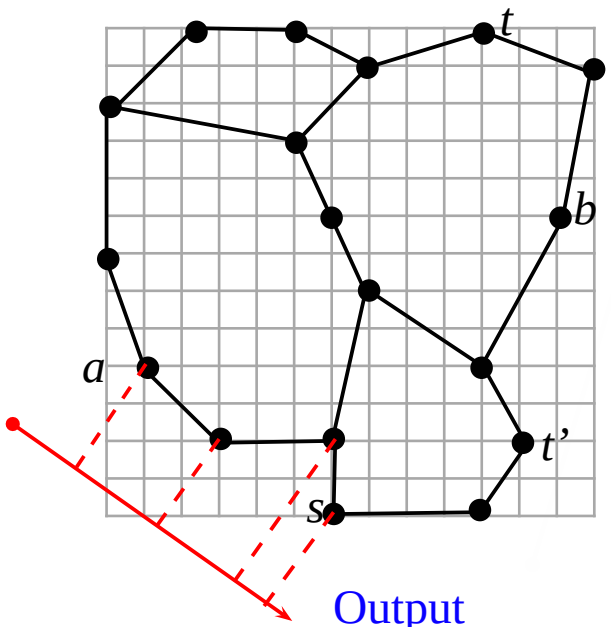

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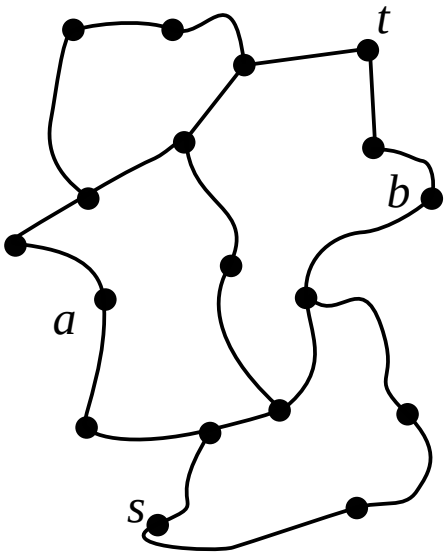

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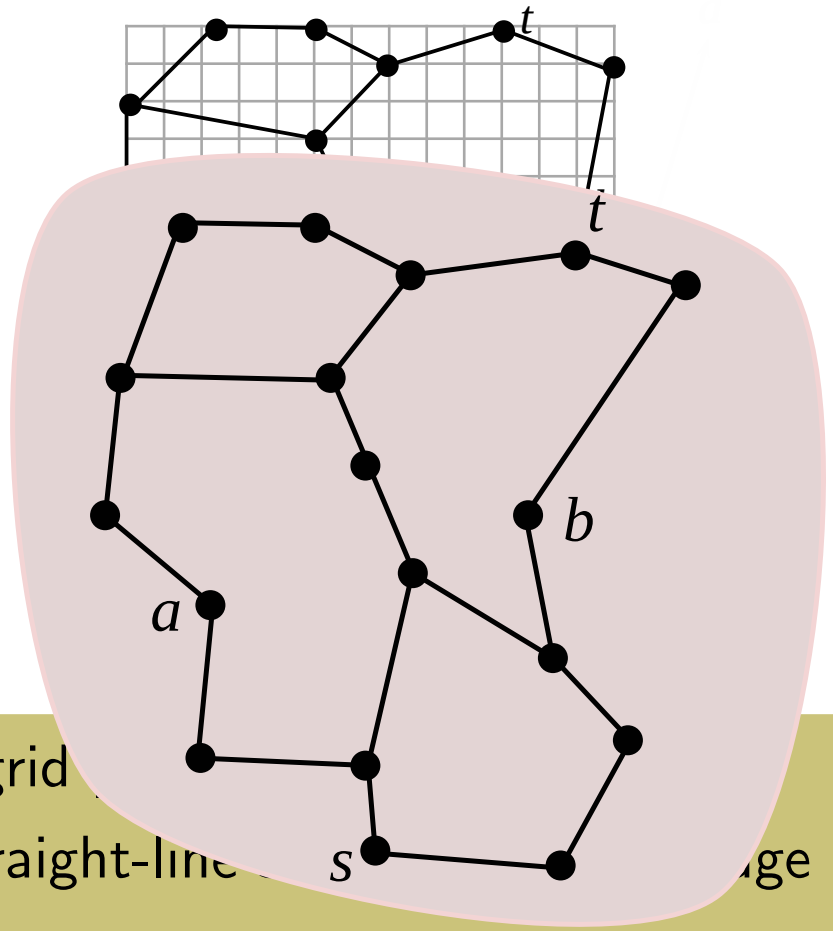

- Each vertex is drawn at a grid
- Each edge is drawn as a straight-line
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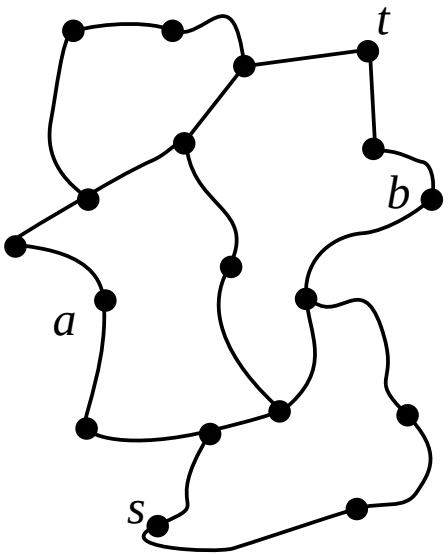

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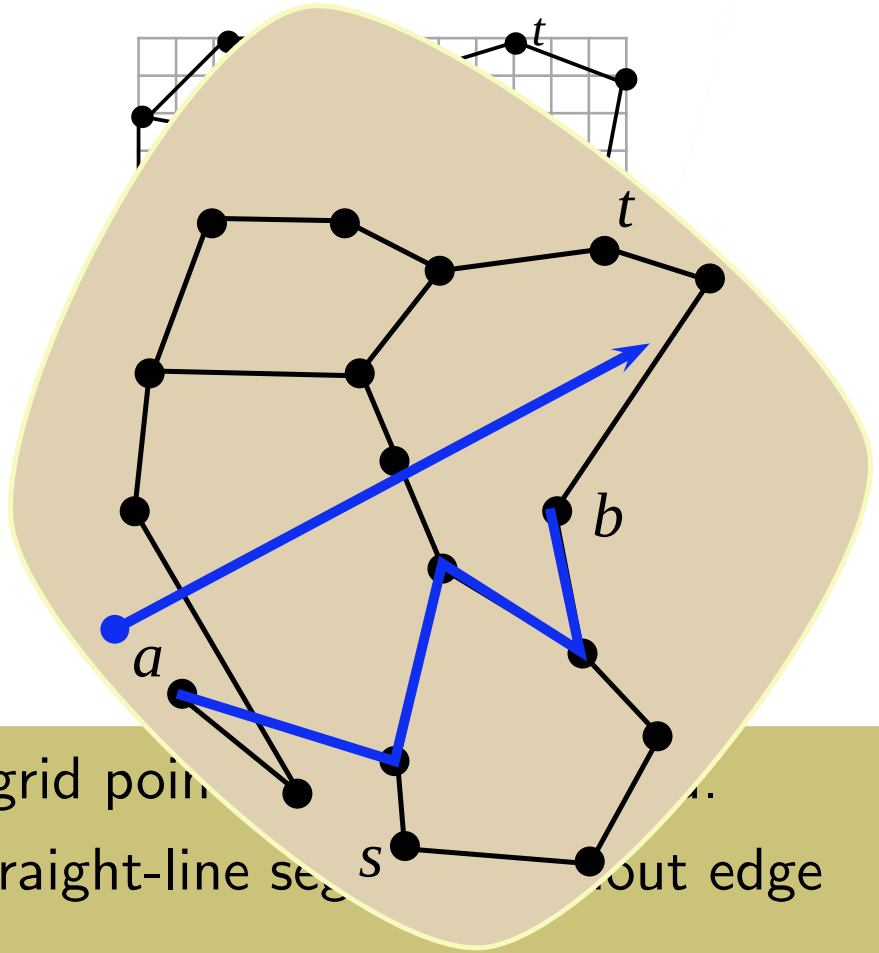

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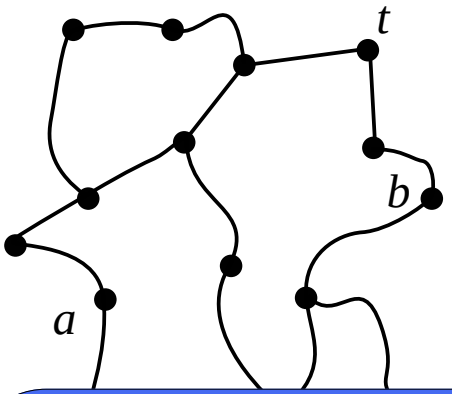

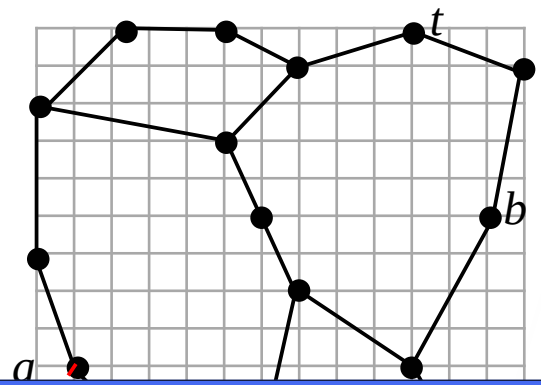

Every planar graph admits a monotone grid drawing on a grid of polyomial size and such a drawing can be found in linear time

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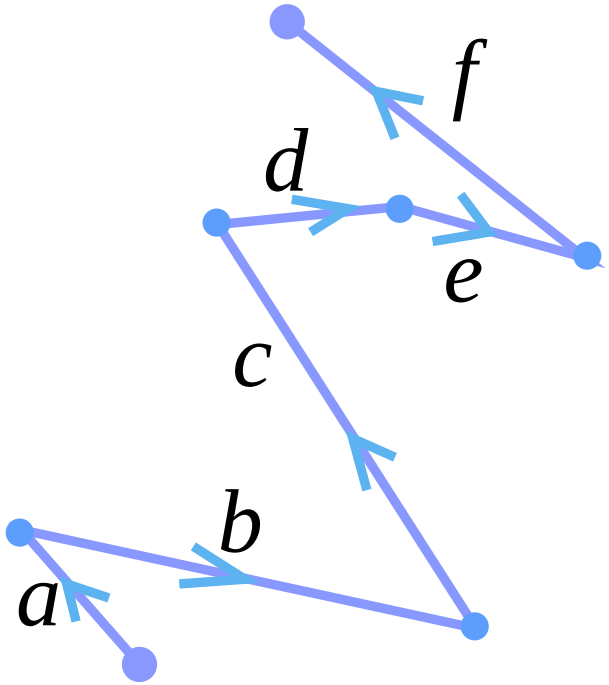
Properties of a monotone path (Angelini et. al. 2011)



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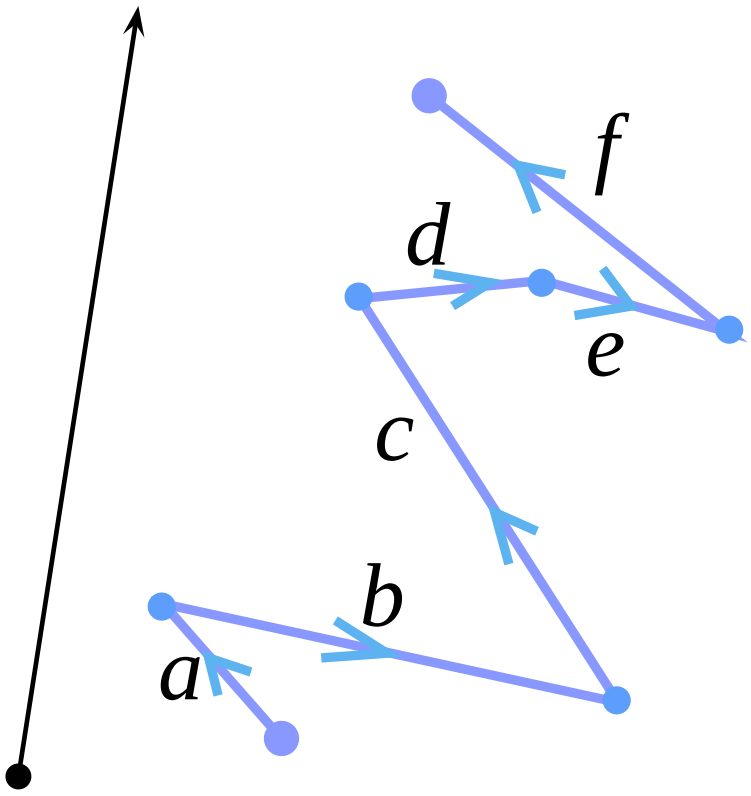

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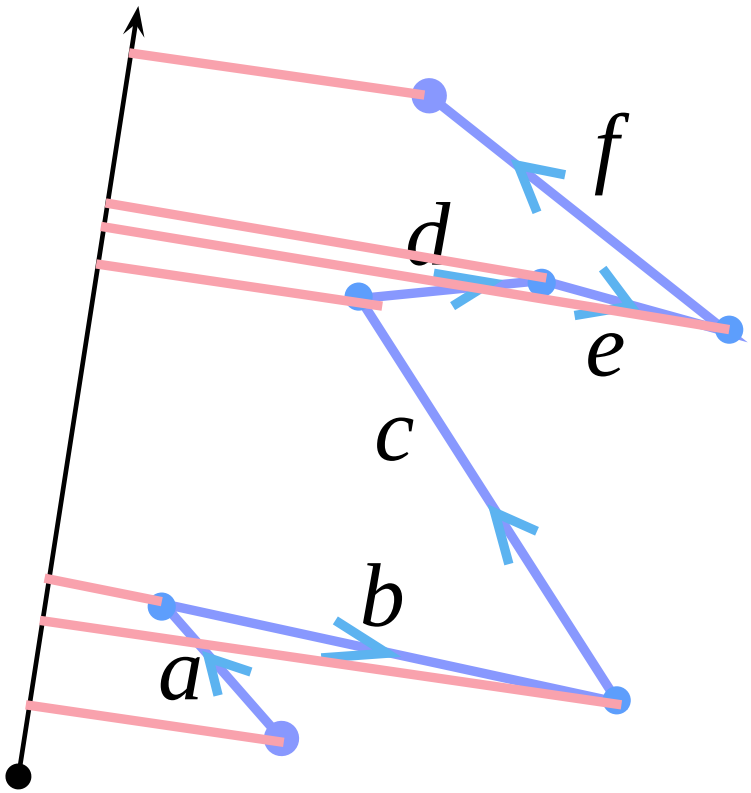

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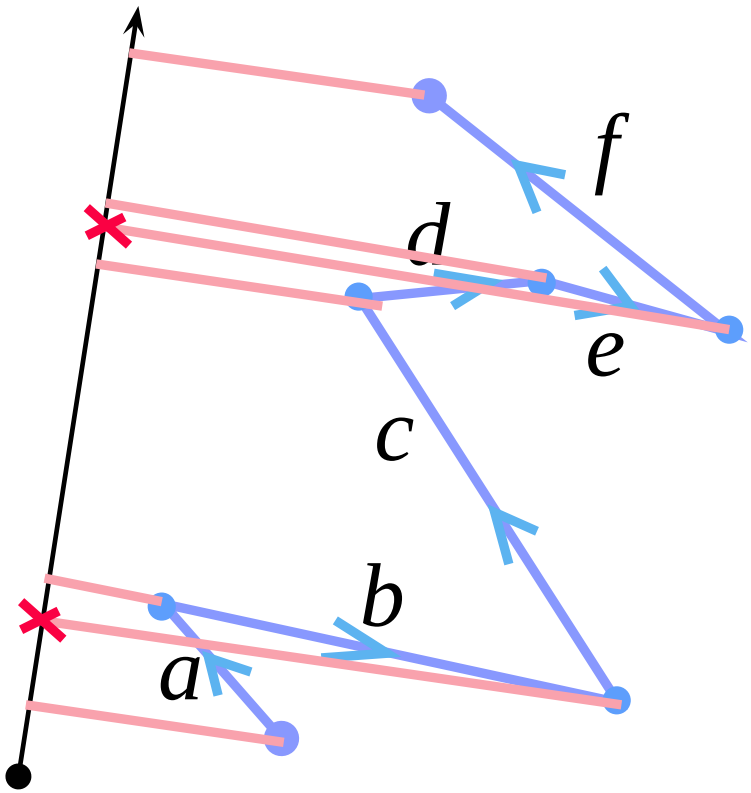

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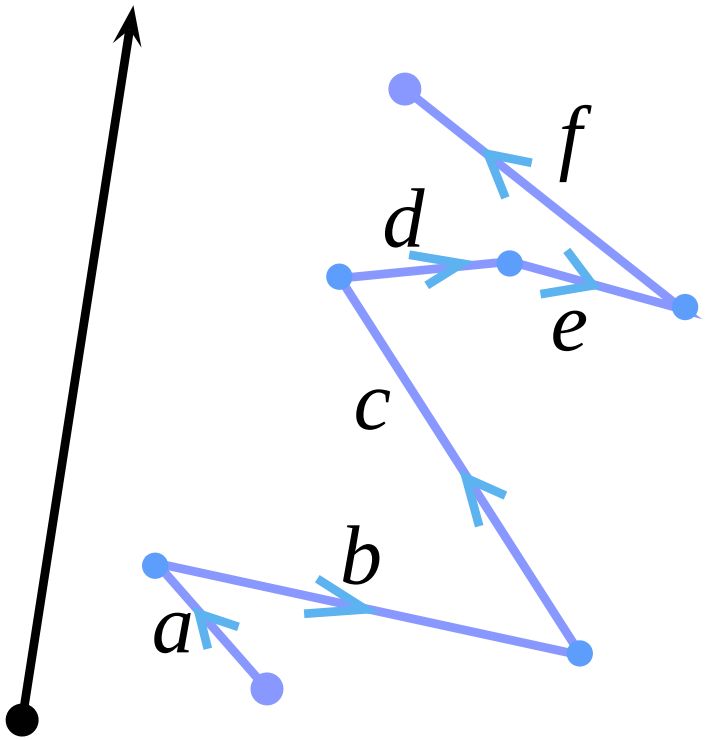

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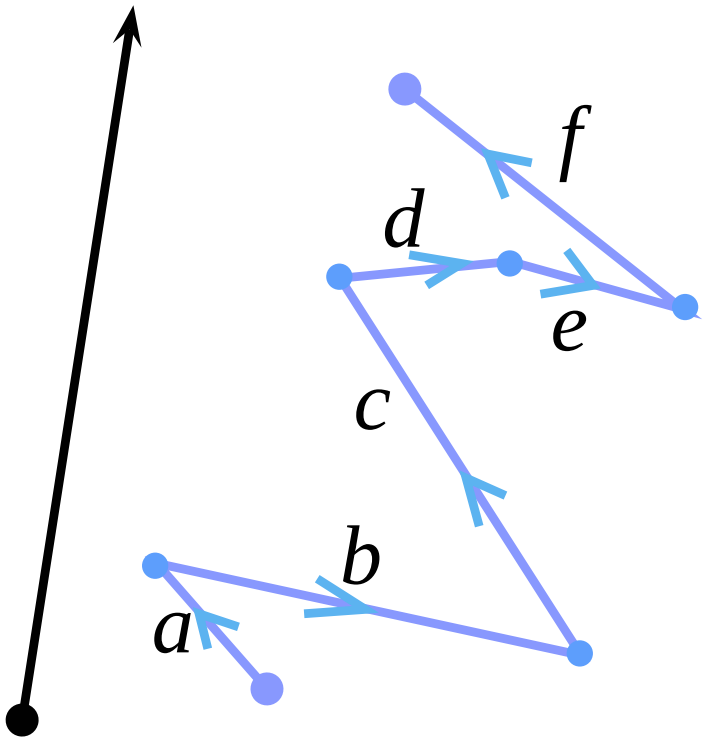

is monotone in any direction?

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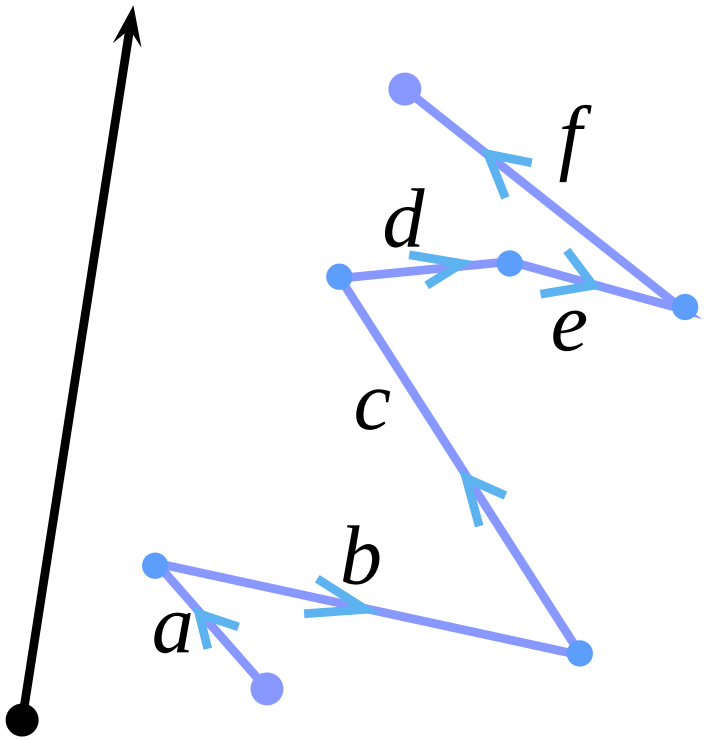

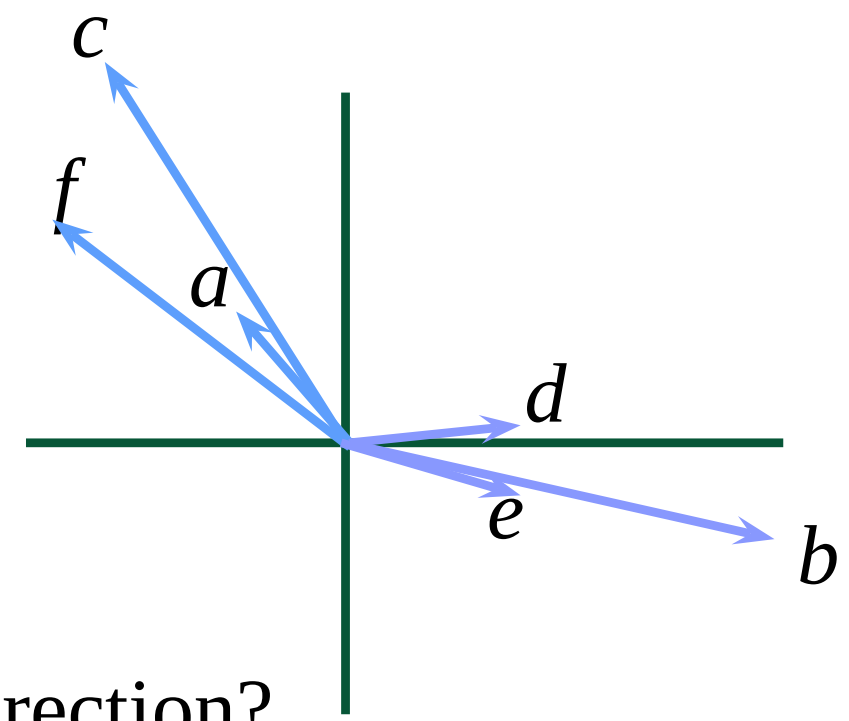

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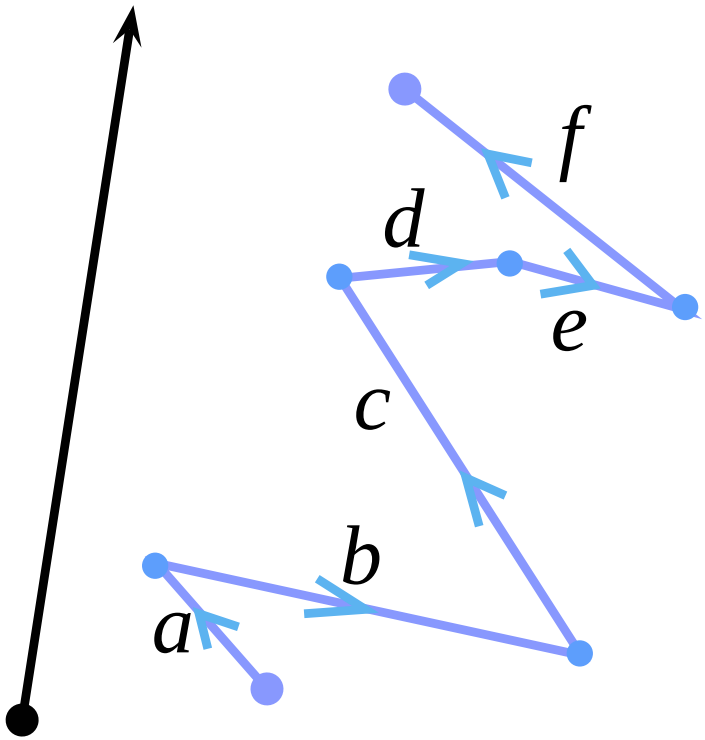

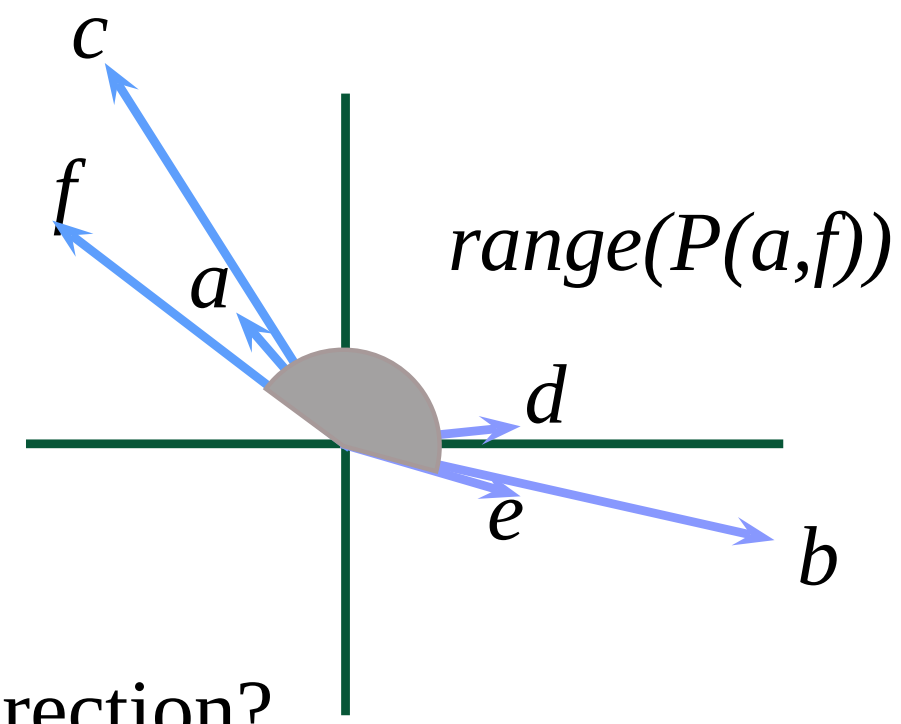

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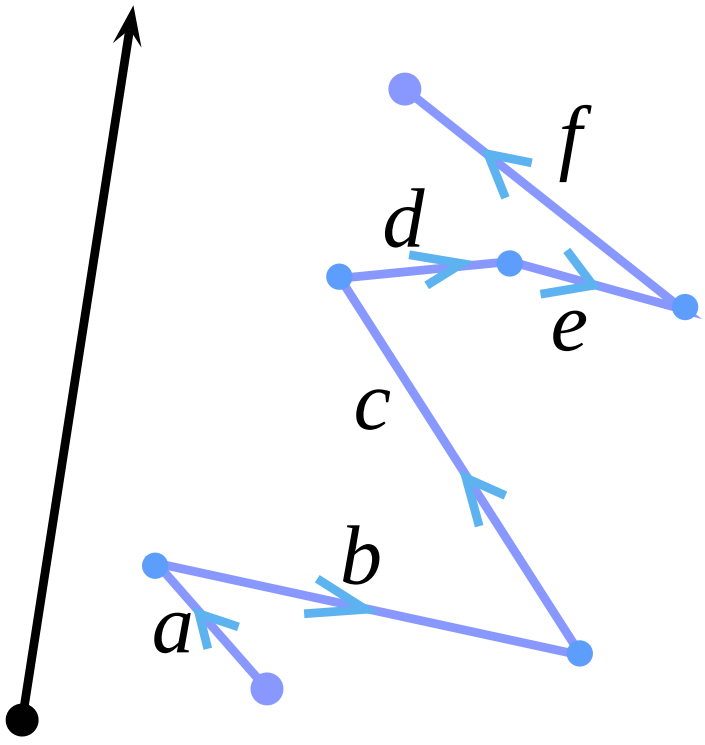

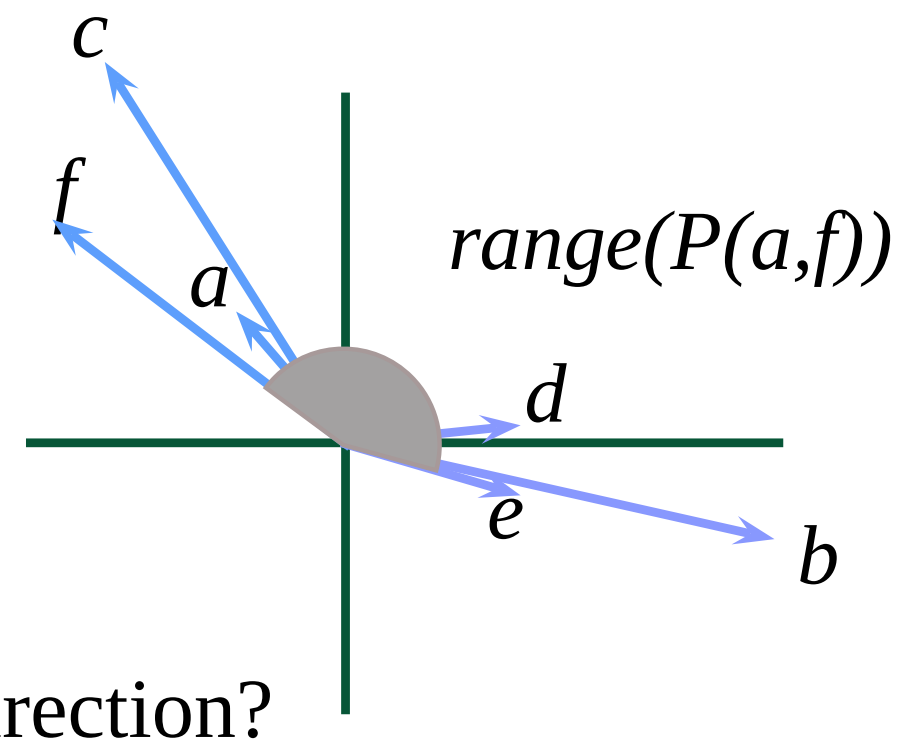

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if $range(P(v,u)) < \pi$ then $P(v,u)$ is monotone.

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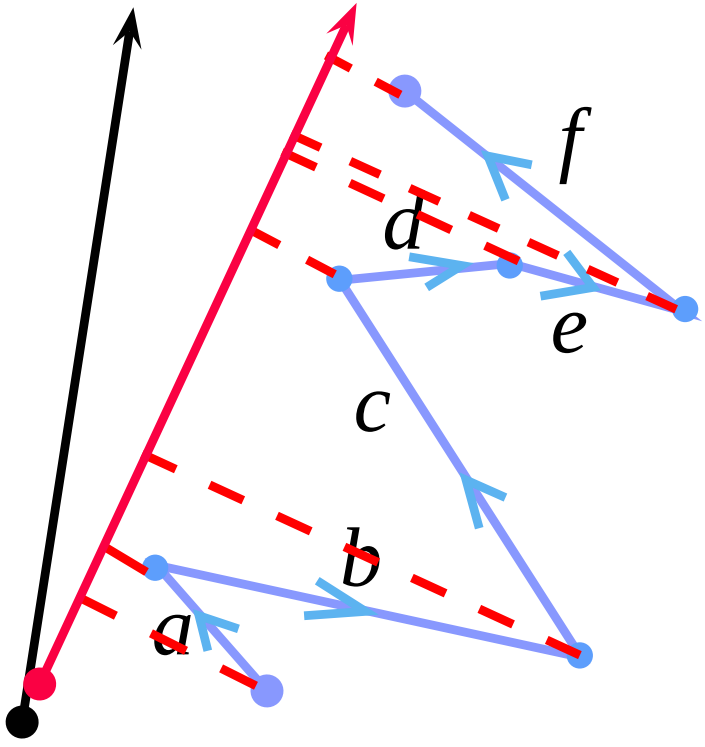

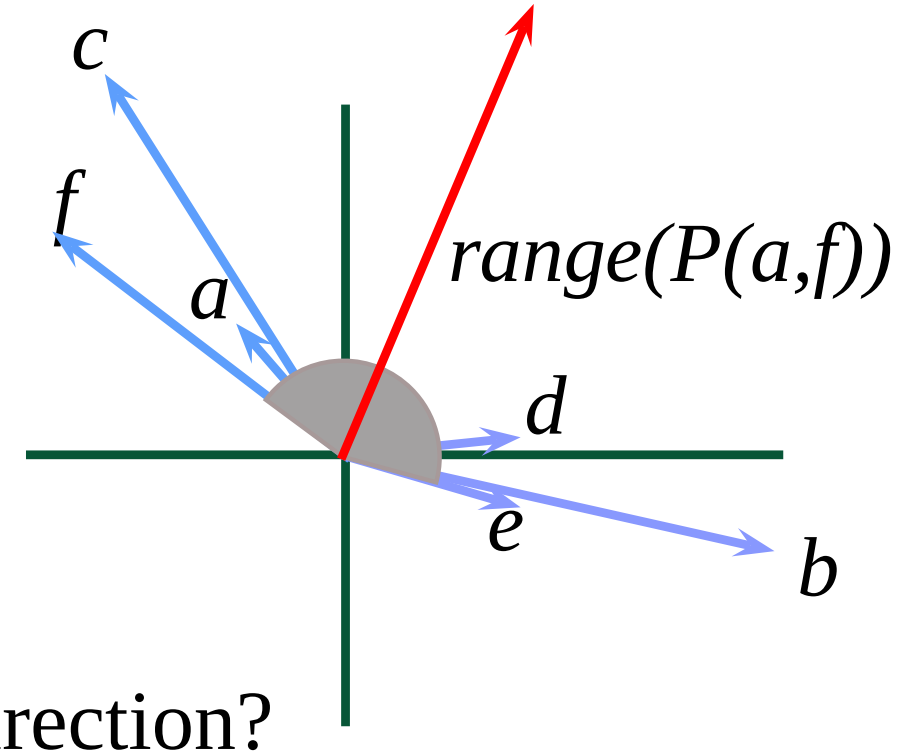

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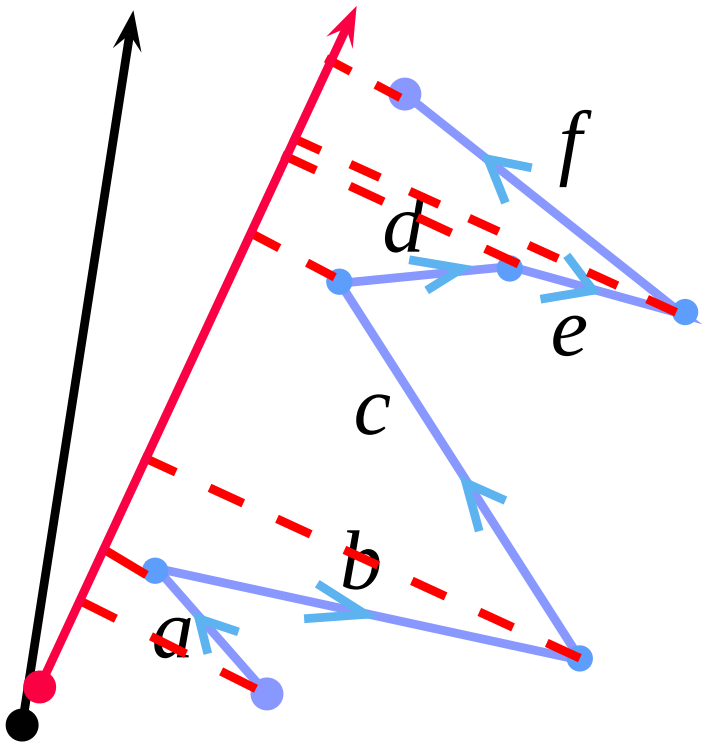

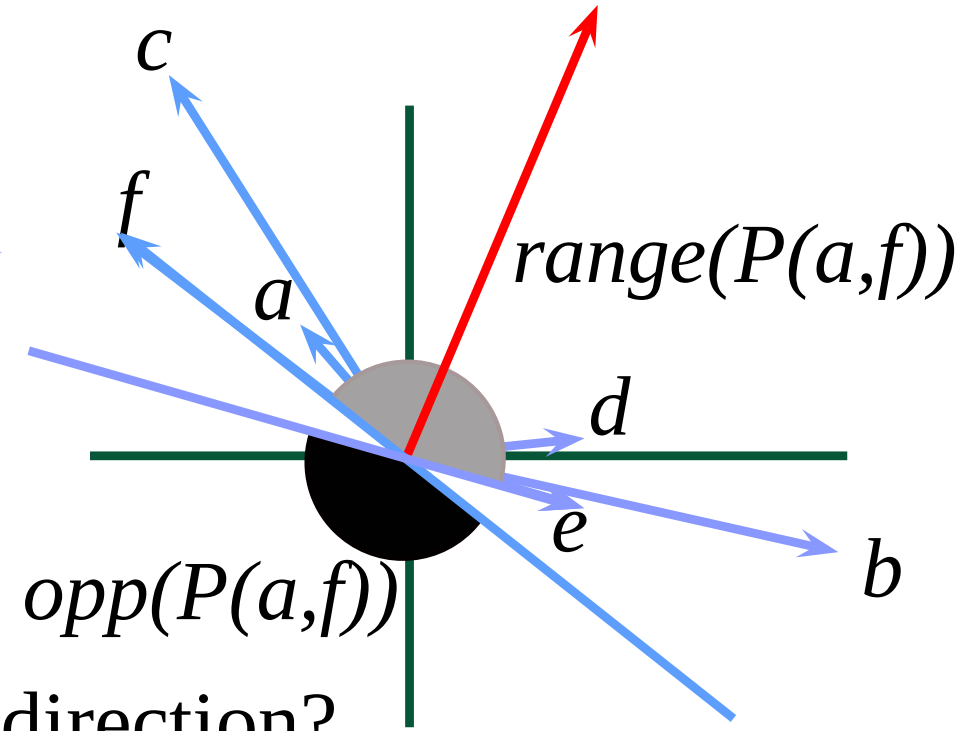

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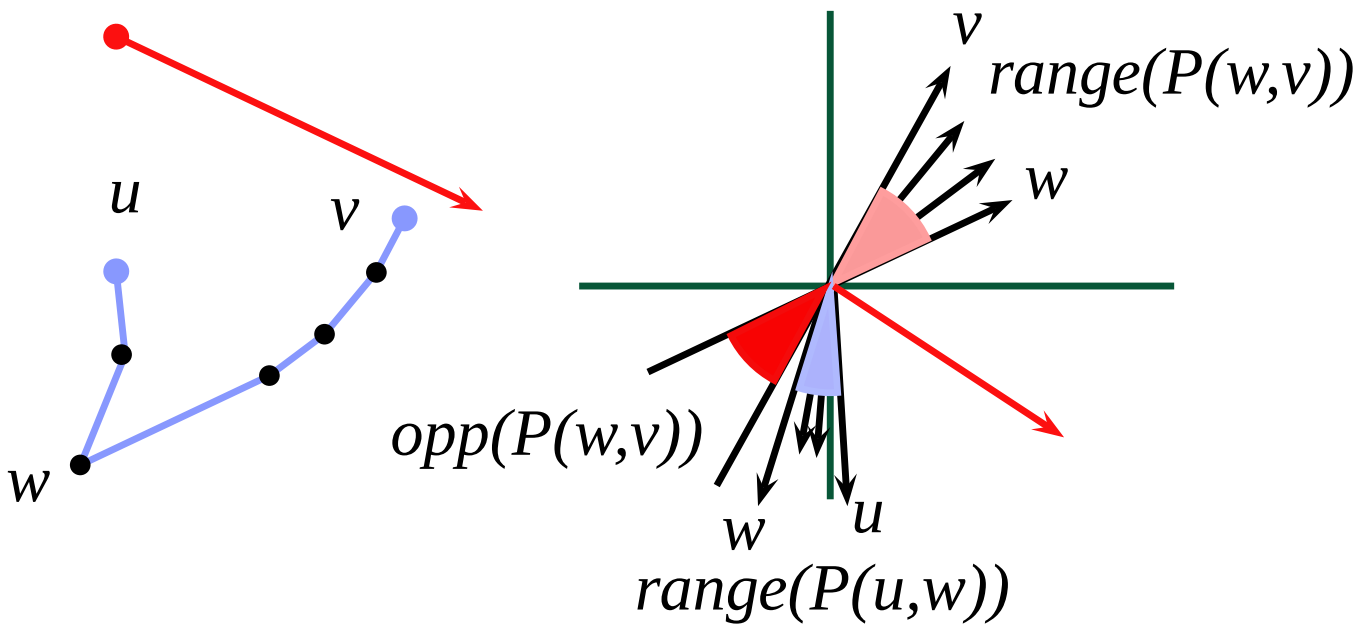

$P(u, v)$ is monotone if

$$range(P(u, w)) \cap opp(P(w, v)) = \phi$$

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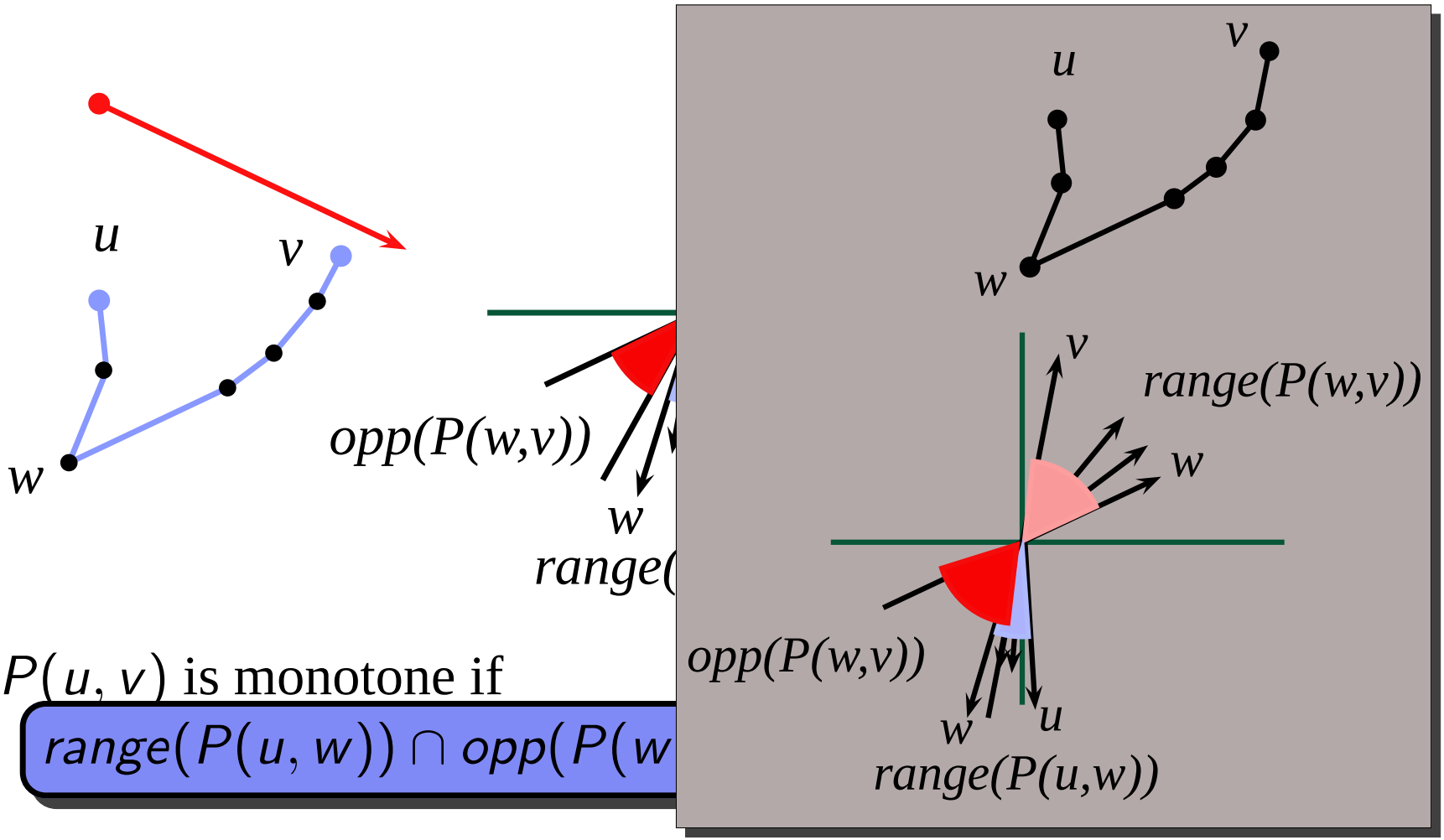

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Result

Previous results: Arkin, Connelly, and Mitchell, 1989

How to find monotone trajectories connecting two given points in the plane avoiding obstacles.

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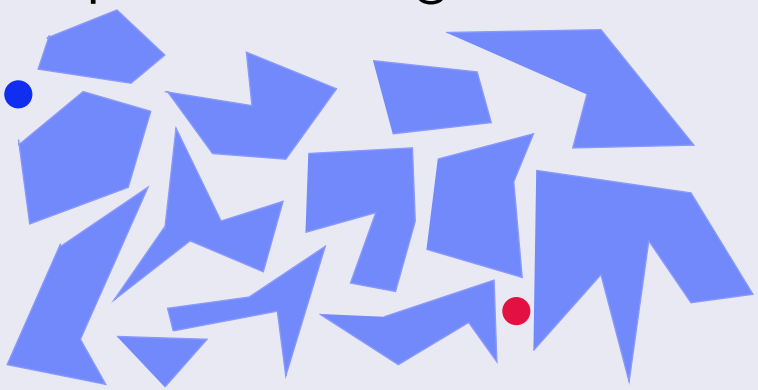

Result

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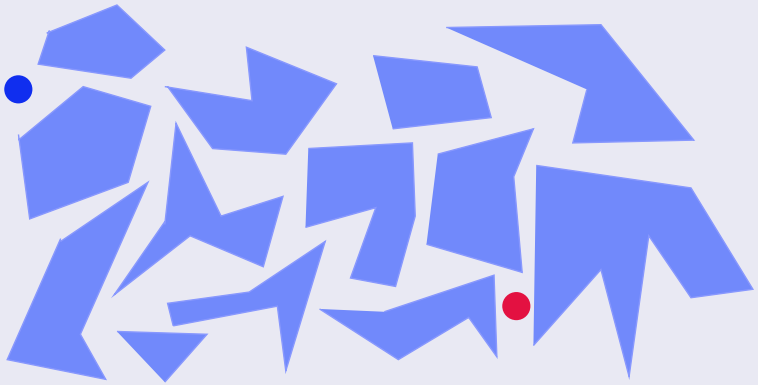

If obstacles are convex then a monotone path can be found in $O(n \log n)$ time.

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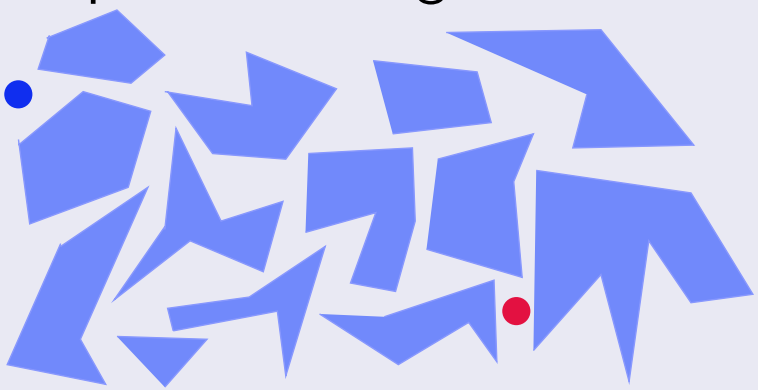

Every convex drawing of a plane graph is a monotone drawing and a monotone path can be found in $O(n \log n)$ time

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Result

Previous results

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- *Hossain and Rahman 2013* **Monotone grid drawings of series-parallel graphs on $O(n) \times O(n^2)$ grid in $O(n \log n)$ time**

Result

Previous results

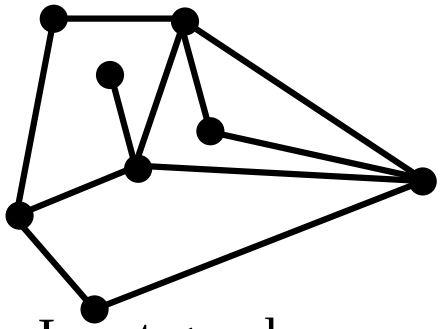
Monotone drawings admits on $O(n) \times O(n^2)$ grid

- trees on $O(n) \times O(n^2)$ grid in linear time
- biconnected plane graphs in real co-ordinate space in linear time
- planar graph on $O(n) \times O(n^2)$ grid with at most two bends per edge
- series-parallel graphs $O(n) \times O(n^2)$ grid in $O(n \log n)$ time

Our Result

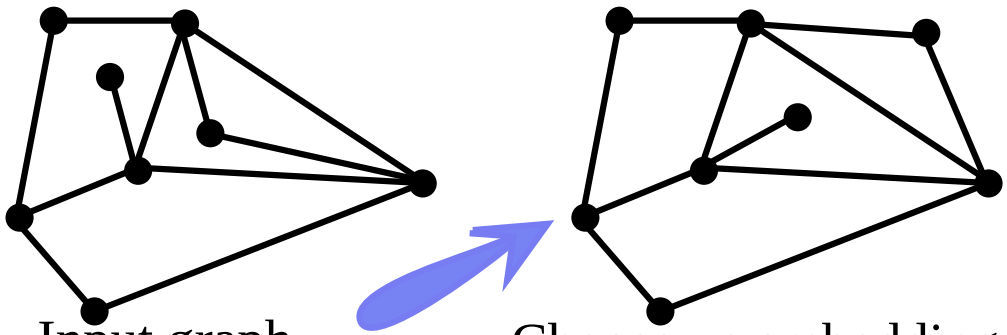
- Monotone grid drawings of planar graphs on $O(n) \times O(n^2)$ grid in $O(n)$ time

Algorithm Overview



Input graph

Algorithm Overview



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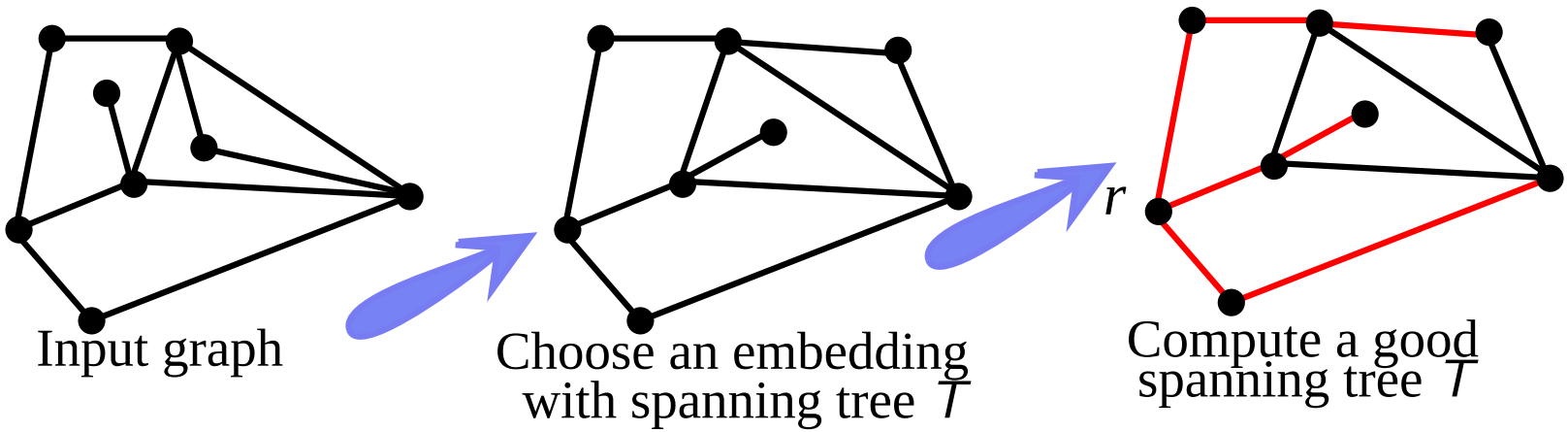

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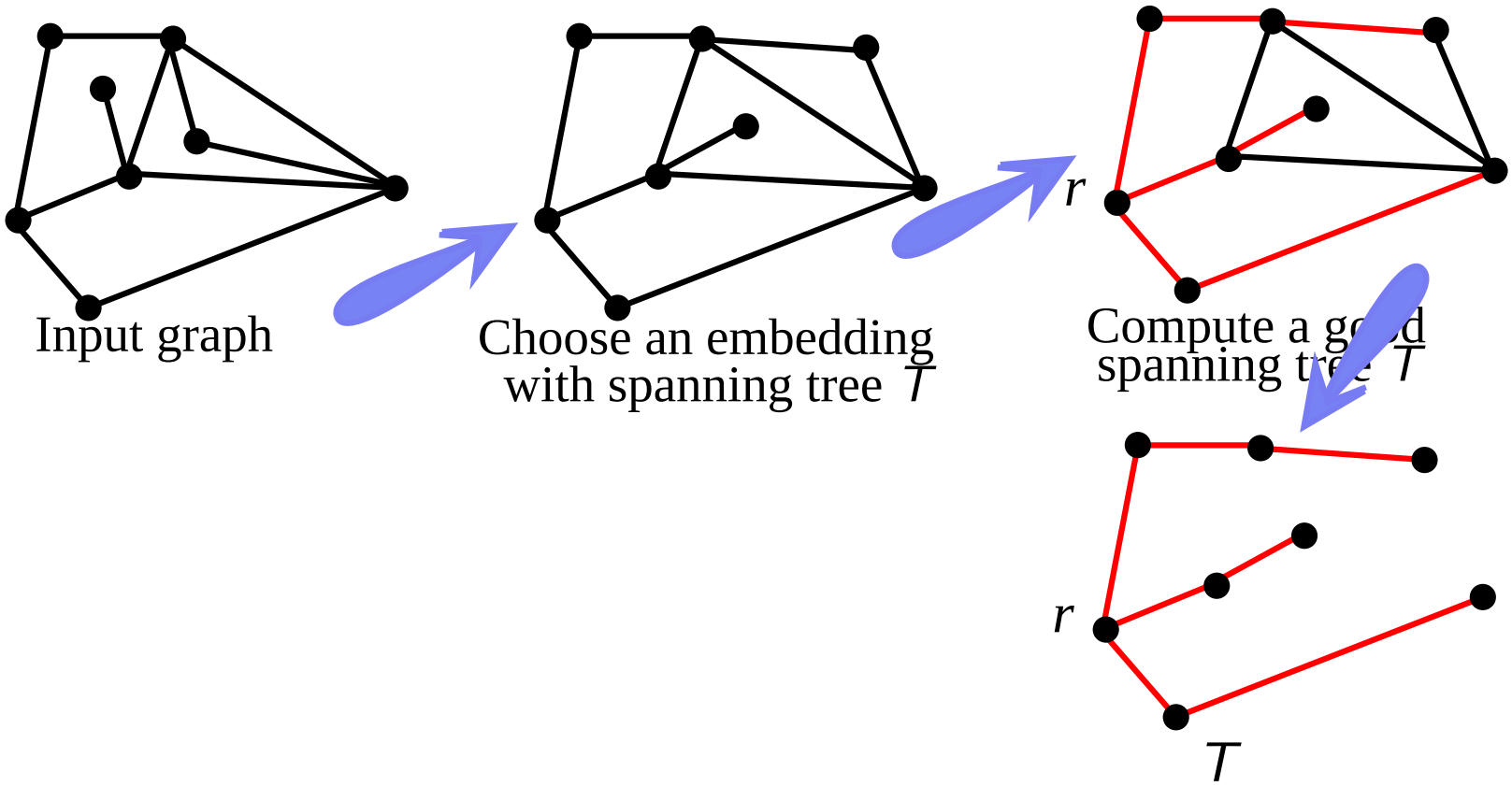

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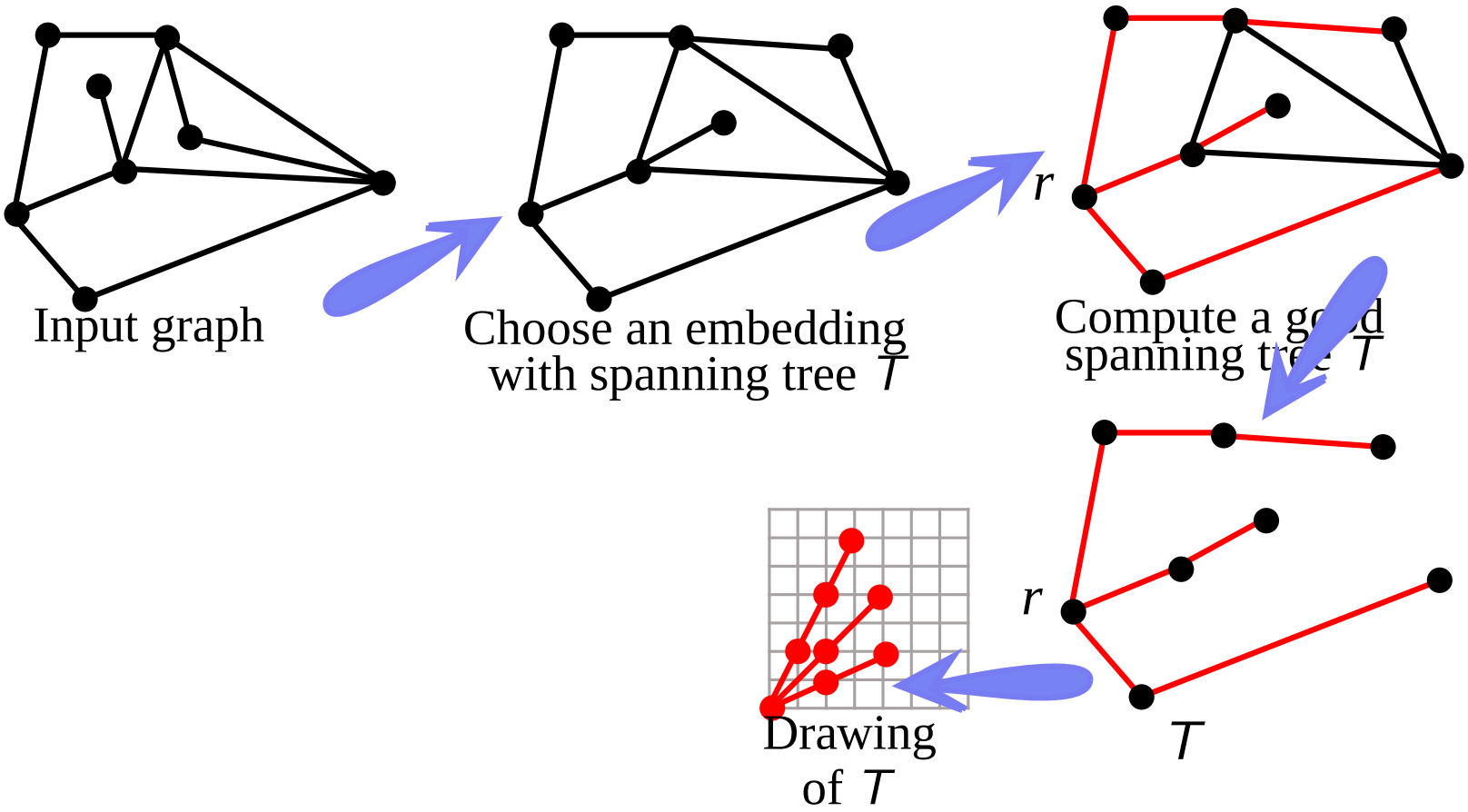

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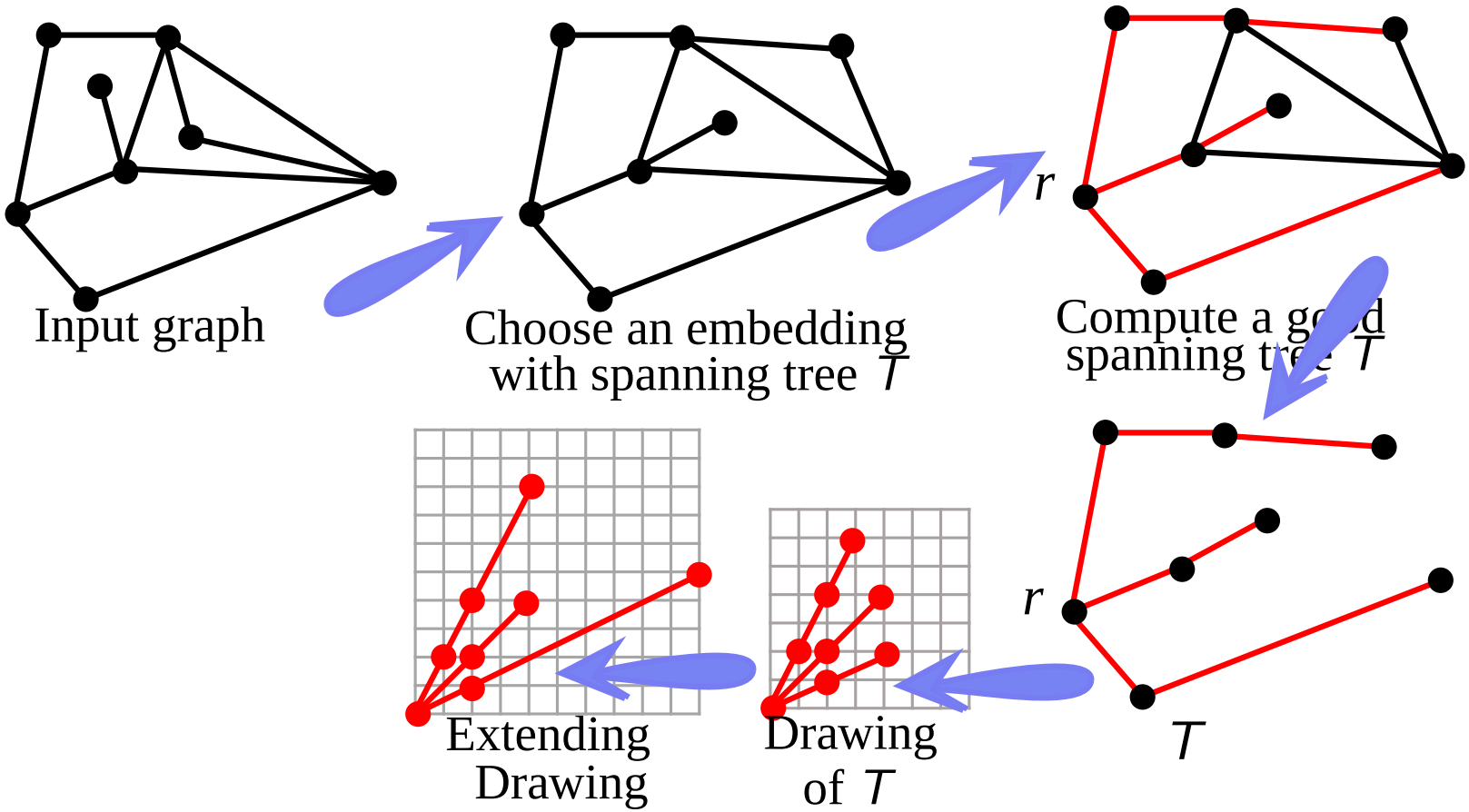

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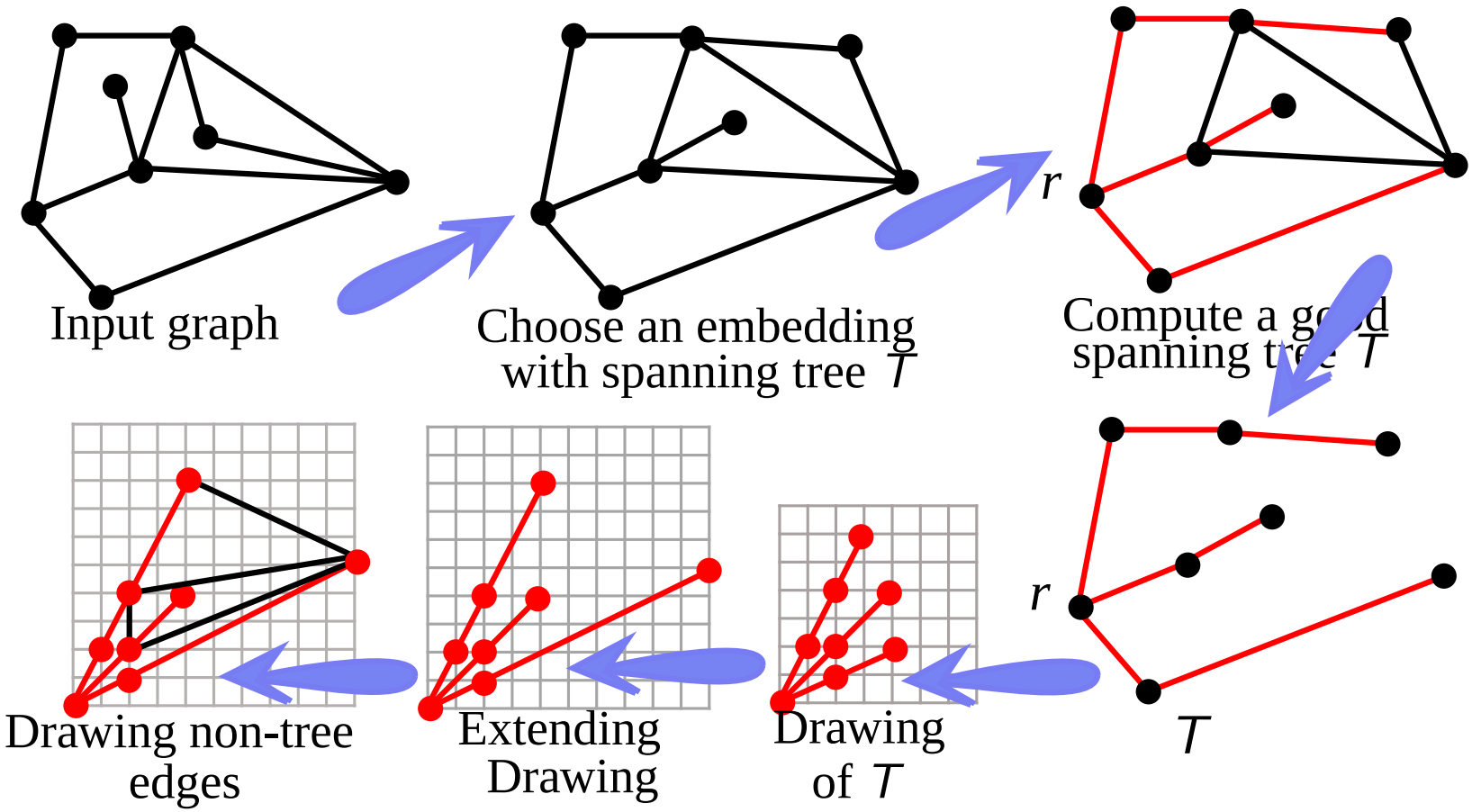

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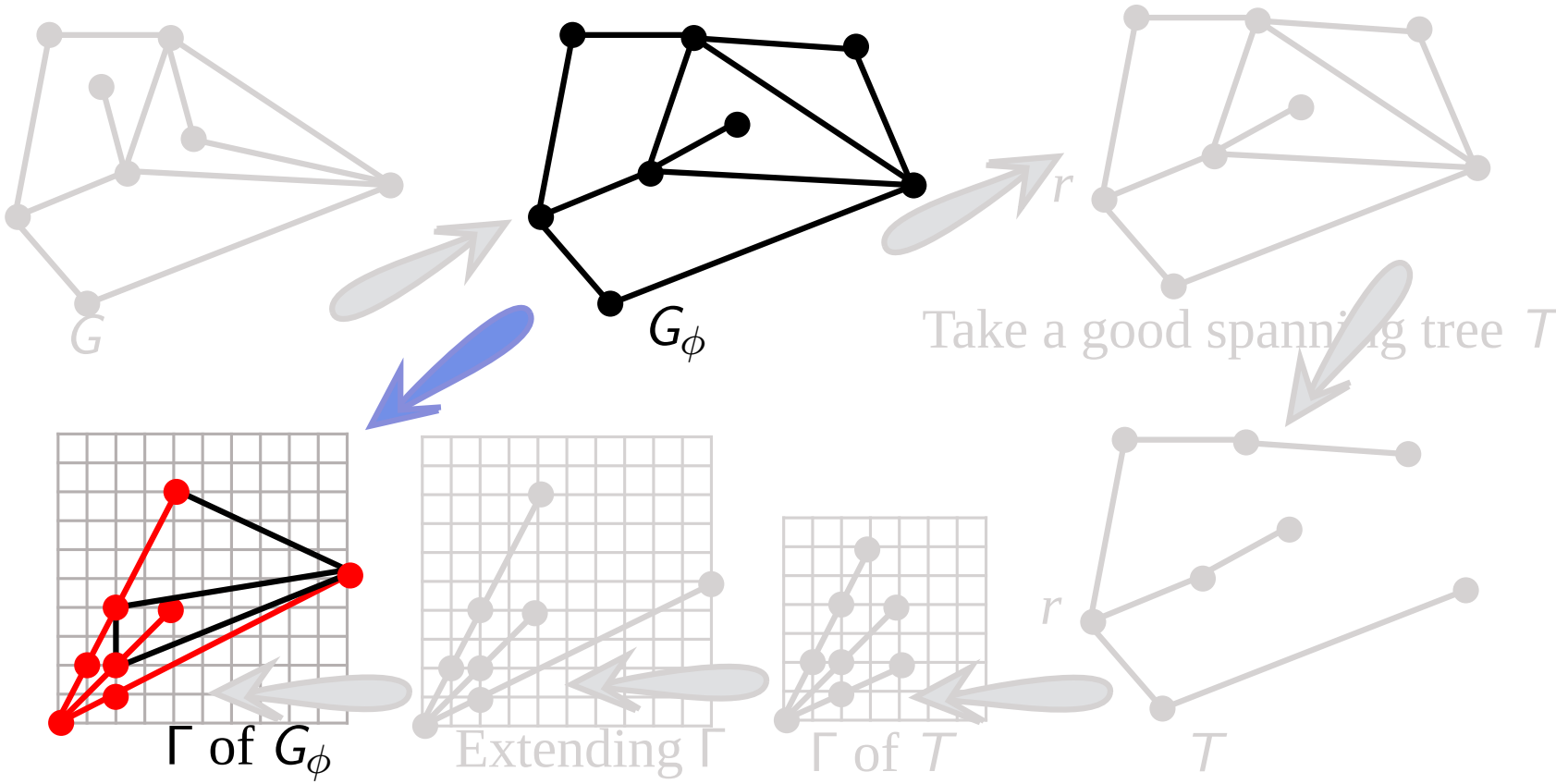
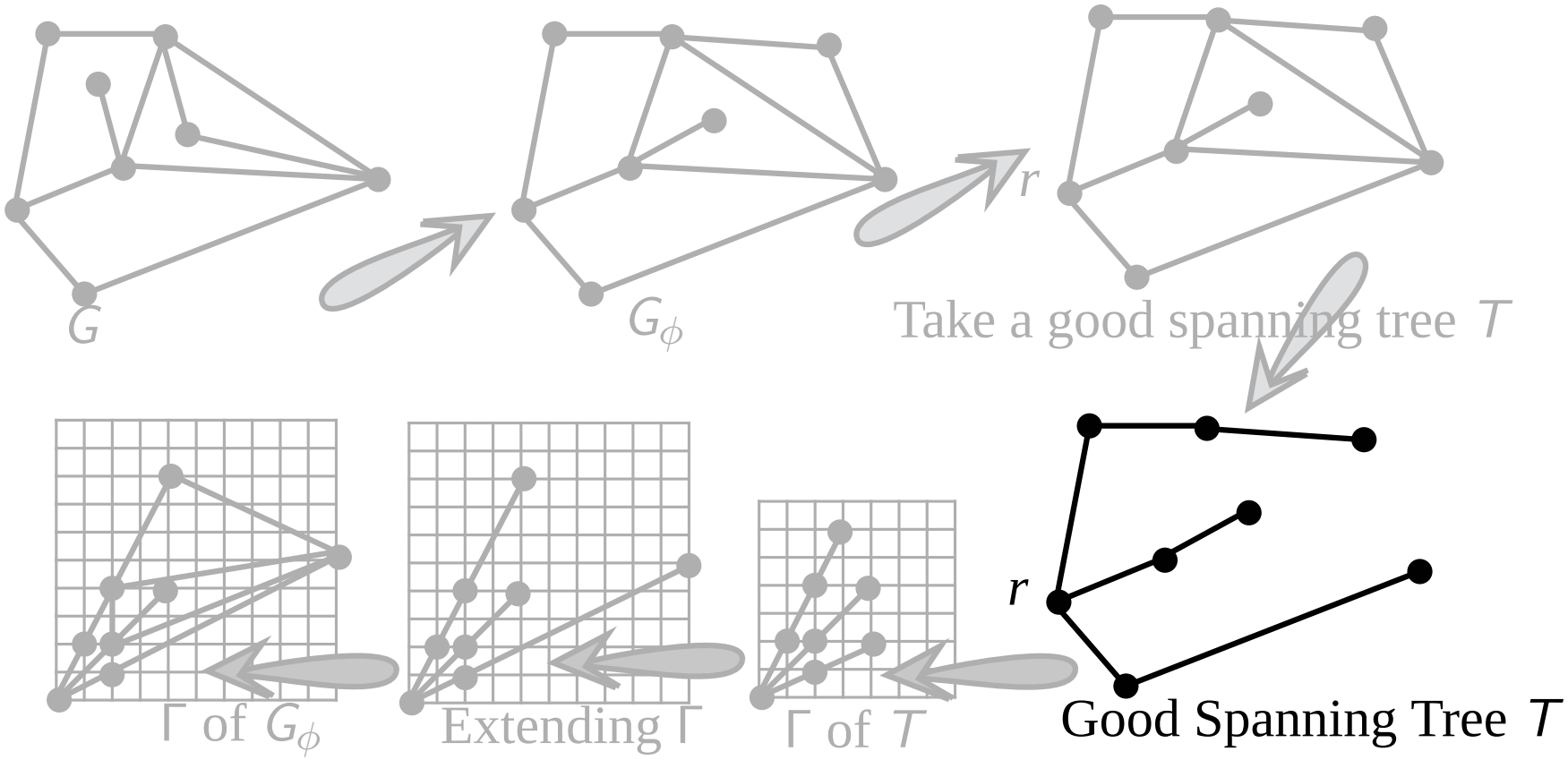
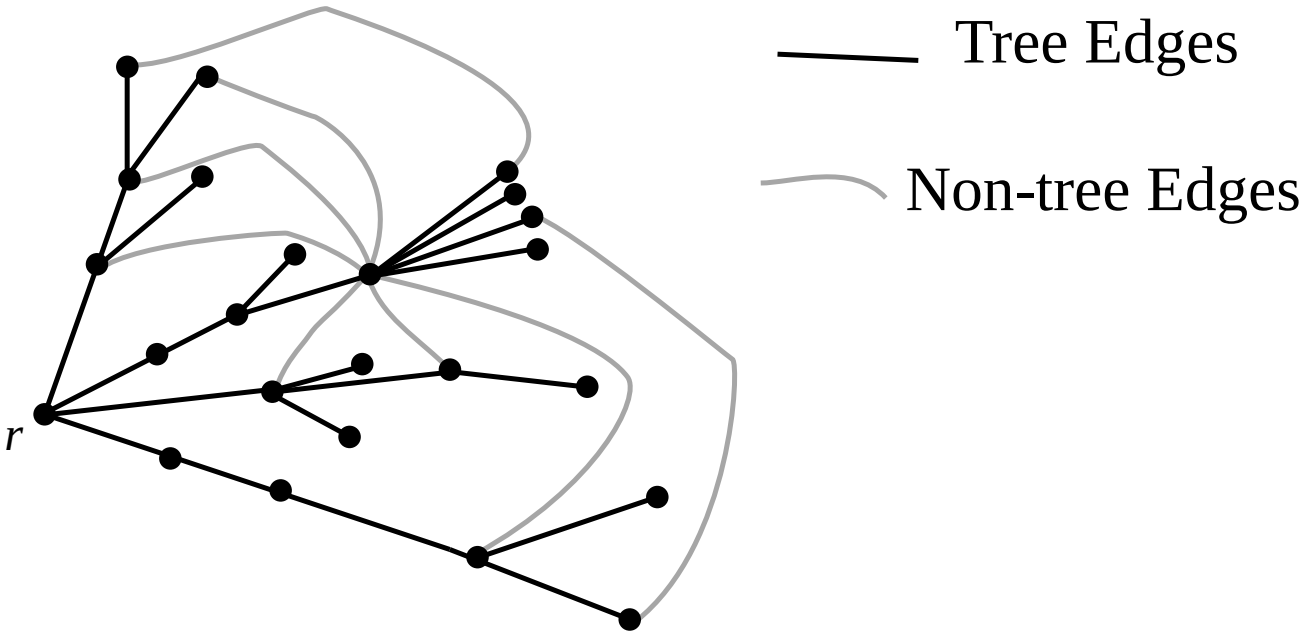


Algorithm Overview



Good Spanning Tree



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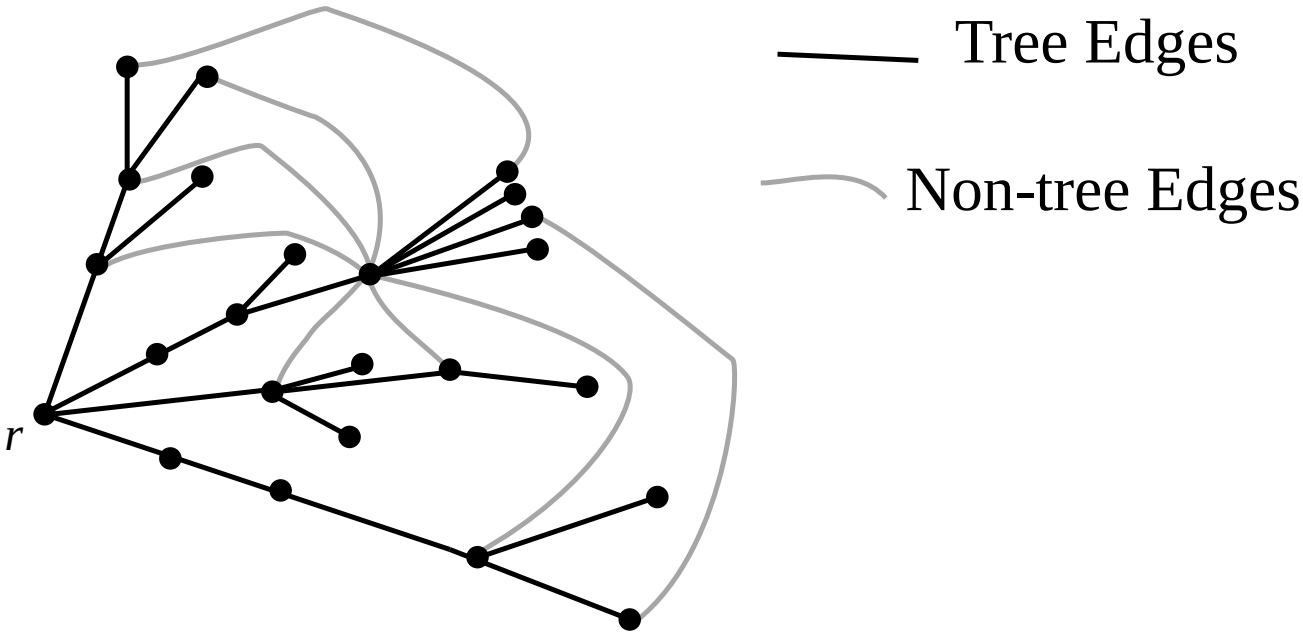
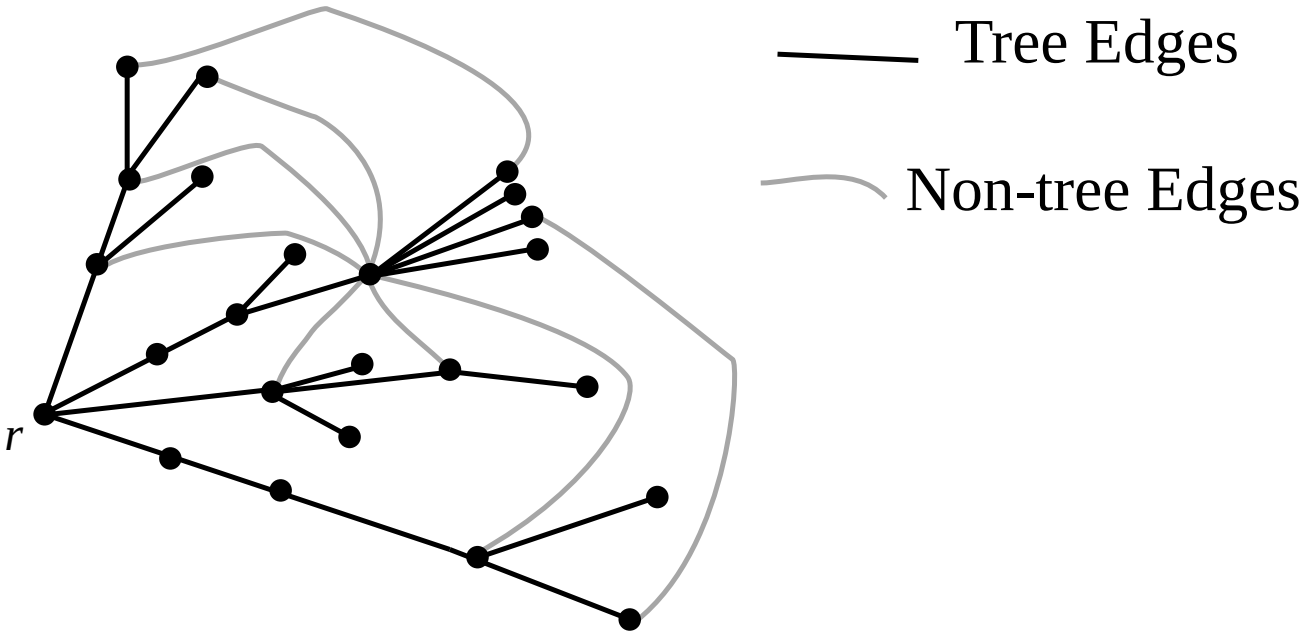


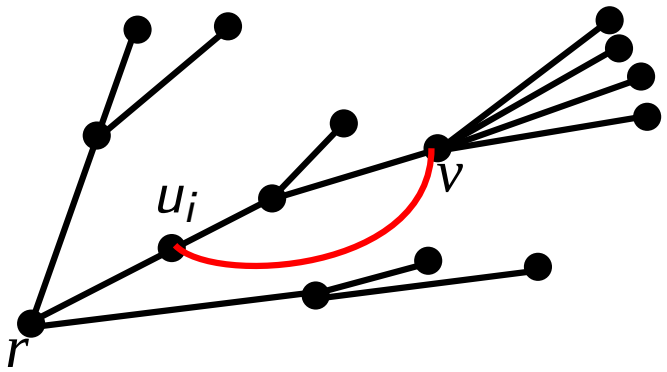
- An ordered rooted spanning tree of an embedded planar graph.

Good Spanning Tree



- An ordered rooted spanning tree of an embedded planar graph.
- rooted at an outer vertex.

Formal Definition of Good Spanning Tree

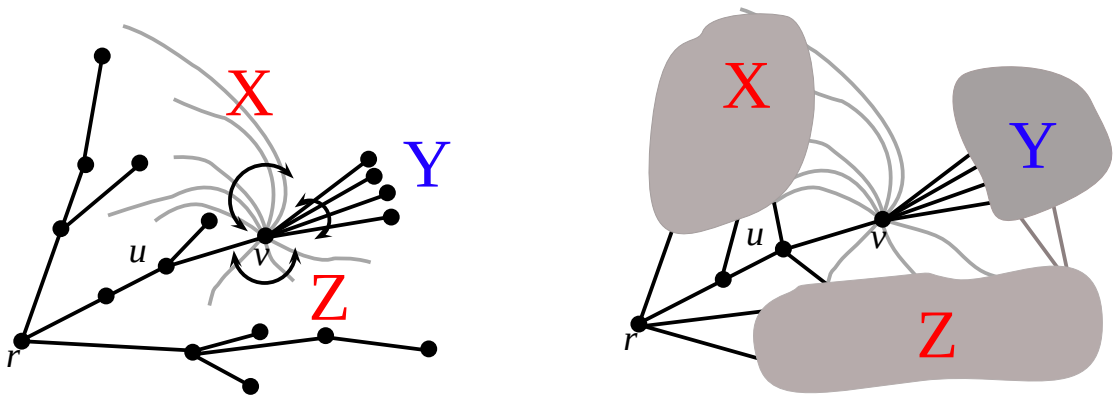


$$p = \{r = u_0, u_1, \ldots, u_k = v\}$$

(Cond1)

- G does not have a non-tree edge (v, u_i).

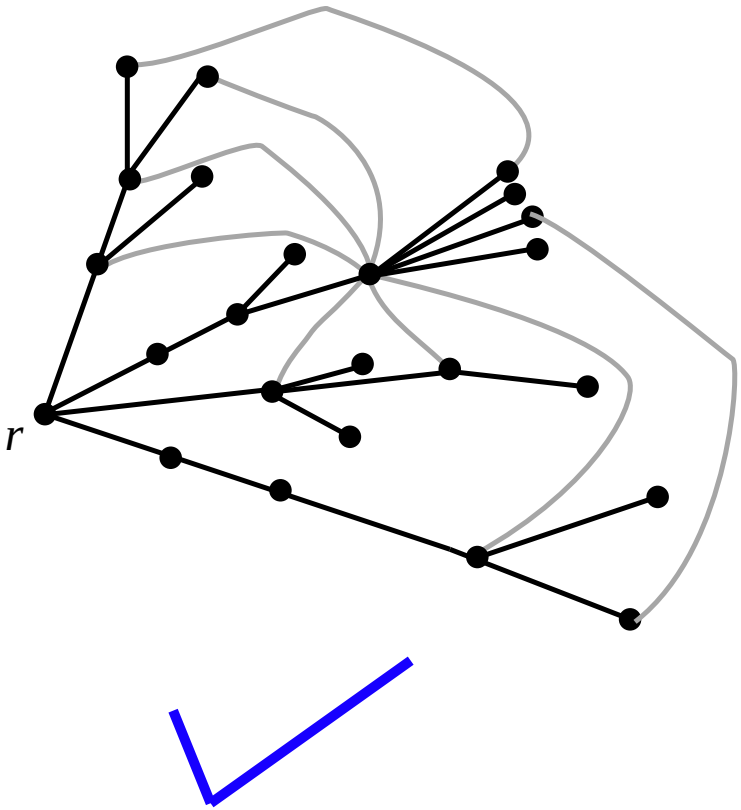
Formal Definition of Good Spanning Tree

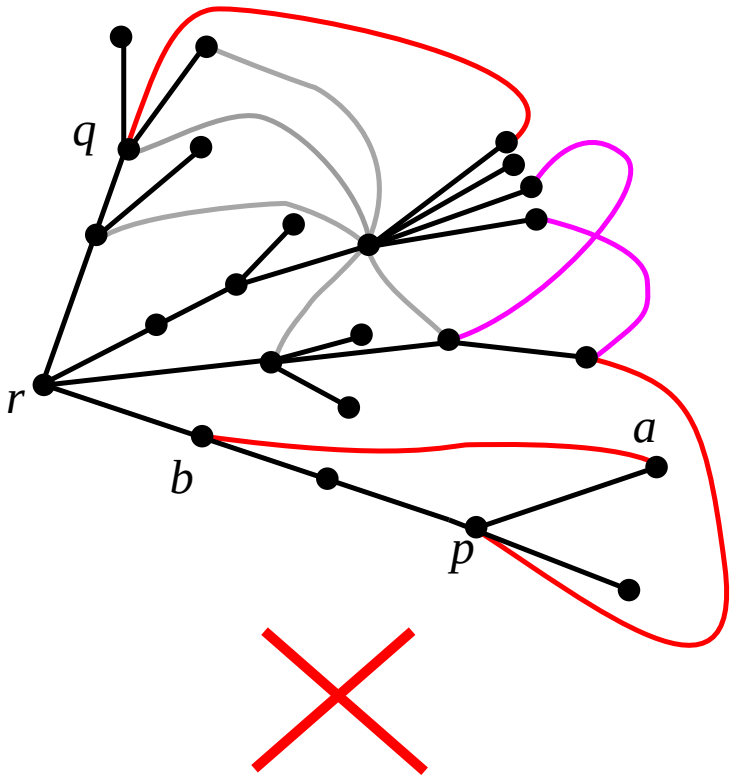


(Cond2)

- X, Y, Z: partition of the edges incident to v.
- X, Z: set of consecutive non-tree edges.
- Y: set of consecutive tree edges.
- X, Y, Z appear in clock wise order from (u, v).
- edges in X, Y, Z terminates in the respective shaded region.

Example of Good Spanning Tree





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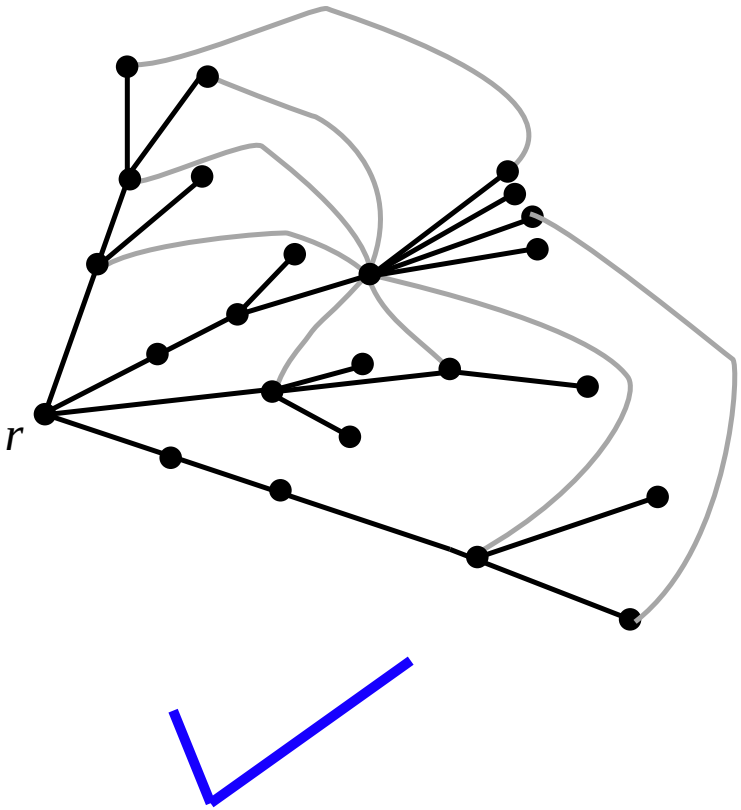

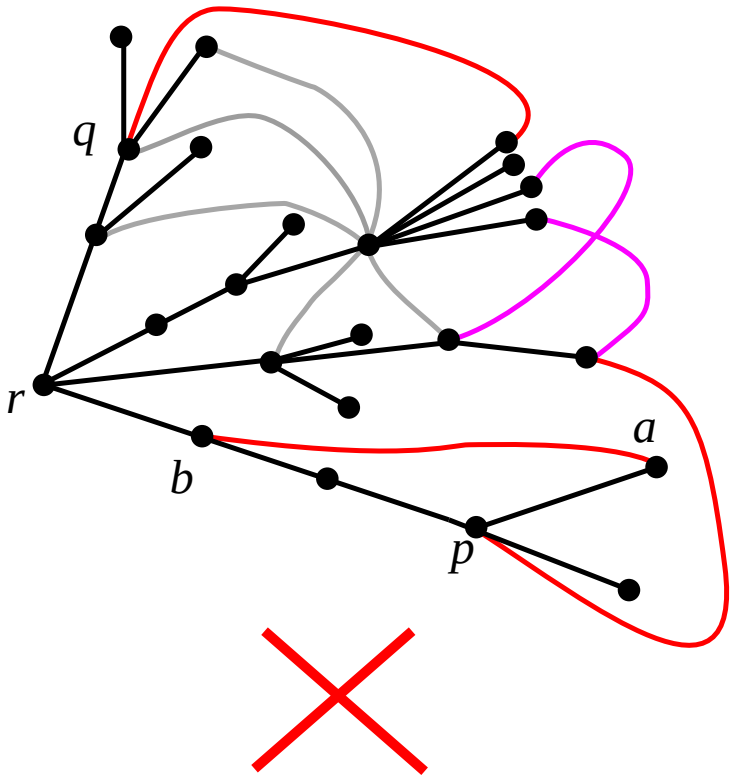

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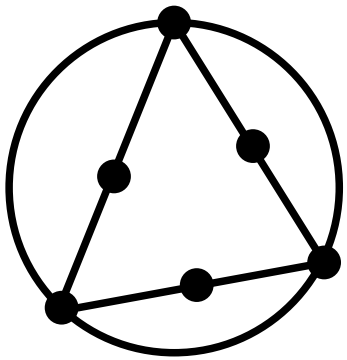
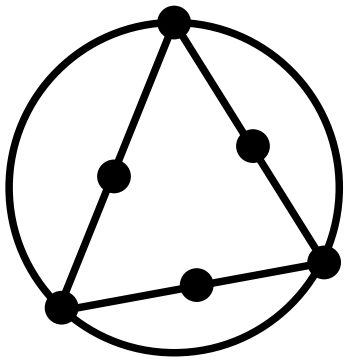


G_1

Example of Good Spanning Tree



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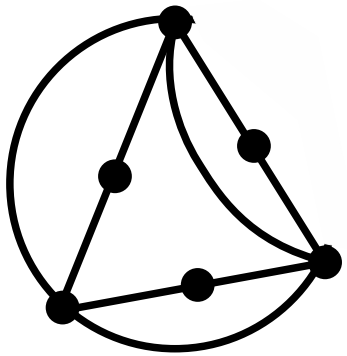
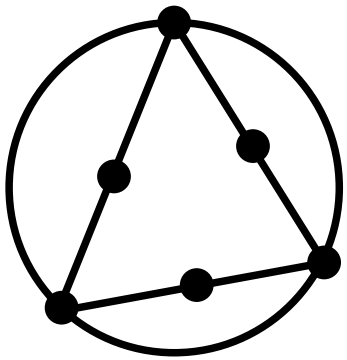


G_2

Example of Good Spanning Tree



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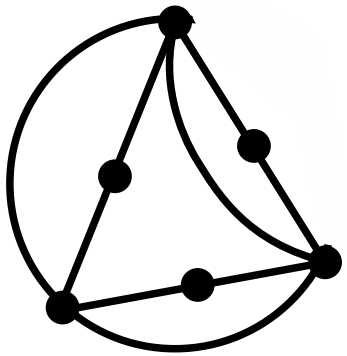
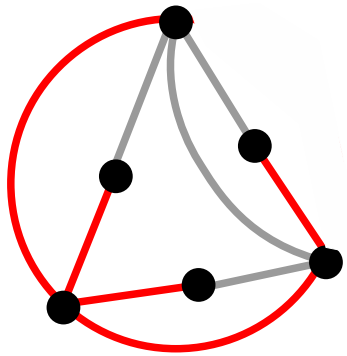


G_2



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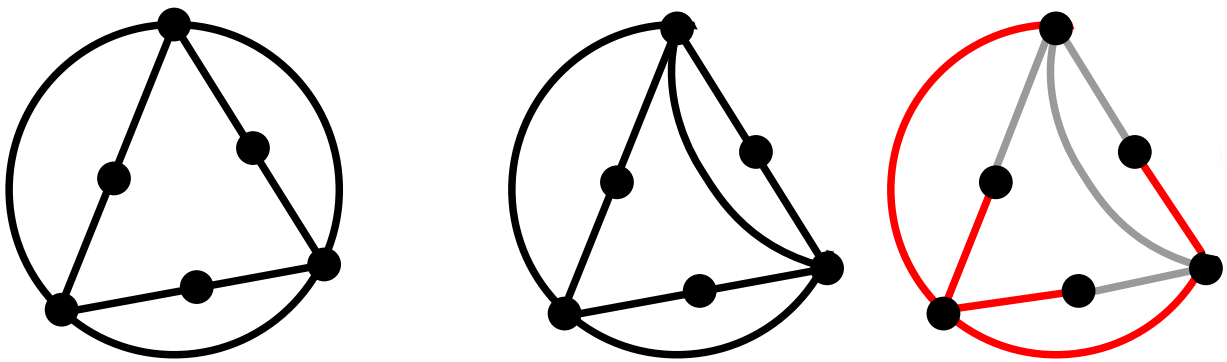
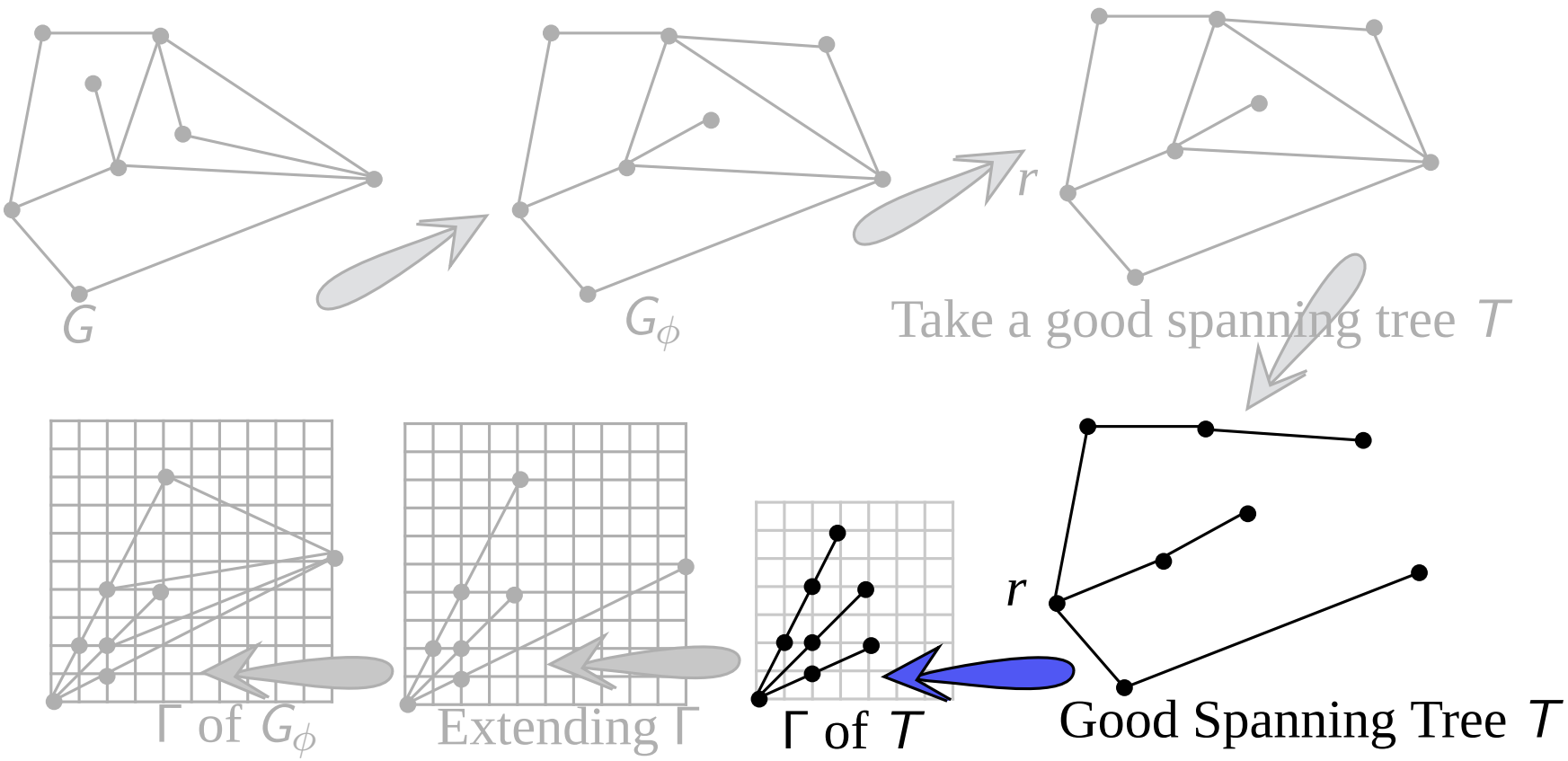


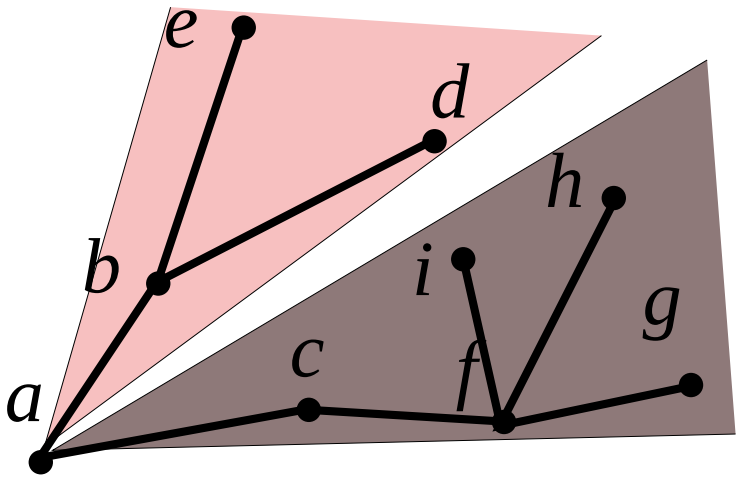
G_1 G_2

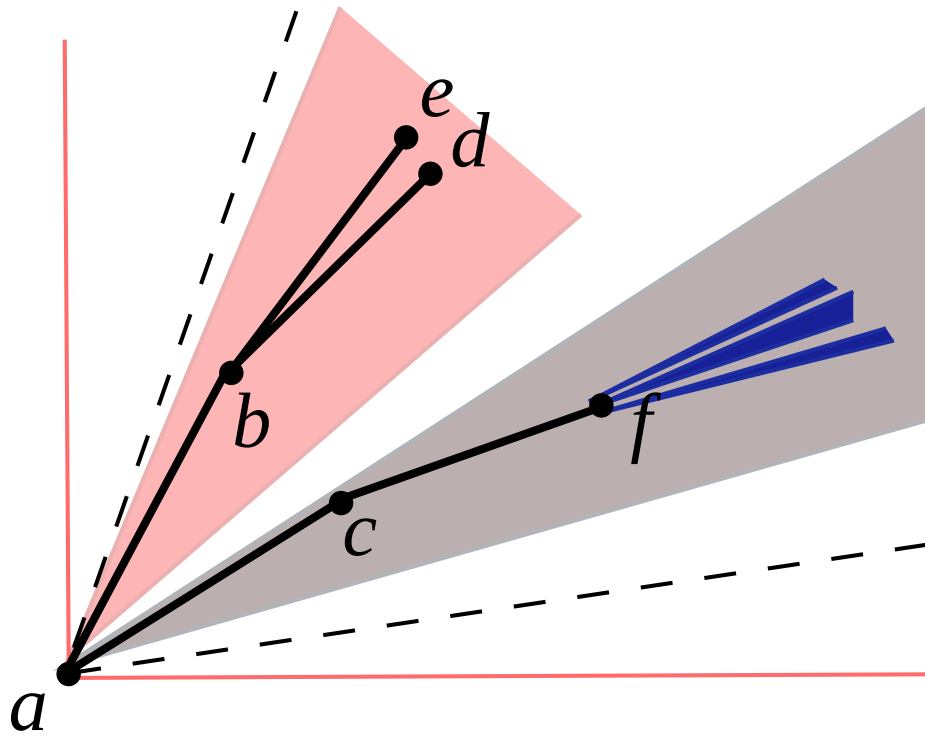
Every planar graph has an embedding with a good spanning tree

Algorithm Overview



Slope Disjoint Drawing of a Tree, [Angelini *et al.* 2012]





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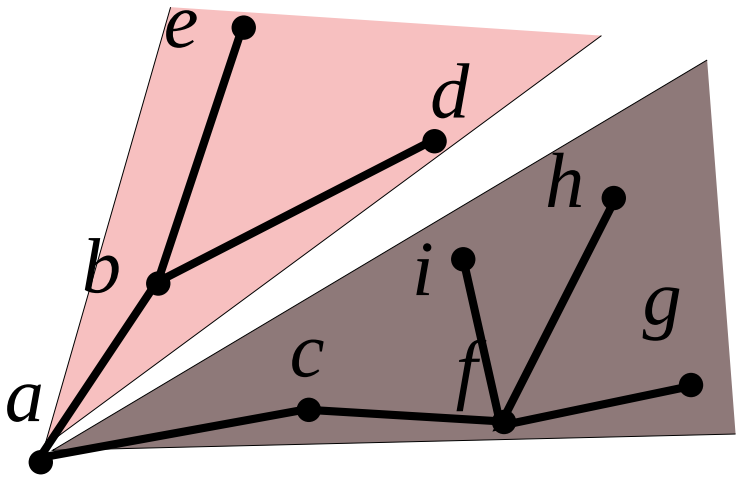
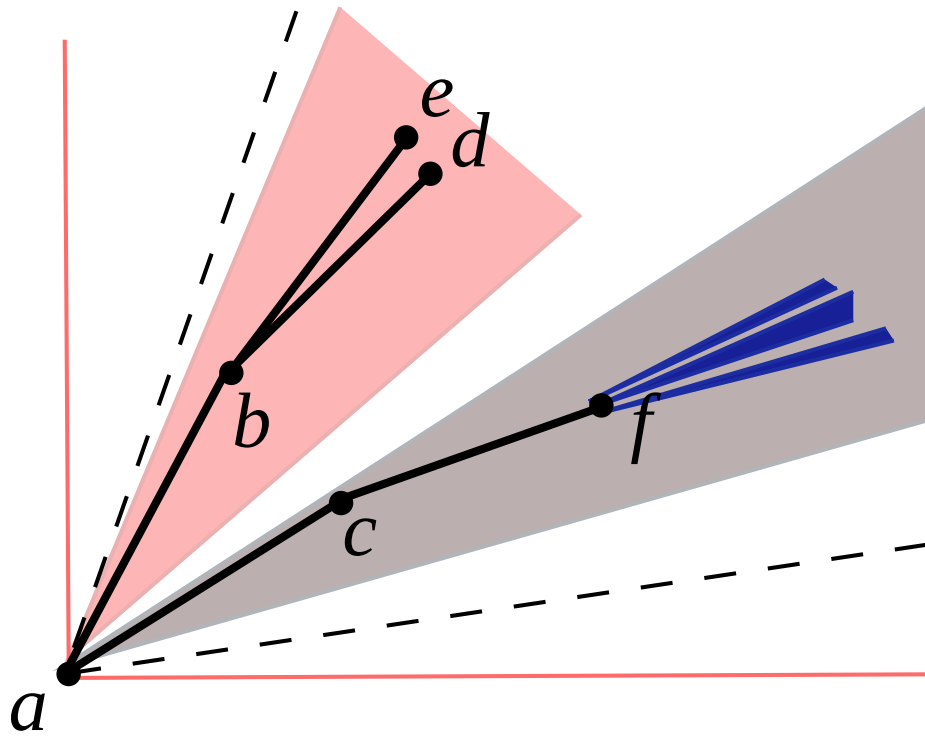
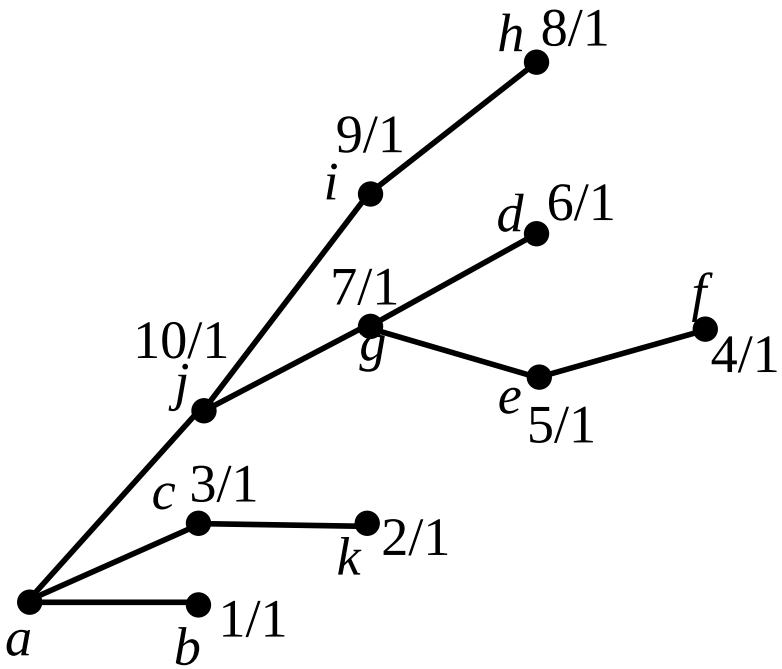
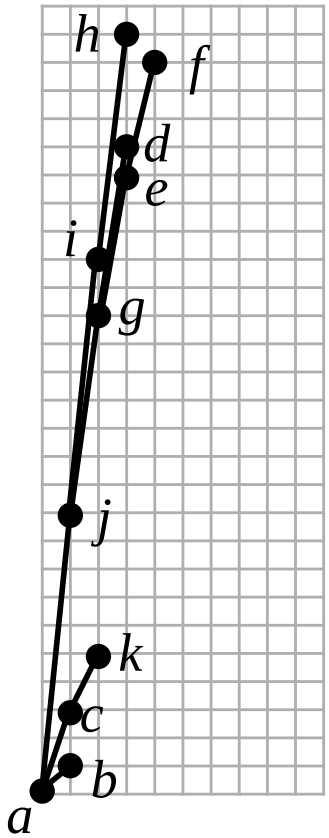




- A disjoint slope range will be assigned to each subtree.

Slope Disjoint Grid Drawing of a Tree, [Angelini *et al.*]





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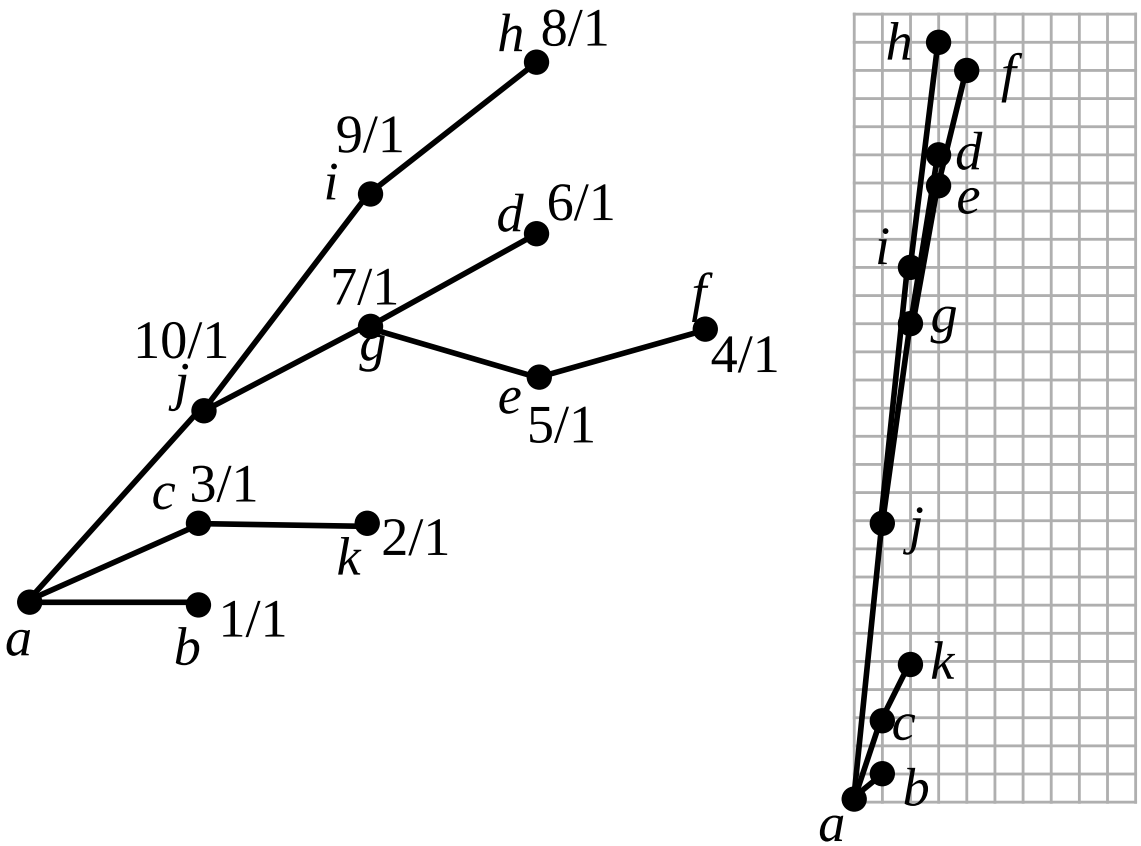
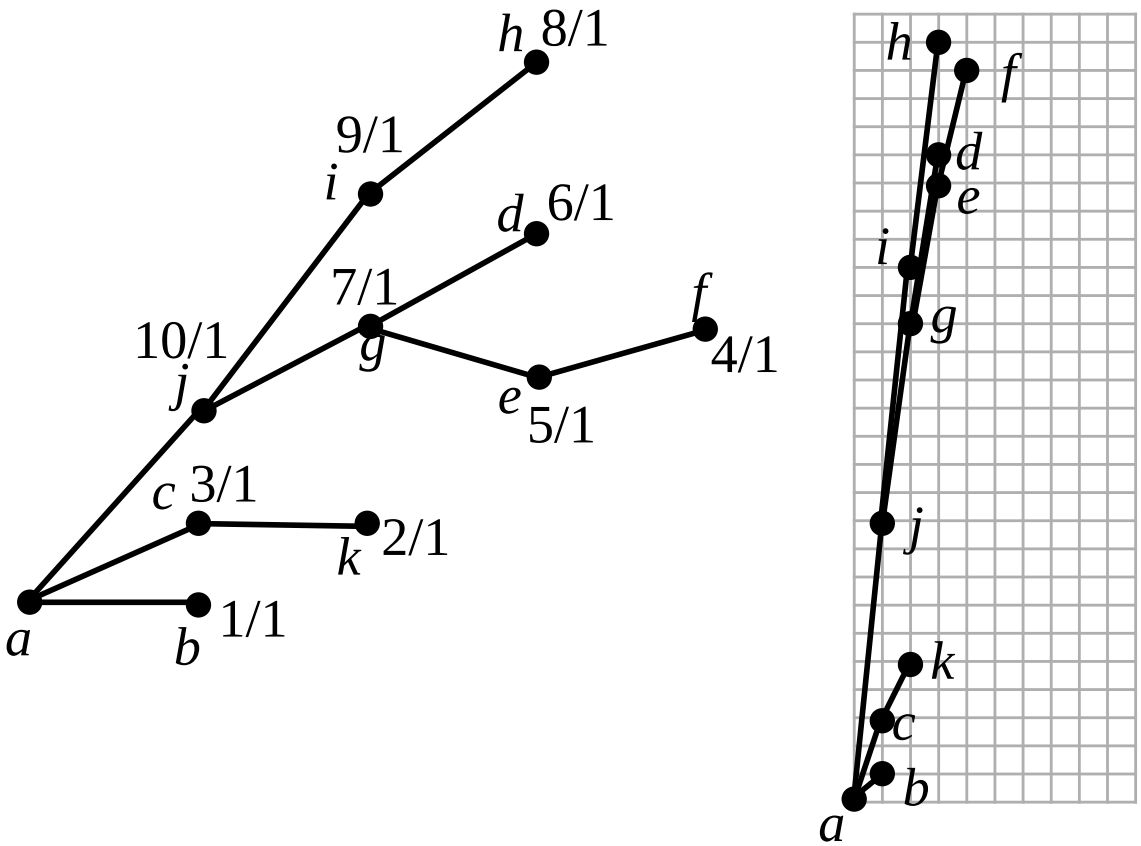


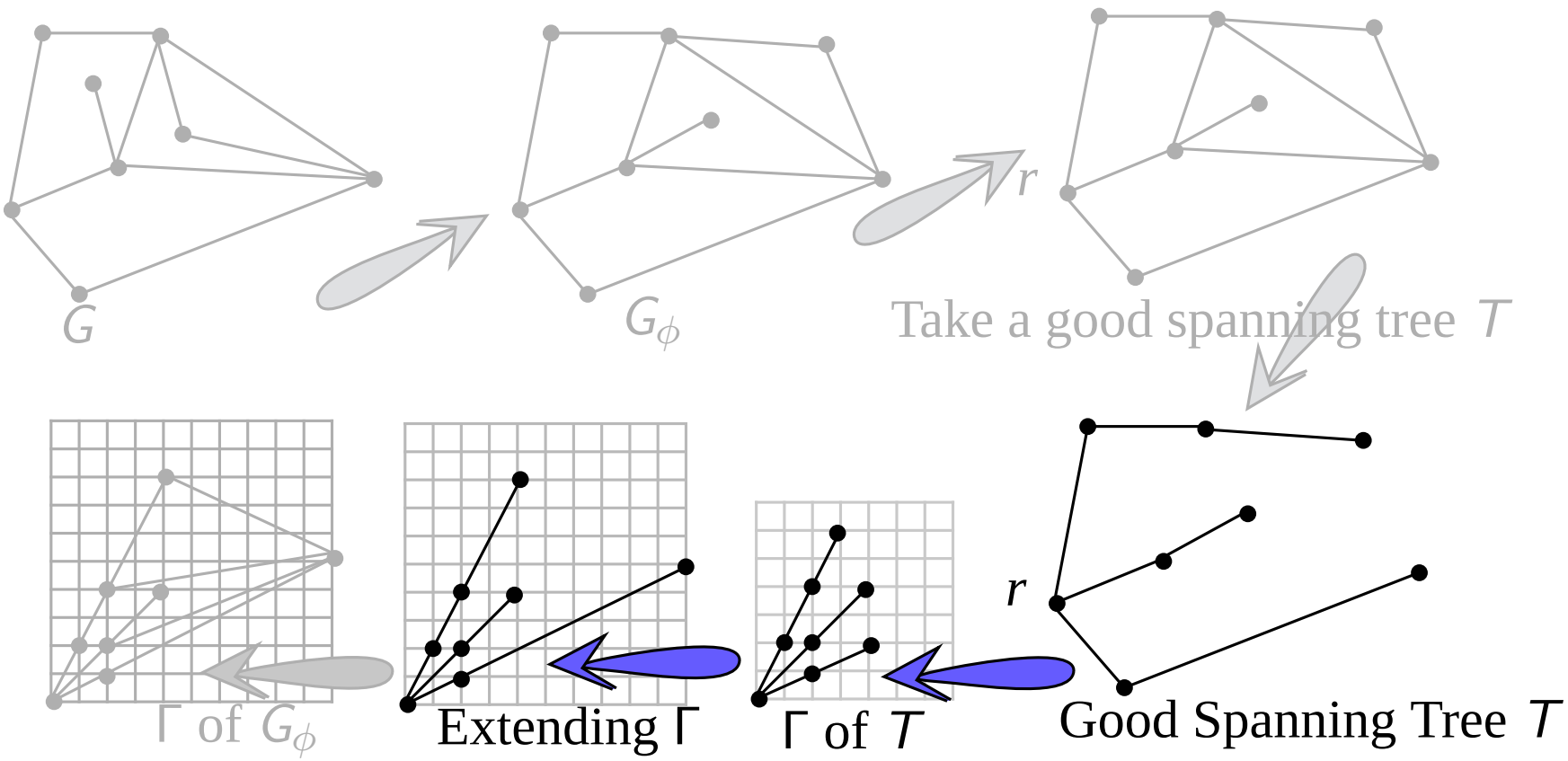
- Takes grid size $O(n) \times O(n^2)$

Slope Disjoint Grid Drawing of a Tree, [Angelini *et al.*]

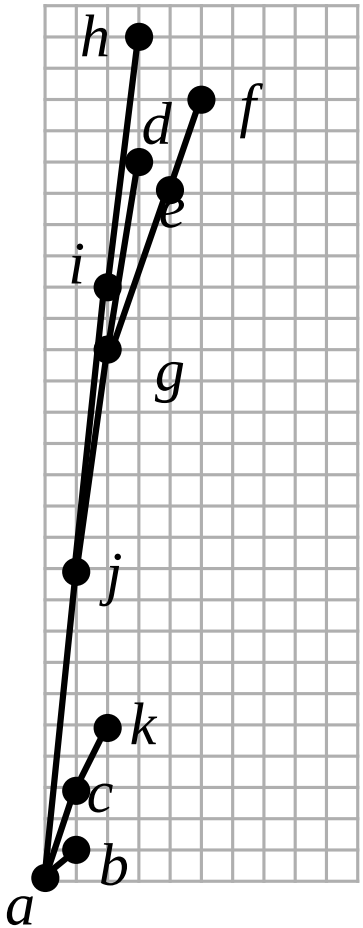


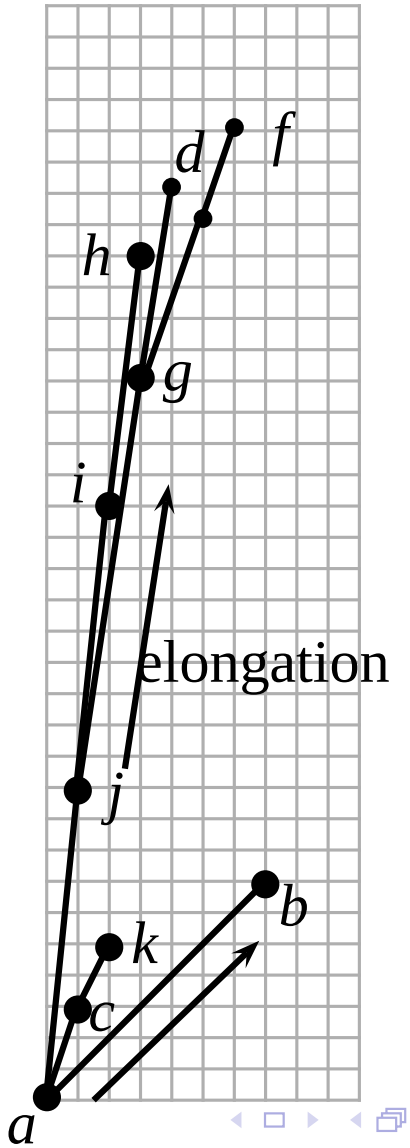
- Takes grid size $O(n) \times O(n^2)$
- Leaves can be extended upto infinity without any edge crossing, due to distinct slope

Algorithm Overview

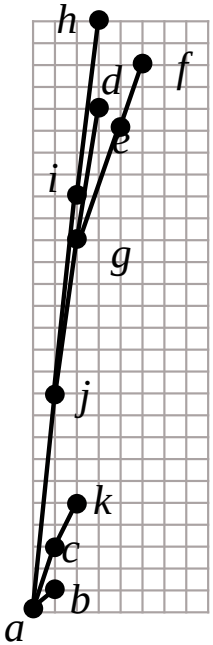


Elongation





Drawing Algorithm



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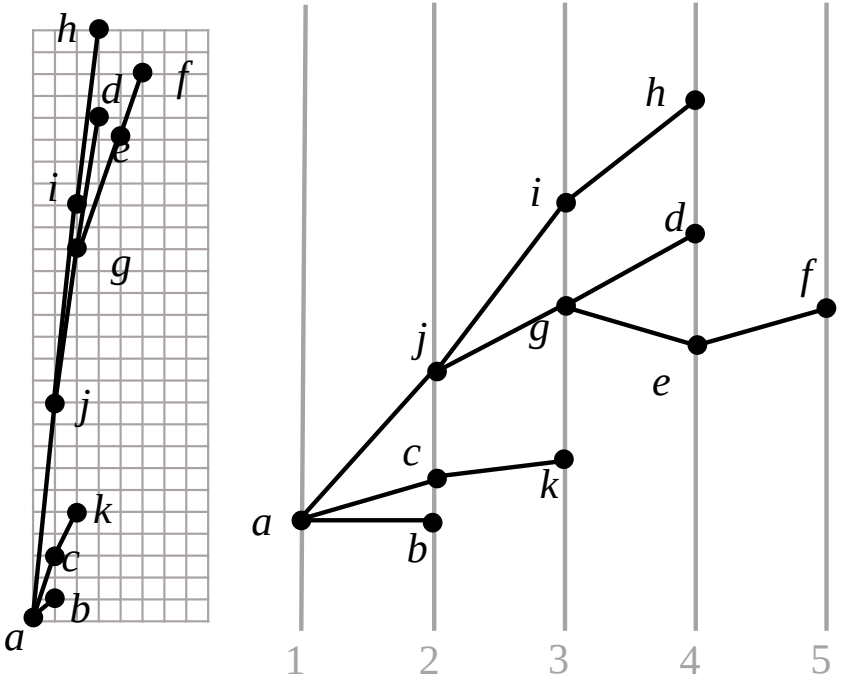

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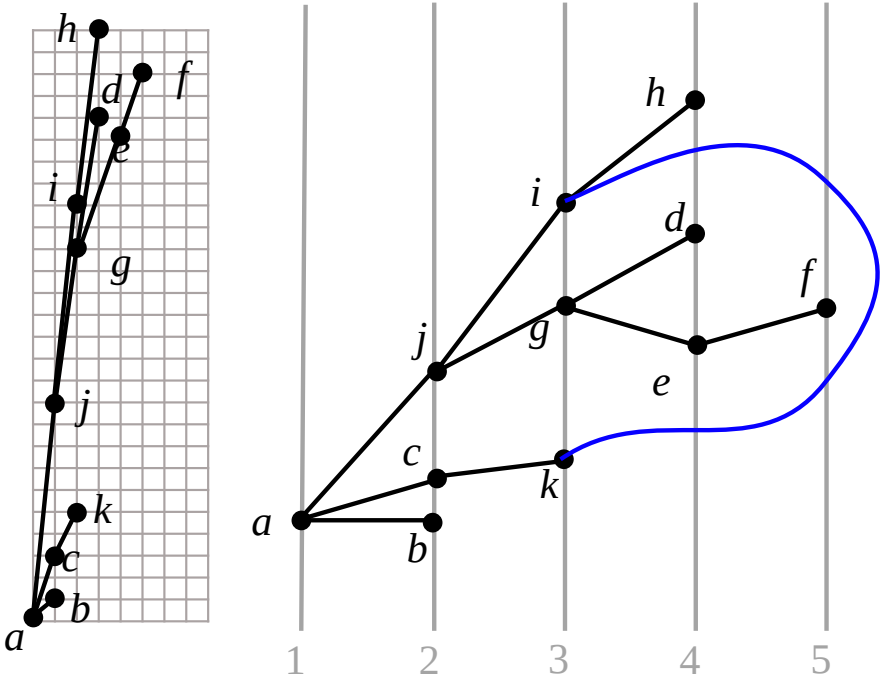

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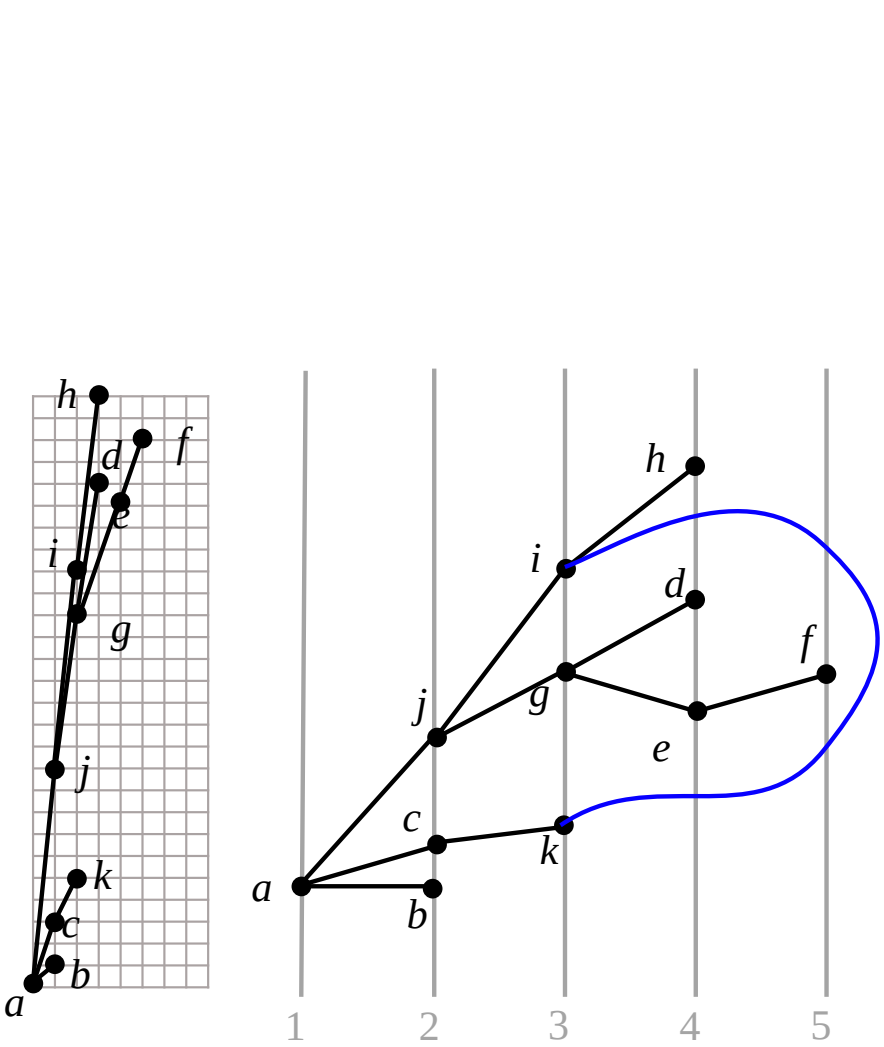
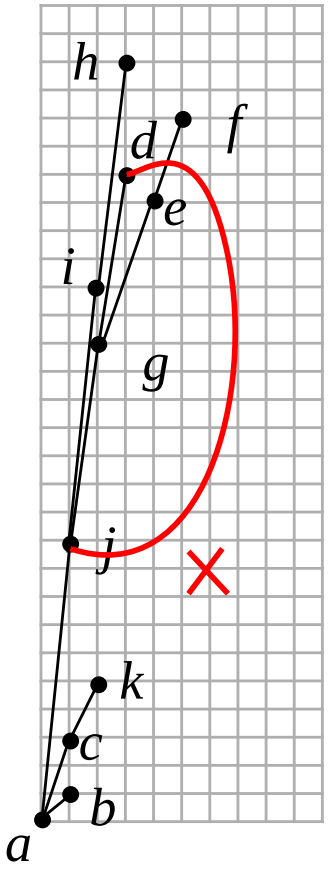


Properties of Good Spanning Tree



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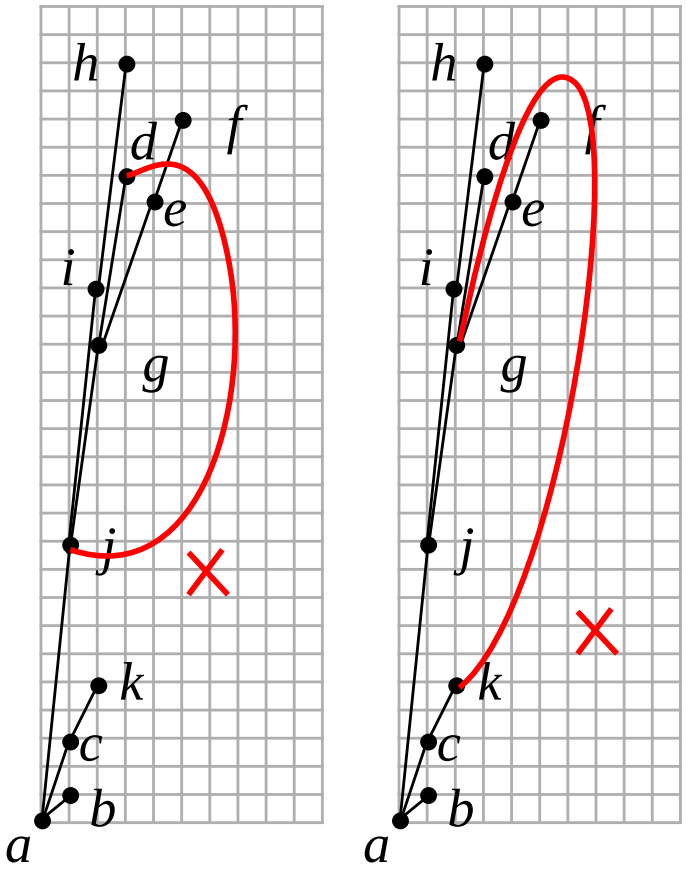

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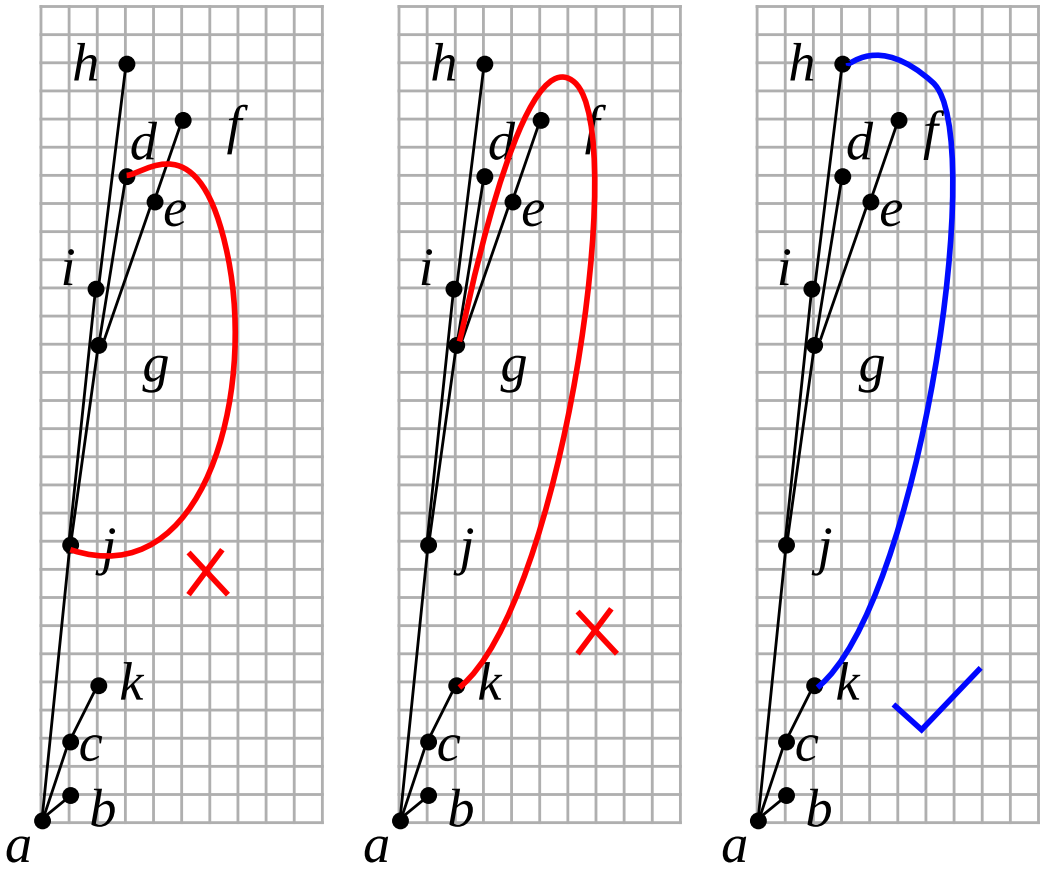

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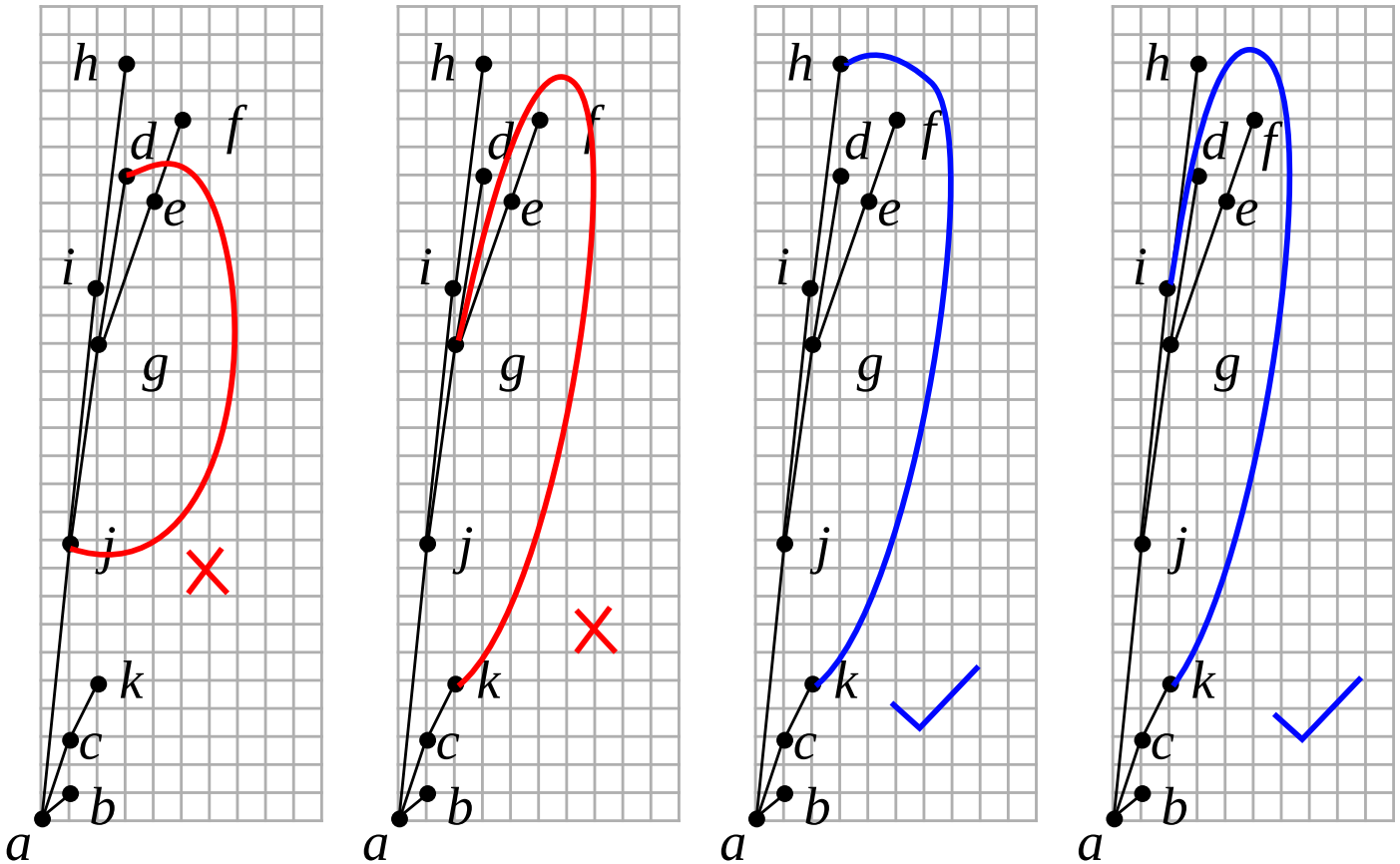
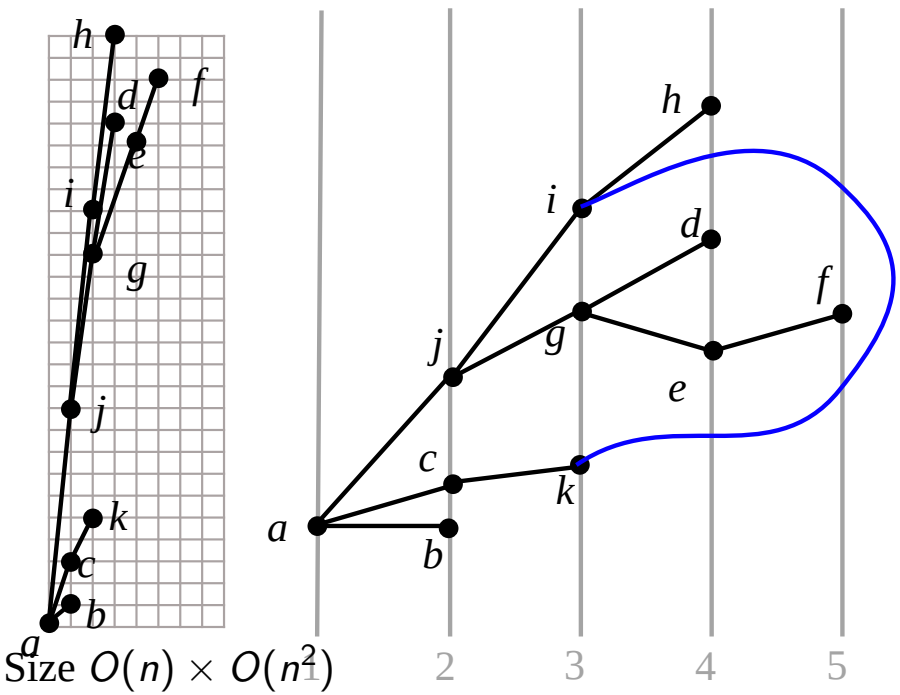


Grid Size

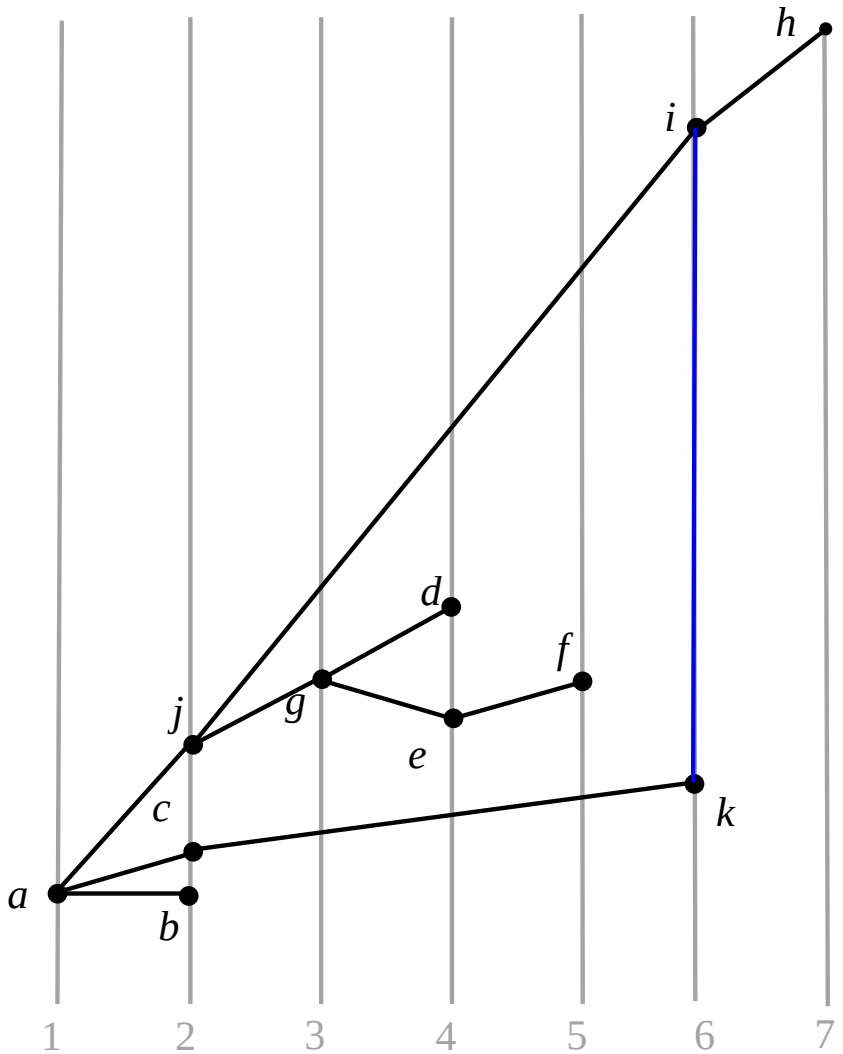
X axis increases by $O(n)$

Y axis increases by $O(n^2)$

Grid Size remains $O(n) \times O(n^2)$



Size $O(n) \times O(n^2)$

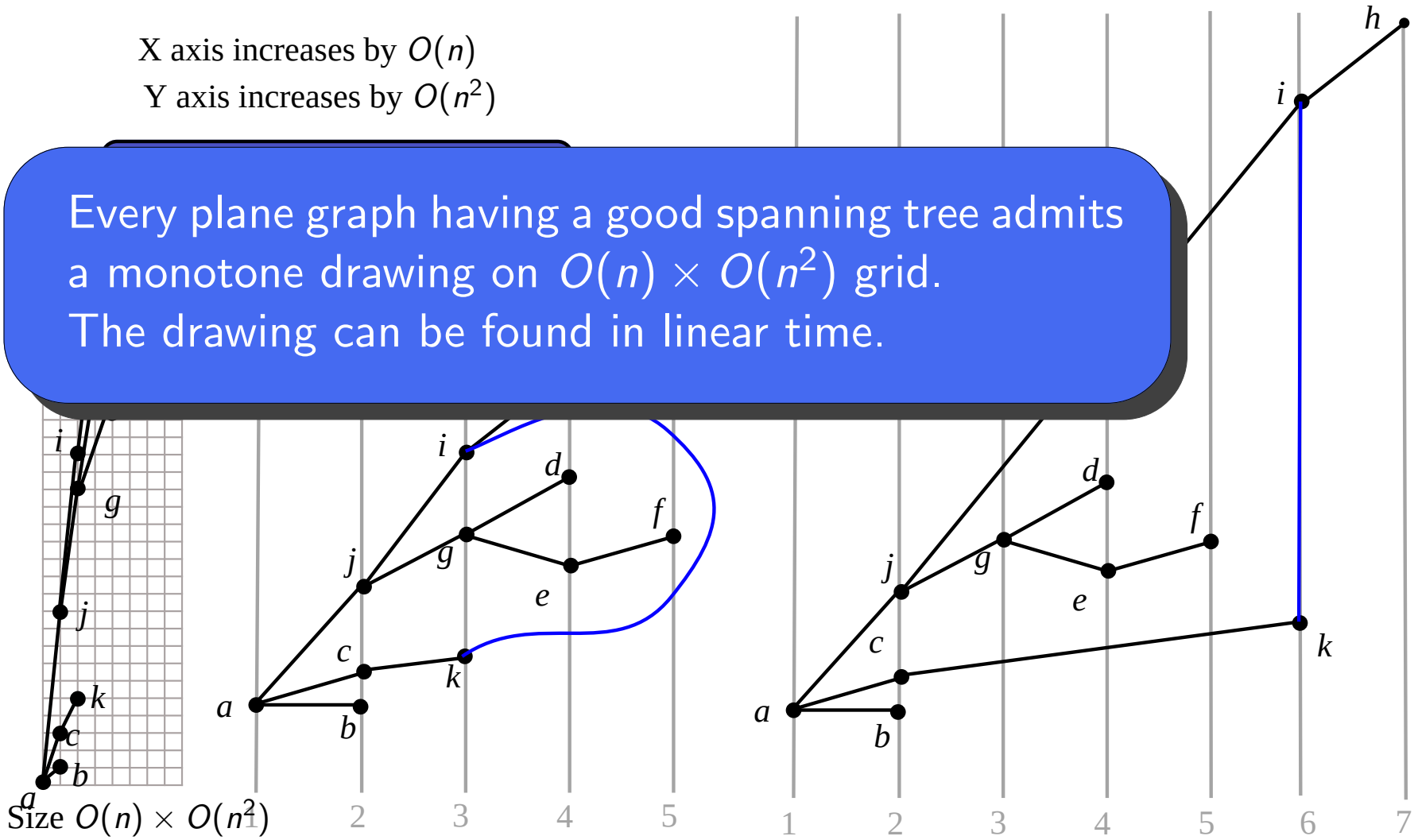


Grid Size

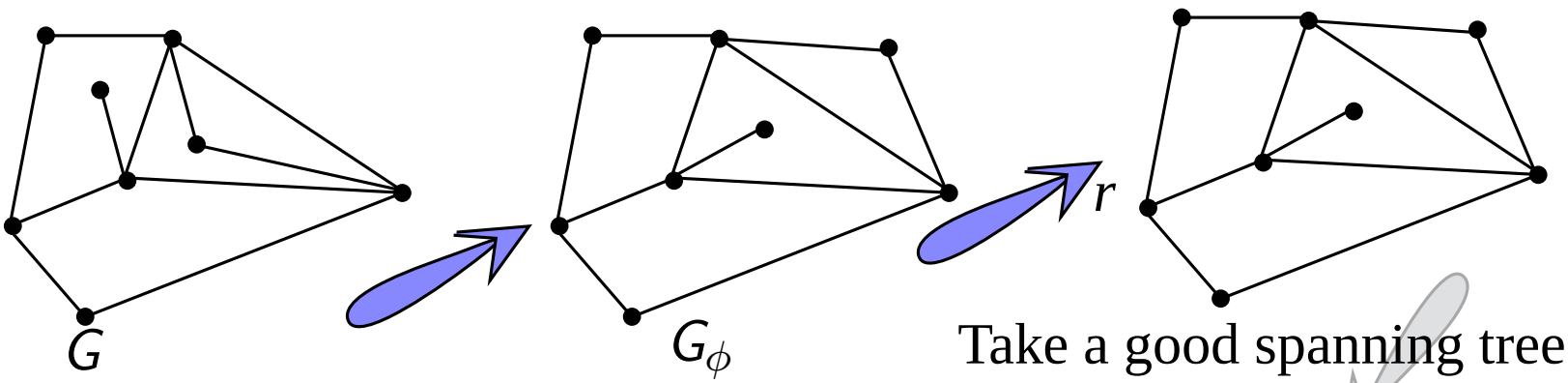
X axis increases by $O(n)$

Y axis increases by $O(n^2)$

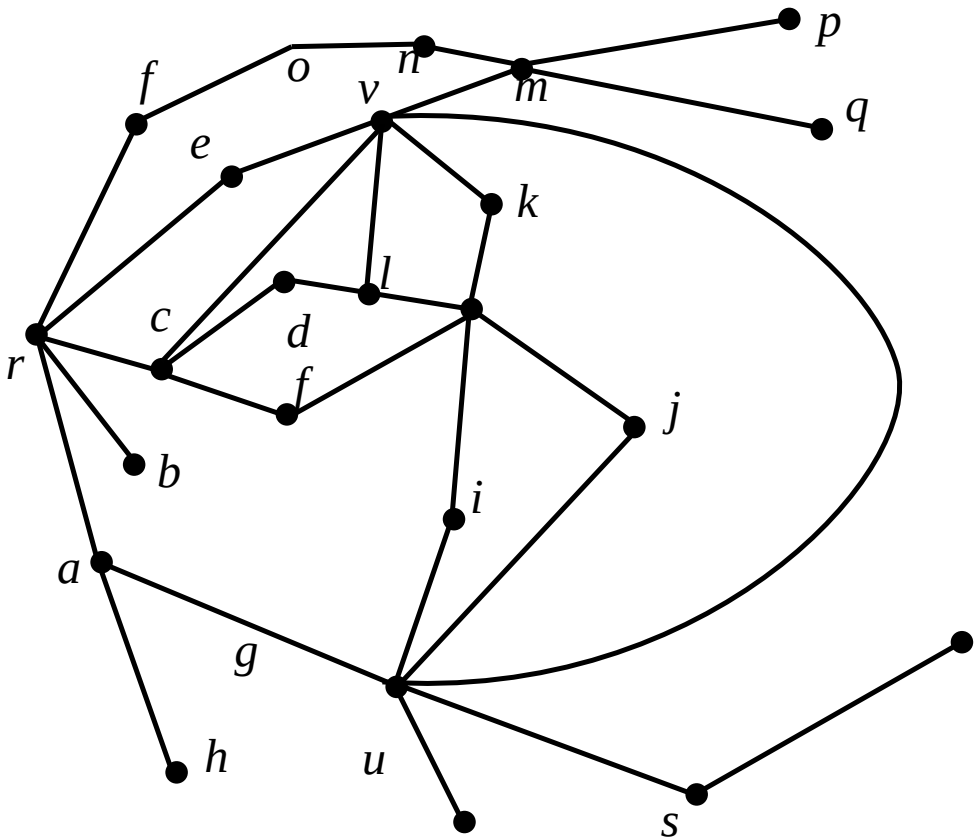
Every plane graph having a good spanning tree admits a monotone drawing on $O(n) \times O(n^2)$ grid.
The drawing can be found in linear time.



Algorithm Overview

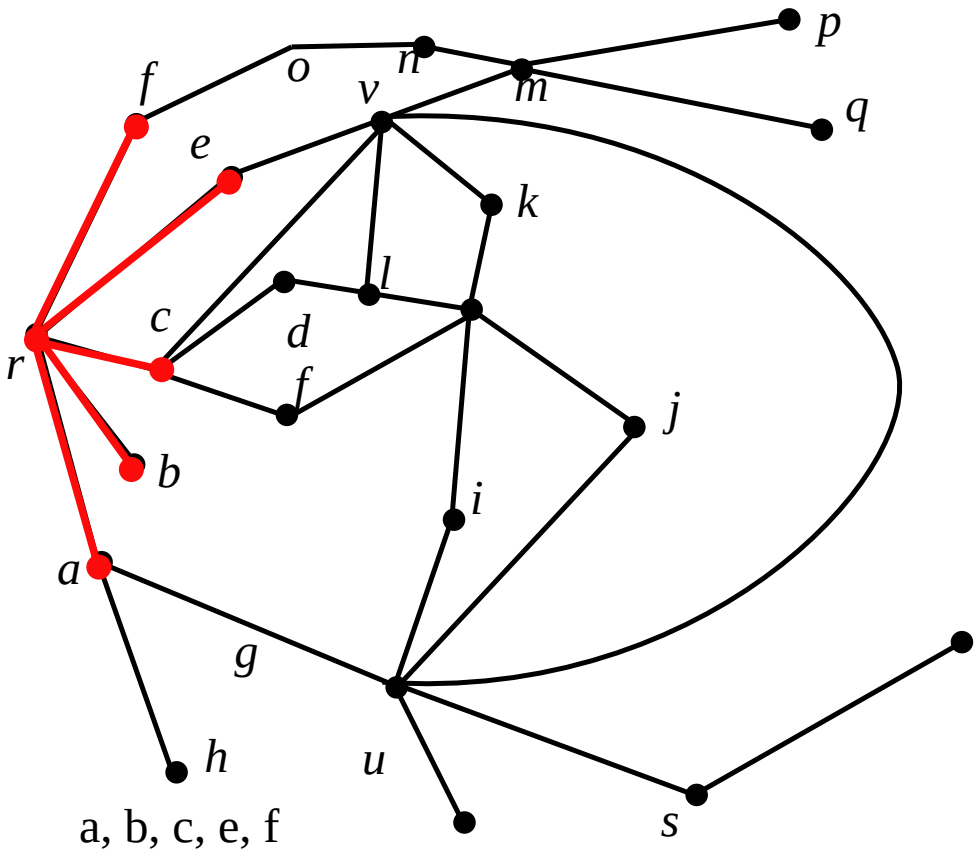


Finding Good Spanning Tree



1. we use BFS from an outer vertex.
2. in intermediate step we change embedding, if needed.

Finding Good Spanning Tree



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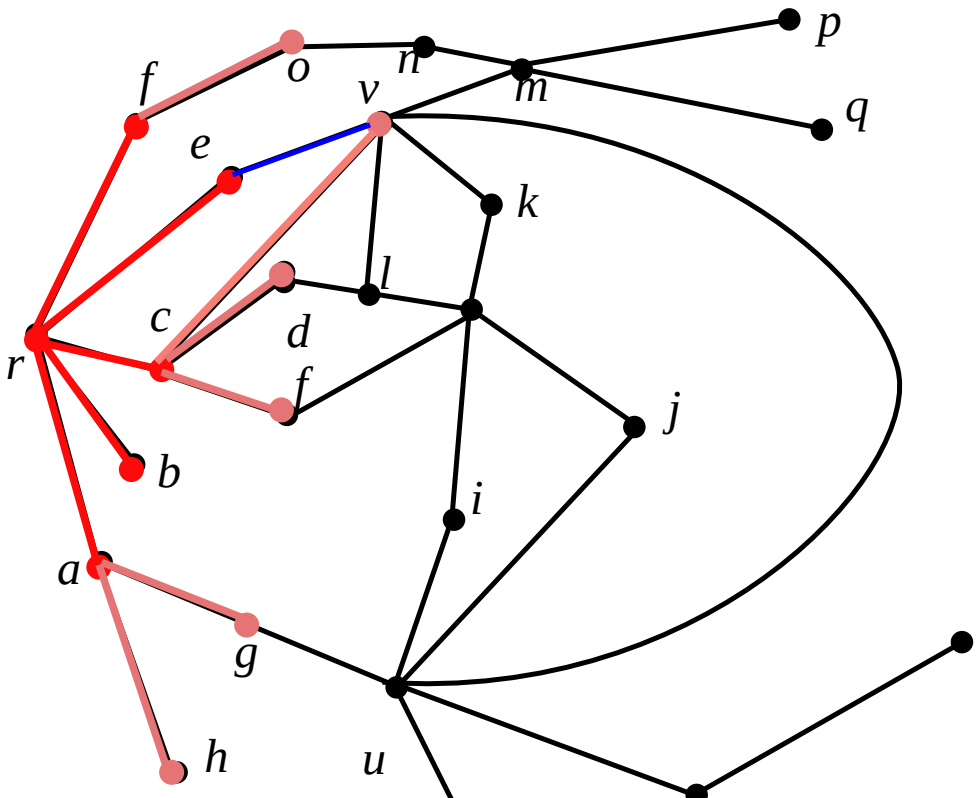
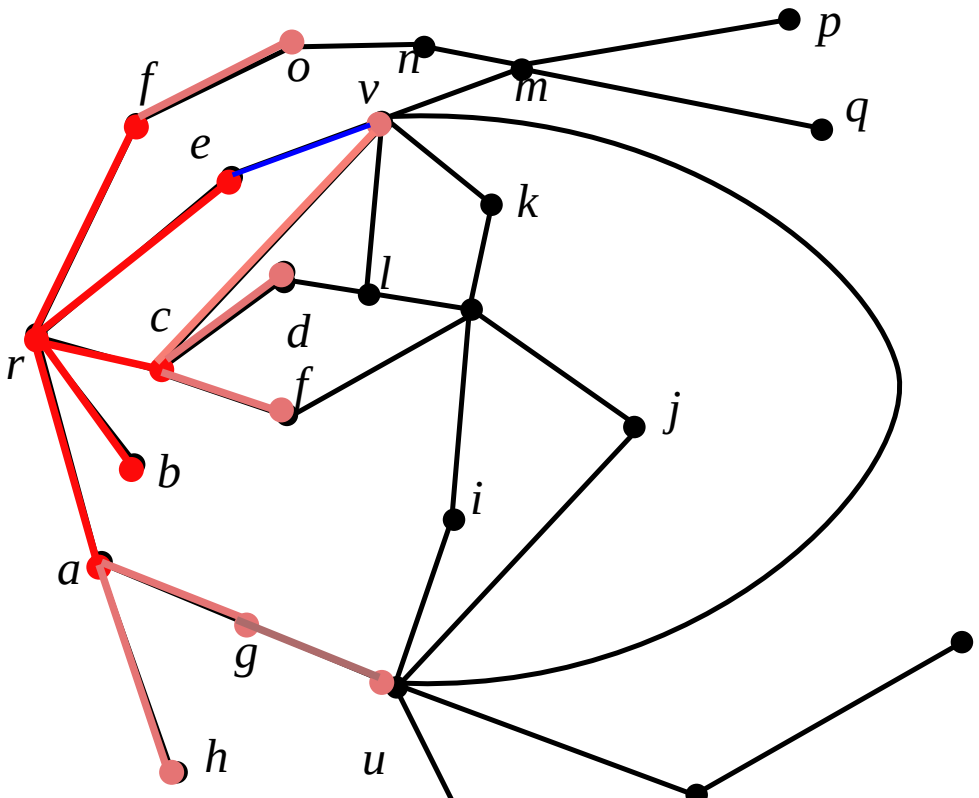


a, b, c, e, f

h, g, f, d, x, o

Finding Good Spanning Tree



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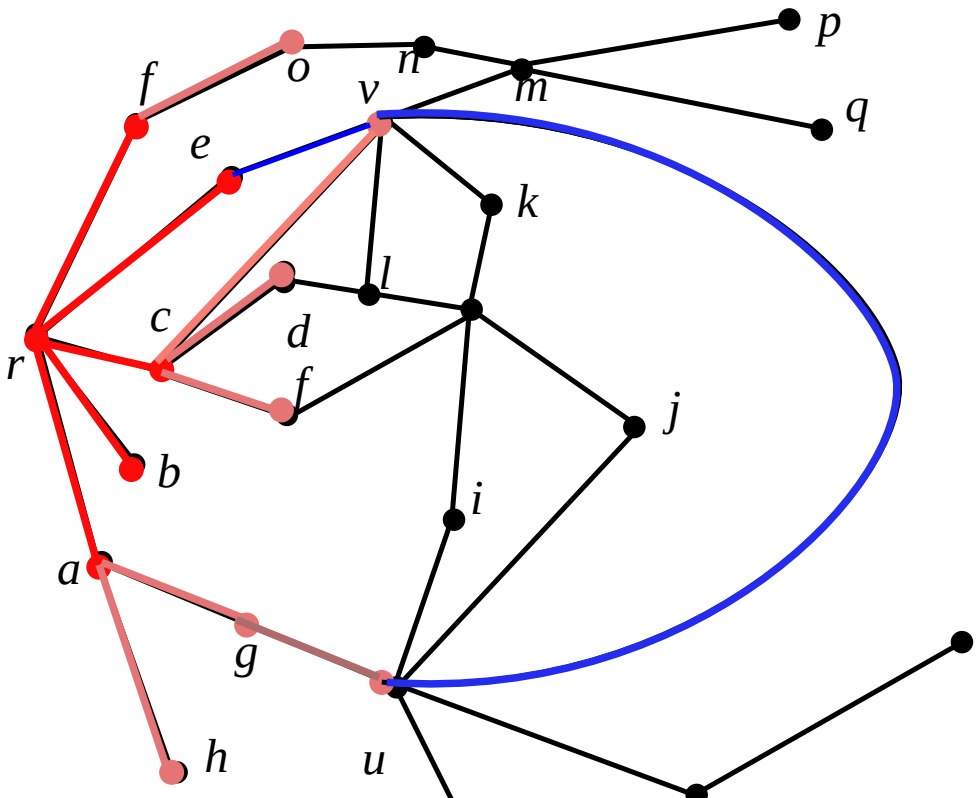
Finding Good Spanning Tree



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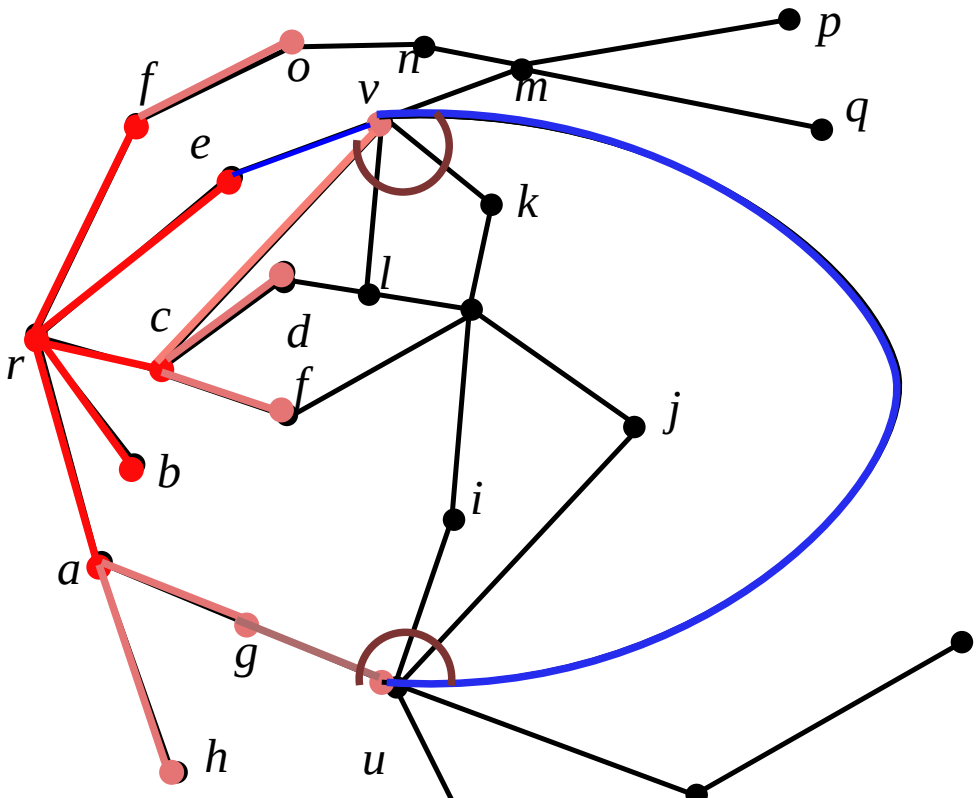

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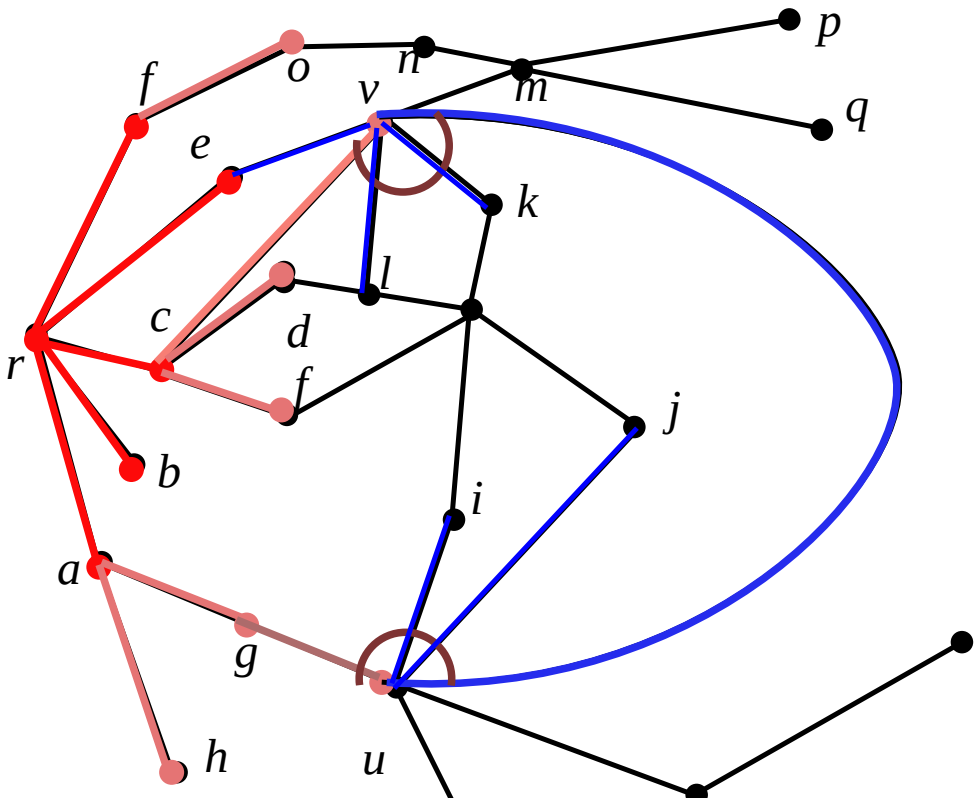

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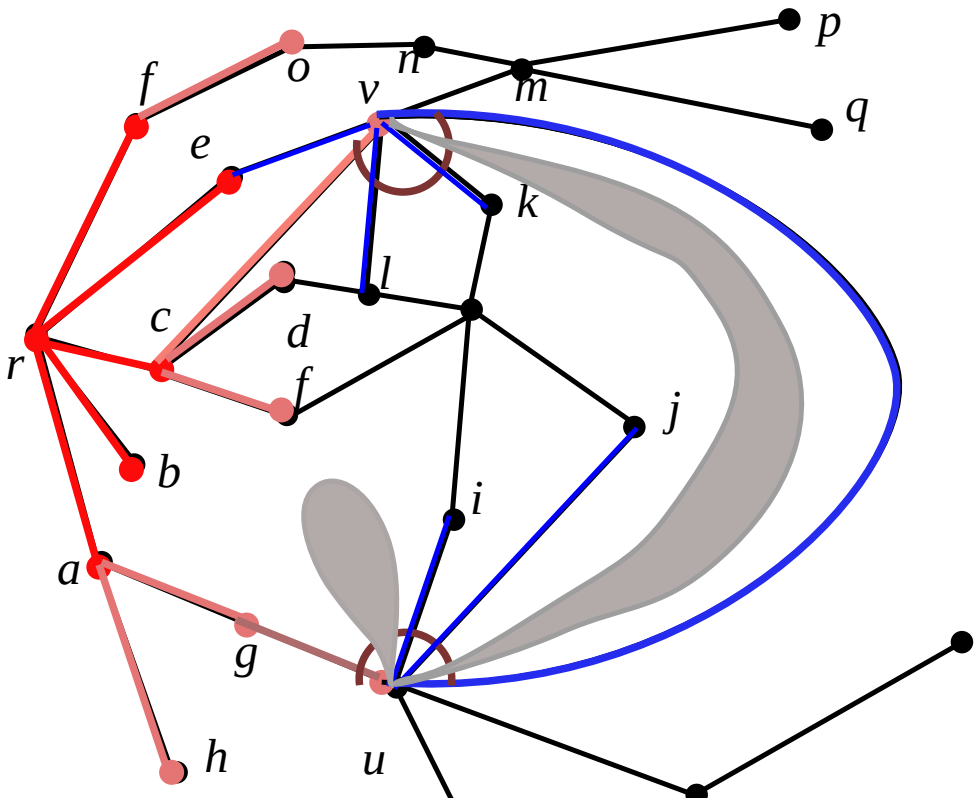

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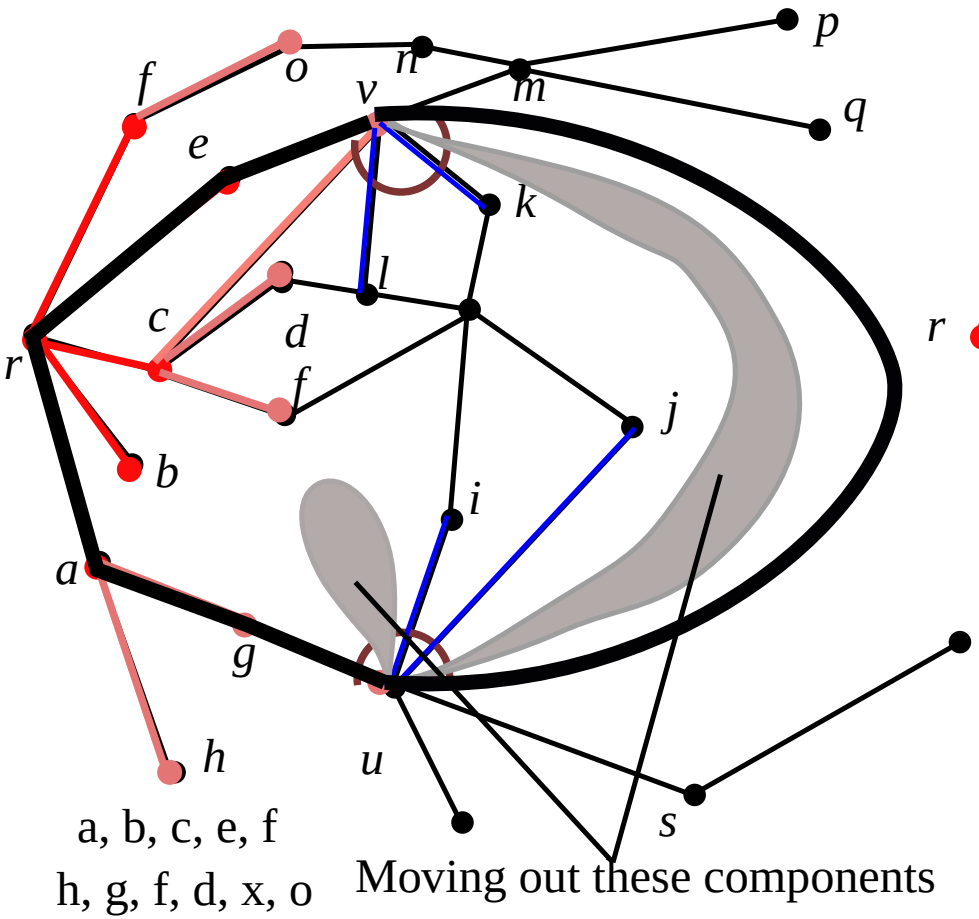

a, b, c, e, f
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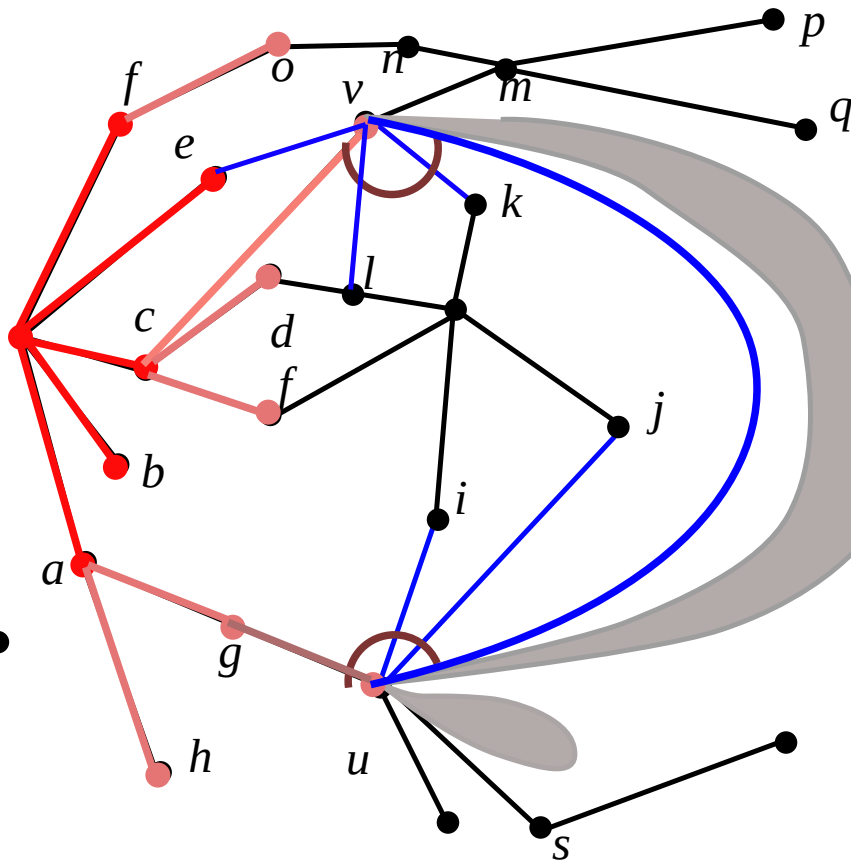
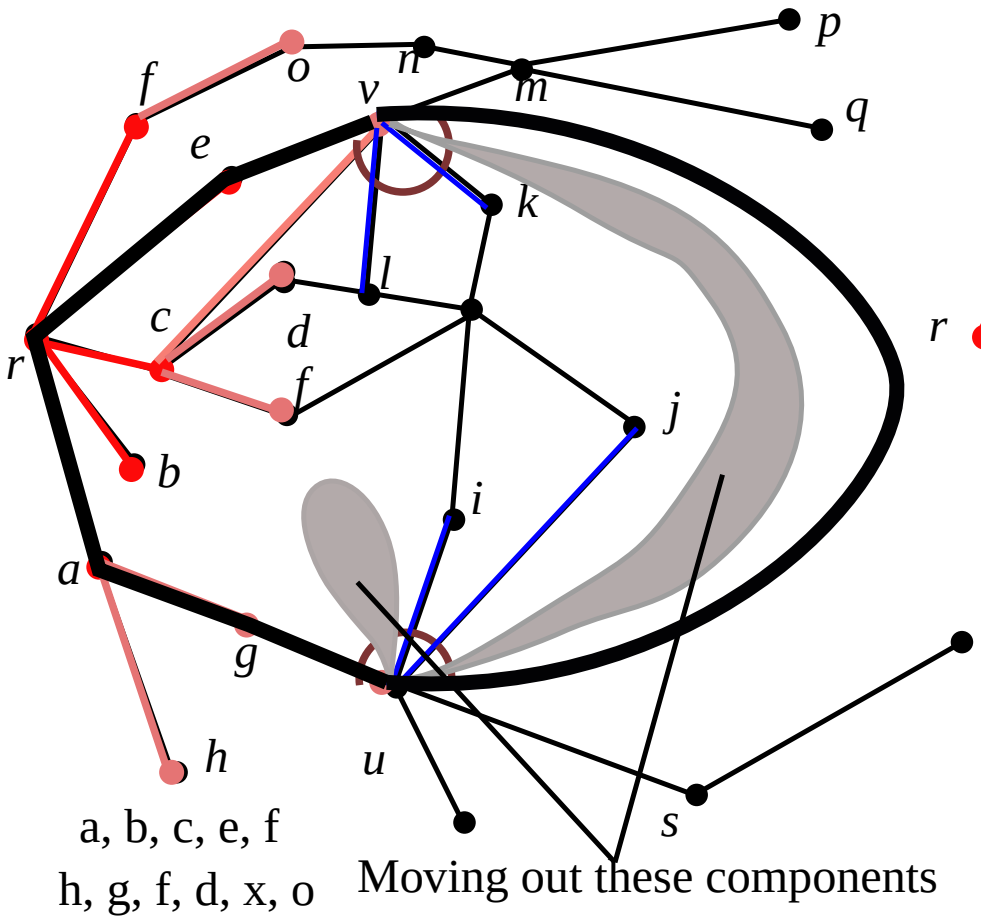


Moving out these components

Finding Good Spanning Tree



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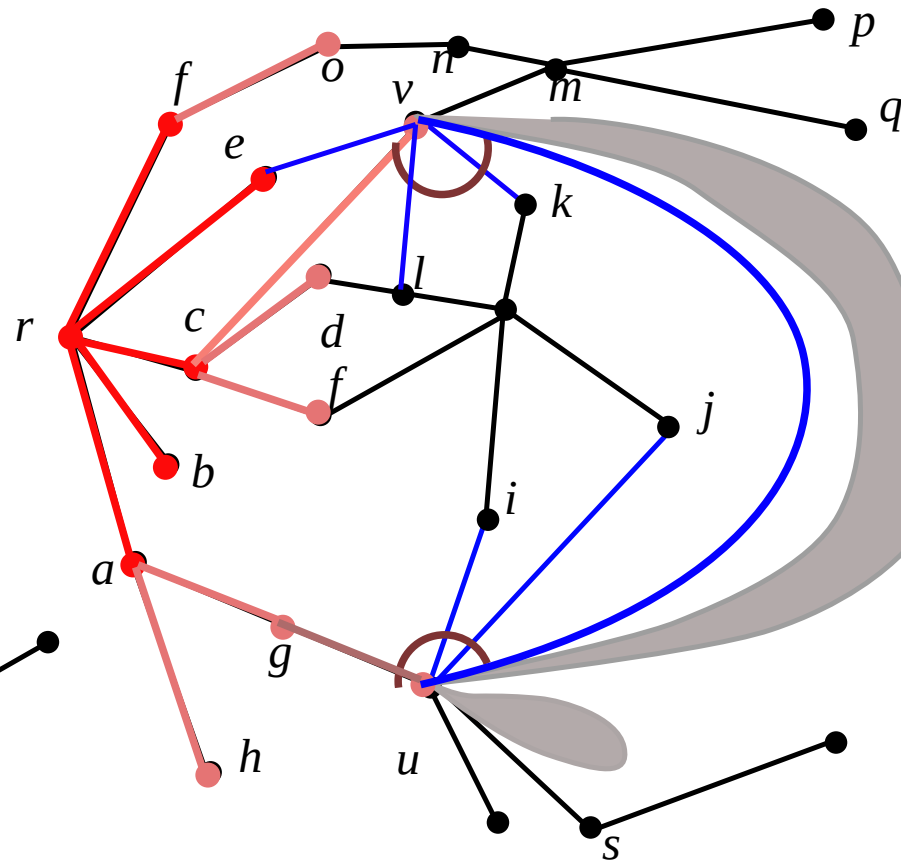
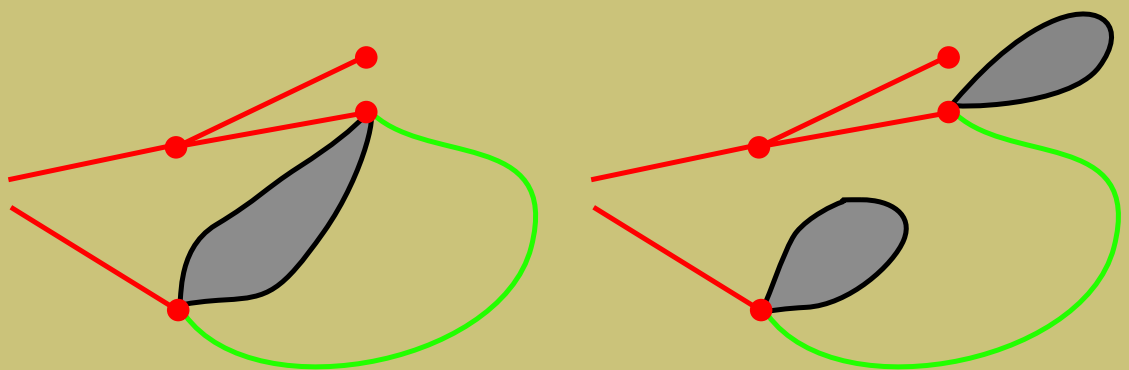


Every planar graph has a good spanning tree

Running Time

Computing all components on $O(n+m)$ time



$x-y$ component

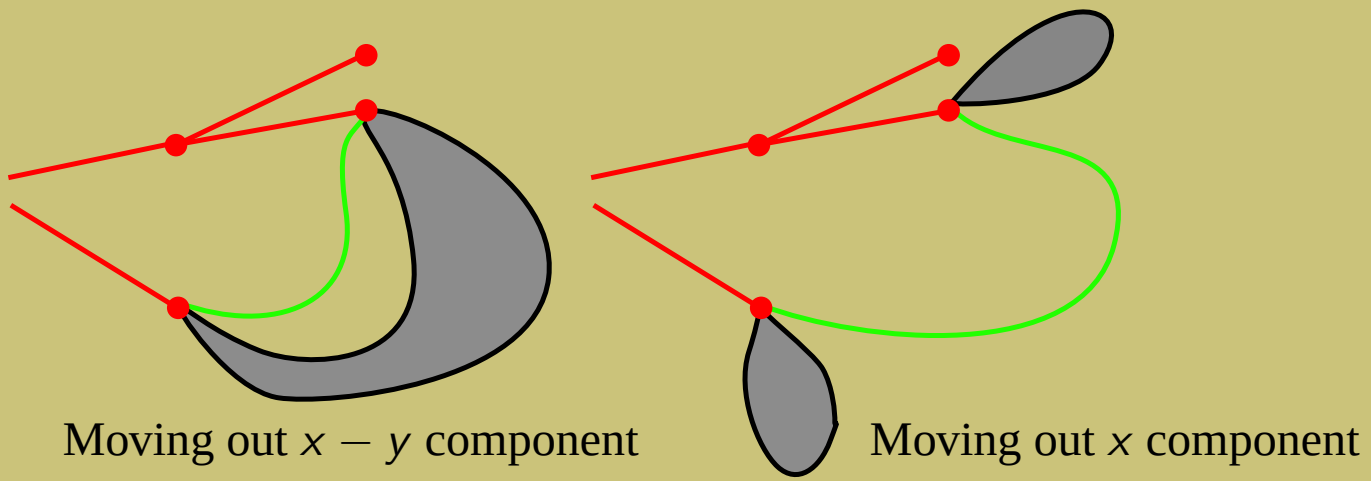
x component

Hopcroft and Tarjan

- All cut vertices and separation pair can be found in $O(n+m)$ time.
- Thus all v-components and all $\{u,v\}$-split components can be found in linear time.

Running Time

Moving per component in $O(1)$ time



- For moving a component outside of cycle takes $O(1)$ time.

Running Time

- All cut vertices can be found in $O(n+m)$ time using DFS.
- v-components and $\{u, v\}$-split components can be found in linear time.
- For moving each component outside of cycle takes $O(1)$ time.
- We maintain a data structure to store each cut vertex or every pair of vertices with split components.
- Since G is a planar graph, G_ϕ can be constructed in $O(n)$ time.

Conclusions

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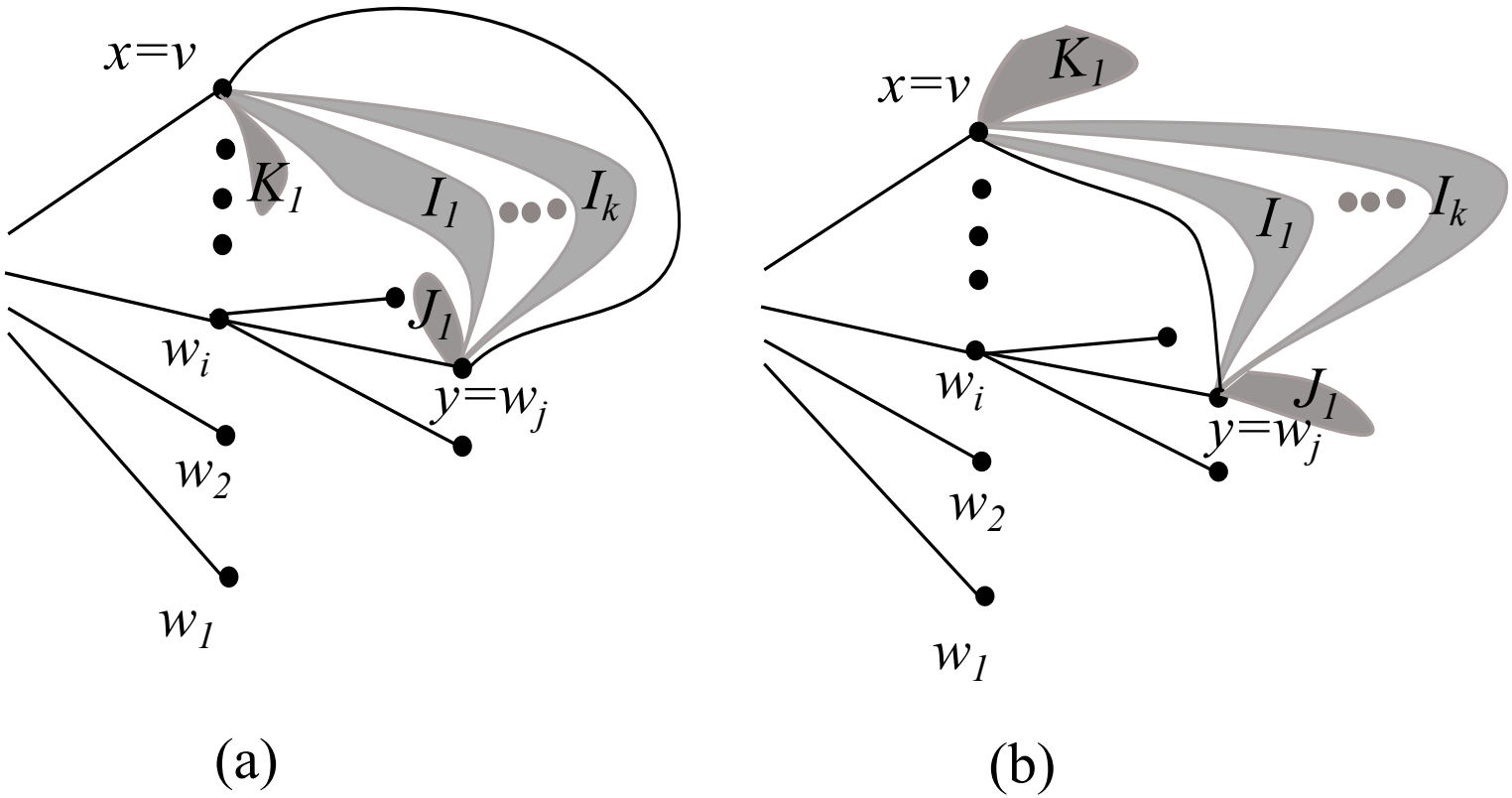
- We have shown that every planar graph has an embedding which have a good spanning tree.
- Good spanning tree can be considered as a generalization of Schyder's realizer. Thus we are hopeful that good spanning tree will find many applications in graph drawing area.

Open Problem

How to determine whether a given plane graph has a good spanning tree or not?

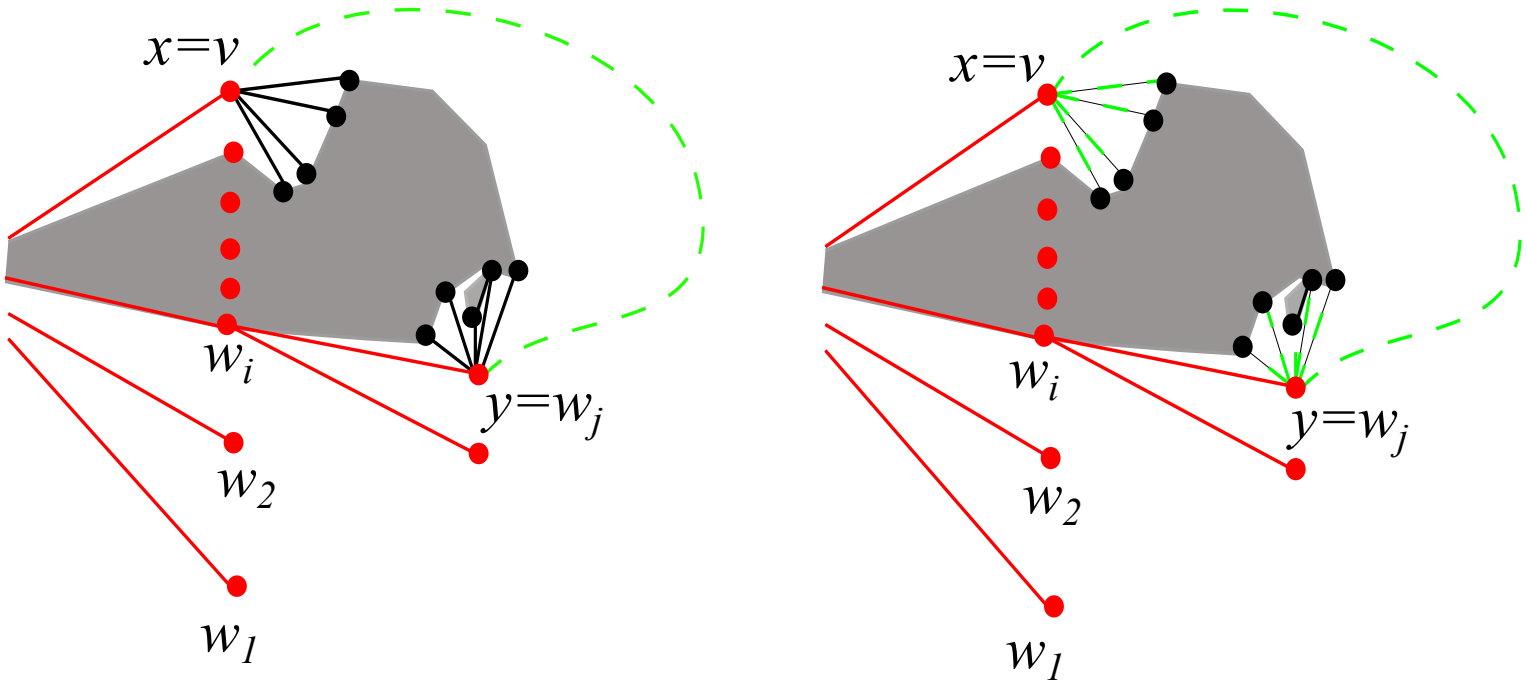
Thank You

Finding Good Spanning Tree: x-component, $\{x, y\}$-component



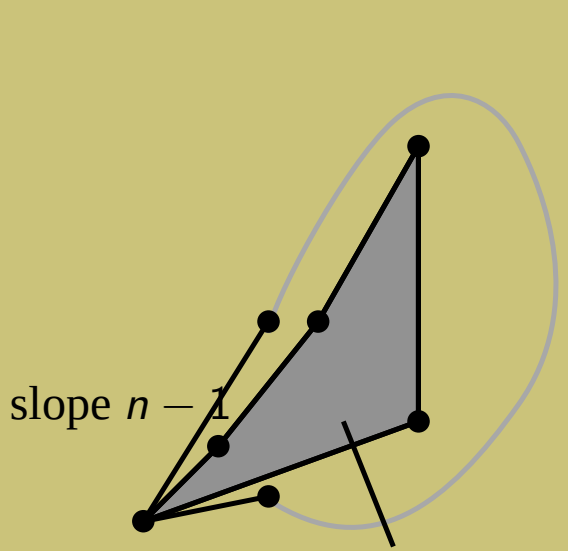
Moving x-components and $\{x, y\}$-components out of cycle.

Finding Good Spanning Tree: non-tree edge by force

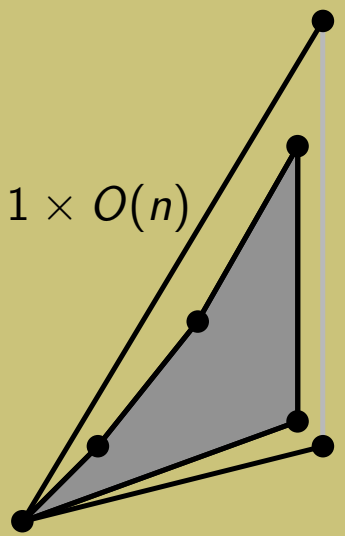


Every planar graph has a good spanning tree

Explanation of size of Y axis

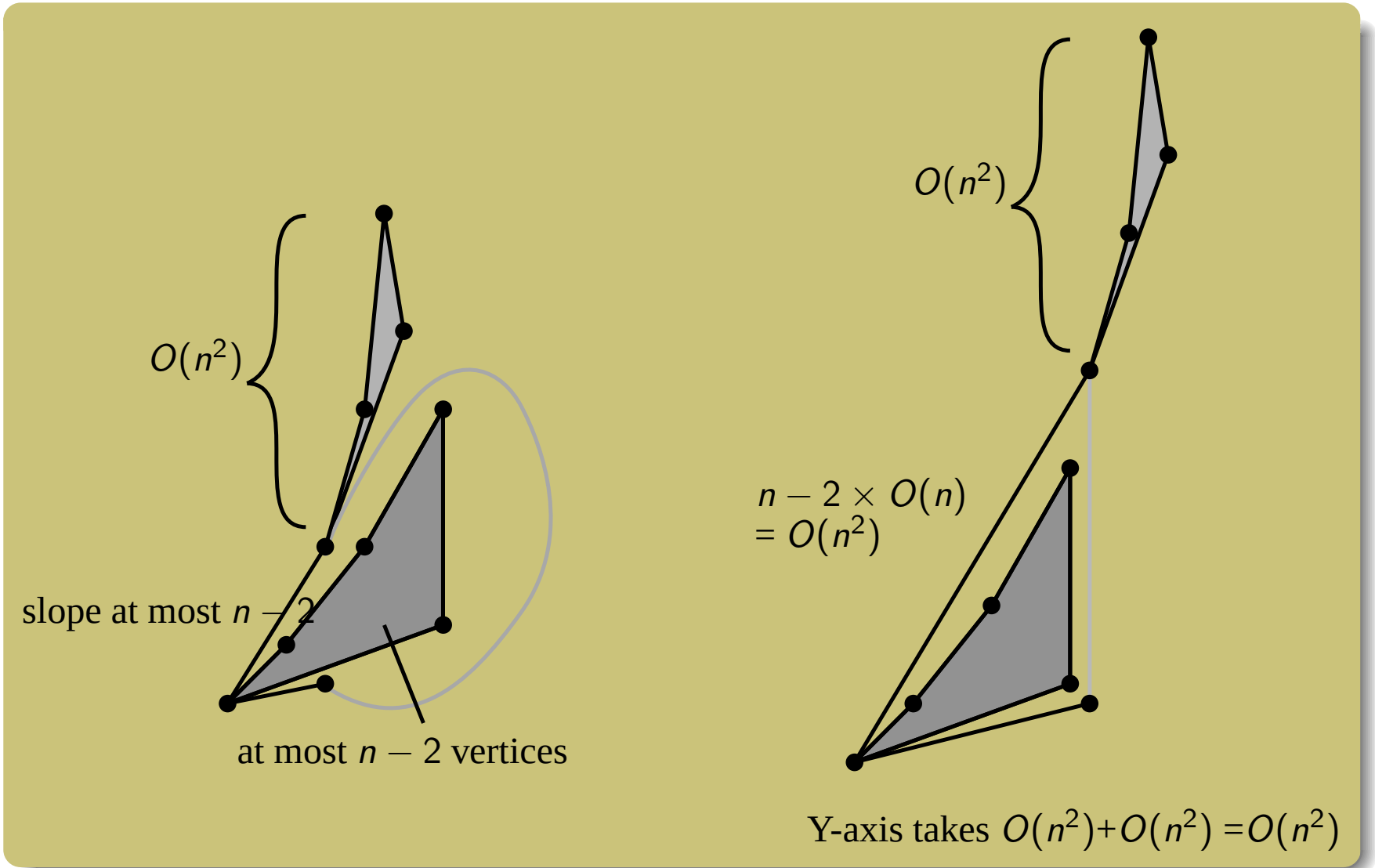


at most $n-1$ vertices, x axis $O(n)$



Y-axis takes $O(n^2)$

Explanation of size of Y axis



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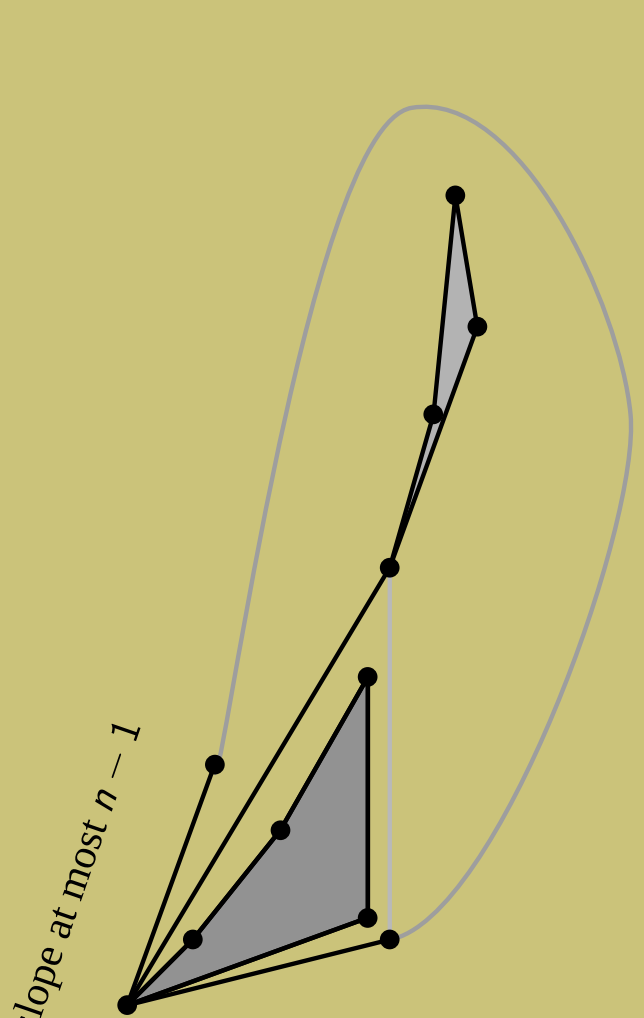
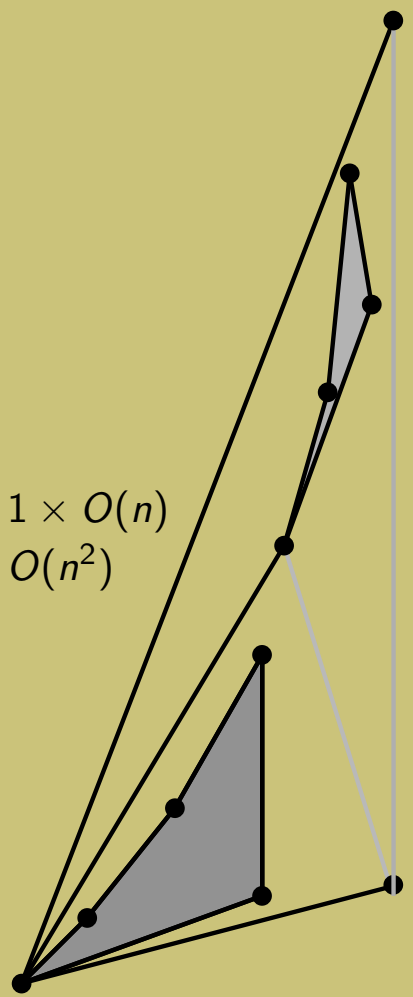


at most n-1 vertices inside of cycle



Y-axis takes $O(n^2)$