
MPO

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TUROTIAL

1	Installation	3
2	Tutorial	5
2.1	Solving a MPQP Program	5
3	API	9
3.1	mpo package	9
4	Indices and tables	35
	Python Module Index	37
	Index	39

MPO is a multiparametric programming solver written in Python, meant for solving general mpQPs and mpLPs with support for mixed integer and Quadratically constrained problems prospectively in the future. Optimized implementations of combinatorial algorithms and graph-based algorithms have been implemented. A focus of this solver is to implement parallel and scalable algorithms for multithreading compute.

INSTALLATION

All you need to do is the following pip command in the relevant console.

```
pip install git+https://github.com/dkenefake/mpo.git
```


TUTORIAL

2.1 Solving a MPQP Program

Here we are going to solve a classic transportation problem with multiparametric uncertainty. We have a set of plants and a set of markets with corresponding supplies and demand, and we want to minimize the transport cost between the plants and ensuring we satisfy all market demand. The multiparametric formulation is fleshed out in more detail in Multiparametric Optimization and Control by Pistikopoulos et al.

This optimization problem leads to the following multiparametric optimization problem, with θ representing the markets' uncertain demands.

$$\begin{aligned} \min_x \quad & \frac{1}{2} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}^T \begin{bmatrix} 306.0 & 0 & 0 & 0 \\ 0 & 324.0 & 0 & 0 \\ 0 & 0 & 324.0 & 0 \\ 0 & 0 & 0 & 252.0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 25.0 \\ 25.0 \\ 25.0 \\ 25.0 \end{bmatrix}^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \\ \text{s.t.} \quad & \begin{bmatrix} 1.0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 1.0 \\ -1.0 & 0 & -1.0 & 0 \\ 0 & -1.0 & 0 & -1.0 \\ -1.0 & 0 & 0 & 0 \\ 0 & -1.0 & 0 & 0 \\ 0 & 0 & -1.0 & 0 \\ 0 & 0 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 350.0 \\ 600.0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1.0 & 0 \\ 0 & -1.0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \\ & \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -1.0 & 0 \\ 0 & -1.0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \leq \begin{bmatrix} 1e+03 \\ 1e+03 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

```

1.0
00 0
1.0 0
0 1.0 - 1.0
00 -1.0
0 -1.0
0 -1.0 - 1.0
00 0
0 -1.0
0 00
00 -1.0
0 0
-1.0

```

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 350.0 \\ 600.0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1.0 & 0 \\ 0 & -1.0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -1.0 & 0 \\ 0 & -1.0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \leq \begin{bmatrix} 1e+03 \\ 1e+03 \\ 0 \\ 0 \end{bmatrix}$$

Using MPO, this is translated as the following python code. (The latex above was generated for me with `prog.latex()` if you were wondering if I typed that all out by hand.)

```
A = numpy.array([[1, 1, 0, 0], [0, 0, 1, 1], [-1, 0, -1, 0], [0, -1, 0, -1], [-1, 0, 0, 0], [0, 0, -1, 0], [0, 0, 0, -1]])
b = numpy.array([350, 600, 0, 0, 0, 0, 0, 0]).reshape(8, 1)
c = 25 * make_column([1, 1, 1, 1])
F = numpy.array([[0, 0], [0, 0], [-1, 0], [0, -1], [0, 0], [0, 0], [0, 0], [0, 0]])
Q = 2.0 * numpy.diag([153, 162, 162, 126])

CRa = numpy.vstack((numpy.eye(2), -numpy.eye(2)))
CRb = numpy.array([1000, 1000, 0, 0]).reshape(4, 1)
H = numpy.zeros((F.shape[1], Q.shape[0]))

prog = MPQP_Program(A, b, c, H, Q, CRa, CRb, F)
```

But before you go forward and solve this, I would always recommend processing the constraints. Removing all strongly and weakly redundant constraints and rescaling them leads to significant performance increases and robustifying the numerical stability. In MPO, processing the constraints is a simple task.

```
prog.process_constraints()
```

This results in the following (identical) multiparametric optimization problem. We were able to remove 2 constraints! And we reduced the condition number of the constraints.

$$\min_x \frac{1}{2} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}^T \begin{bmatrix} 306.0 & 0 & 0 & 0 \\ 0 & 324.0 & 0 & 0 \\ 0 & 0 & 324.0 & 0 \\ 0 & 0 & 0 & 252.0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 25.0 \\ 25.0 \\ 25.0 \\ 25.0 \end{bmatrix}^T \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\text{s.t.} \begin{bmatrix} 0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0.7071 & 0.7071 \\ -0.5774 & 0 & -0.5774 & 0 \\ 0 & -0.5774 & 0 & -0.5774 \\ -1.0 & 0 & 0 & 0 \\ 0 & -1.0 & 0 & 0 \\ 0 & 0 & -1.0 & 0 \\ 0 & 0 & 0 & -1.0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 247.5 \\ 424.3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.5774 & 0 \\ 0 & -0.5774 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -1.0 & 0 \\ 0 & -1.0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \leq \begin{bmatrix} 1e+03 \\ 1e+03 \\ 0 \\ 0 \end{bmatrix}$$

0.7071
 00 0
 0.7071 0
 0 0.7071 - 0.5774
 00 -0.5774
 0 -0.5774
 0 -0.5774 - 1.0
 00 0
 0 -1.0
 00 0
 00 -1.0
 0 0
 -1.0

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 247.5 \\ 424.3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.5774 & 0 \\ 0 & -0.5774 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \\ -1.0 & 0 \\ 0 & -1.0 \end{bmatrix} \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} \leq \begin{bmatrix} 1e+03 \\ 1e+03 \\ 0 \\ 0 \end{bmatrix}$$

That wasn't that bad, and we were able to cut away some constraints that didn't matter in the process! Now we are ready to solve it.

```
solution = mpo.solve(prog)
```

Now we have the solution, we can either export the solution via the micropop module, or we can plot it here. The extra arguments mean we are saving a picture of the plot and displaying it to the user (you can give a file path, so it saves somewhere that is not the current working directory).

```
parametric_plot(solution, 'transport.png' , show = True)
```

3.1 mpo package

3.1.1 Subpackages

`mpo.geometry` package

Submodules

`mpo.geometry.polytope` module

`mpo.geometry.polytope_operations` module

```
mpo.geometry.polytope_operations.get_chebyshev_information(region:
                                                             mpo.critical_region.CriticalRegion,
                                                             determinis-
                                                             tic_solver='glpk')

mpo.geometry.polytope_operations.get_vertices(region: mpo.critical_region.CriticalRegion,
                                                deterministic_solver='glpk')

mpo.geometry.polytope_operations.sample_region_convex_combination(region:
                                                                    mpo.critical_region.CriticalRegion,
                                                                    disper-
                                                                    sion=0,
                                                                    num_samples:
                                                                    int = 100,
                                                                    determinis-
                                                                    tic_solver='glpk')
```

Module contents

`mpo.mp_solvers` package

Submodules

`mpo.mp_solvers.mppq_ahmadi` module

mpo.mp_solvers.mppq_combinatorial module

```
class mpo.mp_solvers.mppq_combinatorial.CombinationTester (combos: List[List[int]]
                                                         = <factory>)
```

Bases: object

This keeps track of all of the infeasible active set combinations and filters prospective active set combinations

add (active_set: List[int]) → None

Added an infeasible active set to keep track of so we can cull later

check (active_set: List[int]) → bool

Checks if the provided active set combination is a superset of a previously tested infeasible active set
:param active_set: :return: False if it should be culled and not tested any further, True if the set could be feasible

combos: List[List[int]]

```
mpo.mp_solvers.mppq_combinatorial.check_child_feasibility (program:
                                                             mpo.mp_program.MPQP_Program,
                                                             set_list: List[List[int]],
                                                             combination_checker:
                                                             mpo.mp_solvers.mppq_combinatorial.CombinationTester
                                                             → List[List[int]])
```

Checks the feasibility of a list of active set combinations, if infeasible add to the combination checker and returns all feasible active set combinations

Parameters

- **program** – An MPQP Program
- **set_list** – The list of active sets
- **combination_checker** – The combination checker that prunes

Returns The list of all feasible active sets

```
mpo.mp_solvers.mppq_combinatorial.generate_children (program_active_set:
                                                         List[int],      num_constraints:
                                                         int,           super_set_checker:
                                                         mpo.mp_solvers.mppq_combinatorial.CombinationTester
                                                         root:          Union[List[int],
                                                         numpy.ndarray] = (), is_root:
                                                         bool = False) → List
```

Takes a child node and then finds all of the feasible (w.r.t. pruning list) active set combinations from this branch :param program_active_set: base active set combinations from the multiparametric program :param num_constraints: number of constraints from the multi parametric program :param super_set_checker: the pruning list that will check and remove all provably infeasible child sets :param root: the base active set of the branch :param is_root: if this root is not active :return: returns all the possibly feasible children of this active set

```
mpo.mp_solvers.mppq_combinatorial.solve (program:      mpo.mp_program.MPQP_Program,
                                           solver='gurobi') → mpo.solution.Solution
```

Solves the MPQP program with a modified algorithm described in Gupta et. al. 2011

url: <https://www.sciencedirect.com/science/article/pii/S0005109811003190>

Parameters

- **program** – MPQP to be solved
- **solver** – Deterministic solver to use

Returns the solution of the MPQP

mpo.mp_solvers.mppq_geometric module

`mpo.mp_solvers.mppq_geometric.solve` (*program:* `mpo.mp_program.MPQP_Program`) → `mpo.solution.Solution`

mpo.mp_solvers.mppq_graph module

`mpo.mp_solvers.mppq_graph.generate_extra` (*candidate:* `tuple`, *expansion_set:* `set`, *attempted:* `set`, *murder_list:* `settrie.SetTrie`) → `list`

`mpo.mp_solvers.mppq_graph.generate_reduce` (*candidate:* `tuple`, *attempted:* `set`, *murder_list:* `settrie.SetTrie`) → `list`

`mpo.mp_solvers.mppq_graph.solve` (*program:* `mpo.mp_program.MPQP_Program`) → `mpo.solution.Solution`

Solves the MPQP program with a modified algorithm described in Oberdieck et. al. 2016

url: <https://www.sciencedirect.com/science/article/pii/S0005109816303971>

Parameters `program` – MPQP to be solved

Returns the solution of the MPQP

mpo.mp_solvers.mppq_parallel_combinatorial module

class `mpo.mp_solvers.mppq_parallel_combinatorial.CombinationTester`

Bases: `object`

This keeps track of all of the infeasible active set combinations and filters prospective active set combinations

add_combos (*set_list:* `Set[Tuple[int]]`) → `None`

check (*active_set:* `Set[int]`) → `bool`

Checks if the provided active set combination is a superset of a previously tested infeasible active set
:param active_set: :return: False if it should be culled and not tested any further, True if the set could be feasible

`mpo.mp_solvers.mppq_parallel_combinatorial.full_process` (*program:* `mpo.mp_program.MPQP_Program`,
active_set: `List[int]`,
murder_list:
gen_children)

`mpo.mp_solvers.mppq_parallel_combinatorial.generate_children` (*program_active_set:* `List[int]`,
num_constraints:
int, *super_set_checker:* `mpo.mp_solvers.mppq_parallel_combinatorial.CombinationTester`,
root: `Union[List[int], numpy.ndarray]`
= (), is_root: `bool`
= False) → `List`

Takes a child node and then finds all of the feasible (w.r.t. pruning list) active set combinations from this branch
:param program_active_set: base active set combinations from the multiparametric program
:param num_constraints: number of constraints from the multi parametric program
:param super_set_checker: the

pruning list that will check and remove all provably infeasible child sets :param root: the base active set of the branch :param is_root: if this root is not active :return: returns all the possibly feasible children of this active set

```
mpo.mp_solvers.mpq_parrallel_combinatorial.is_feasible(program:
    mpo.mp_program.MQP_Program,
    active_set: List[int]) →
    bool
mpo.mp_solvers.mpq_parrallel_combinatorial.is_optimal(program:
    mpo.mp_program.MQP_Program,
    active_set: List[int]) →
    bool
mpo.mp_solvers.mpq_parrallel_combinatorial.solve(program:
    mpo.mp_program.MQP_Program,
    num_cores=-1) →
    mpo.solution.Solution
```

Solves the MPQP program with a modified algorithm described in Gupta et. al. 2011

This is the parallel version of the combinatorial.

url: <https://www.sciencedirect.com/science/article/pii/S0005109811003190>

Parameters

- **num_cores** – Sets the number of cores that are allocated to run this algorithm
- **program** – MPQP to be solved

Returns the solution of the MPQP

mpo.mp_solvers.mpq_parrallel_combinatorial_exp module

mpo.mp_solvers.mpq_parrallel_graph module

```
mpo.mp_solvers.mpq_parrallel_graph.solve(program: mpo.mp_program.MQP_Program,
    num_cores=-1) → mpo.solution.Solution
```

Solves the MPQP program with a modified algorithm described in Oberdieck et. al. 2016

url: <https://www.sciencedirect.com/science/article/pii/S0005109816303971>

Parameters

- **program** – MPQP to be solved
- **num_cores** – specifies numbers of cores to run, default is set to run on all available cores

Returns the solution of the MPQP

mpo.mp_solvers.solve_mplp module

```
class mpo.mp_solvers.solve_mplp.mplp_solver(value)
```

Bases: enum.Enum

An enumeration.

Dustin = '1'

```
mpo.mp_solvers.solve_mplp.solve_mplp(problem: mpo.mp_program.MPLP_Program, algo-
    rithm: mpo.mp_solvers.solve_mplp.mplp_solver =
    <mplp_solver.Dustin: '1'>)
```


mpo.mp_solvers.solve_mqp module

`mpo.mp_solvers.solve_mqp.filter_solution` (*solution:* `mpo.solution.Solution`) → `mpo.solution.Solution`

class `mpo.mp_solvers.solve_mqp.mqp_algorithm` (*value*)

Bases: `enum.Enum`

Enum that selects algorithm to be used

graph = '5'

gupta = '1'

gupta_parallel = '2'

space = '6'

step = '7'

`mpo.mp_solvers.solve_mqp.solve_mqp` (*problem:* `mpo.mp_program.MPQP_Program`, *algorithm:* `mpo.mp_solvers.solve_mqp.mqp_algorithm` = `<mpqp_algorithm.gupta: '1'>`) → `mpo.solution.Solution`

Takes a mpqp programming problem and solves it in a specified manner

default behavior is the algorithm from Gupta et al.

Using `mpqp_algorithm` as the algorithm selector

`mpqp_algorithm.gupta` => Gupta et al. Algorithm

Parameters

- **algorithm** –
- **problem** – MPQP to be solved

Returns the solution of the MPQP

mpo.mp_solvers.solver_utils module

Module contents

mpo.solver_interface package

Submodules

mpo.solver_interface.cvxopt_interface module

`mpo.solver_interface.cvxopt_interface.solve_fully_constraints` (*c:* `numpy.ndarray`, *A:* `numpy.ndarray`, *b:* `numpy.ndarray`, *equality_constraints=()*) → `Optional[mpo.solver_interface.solver_utils.Sc`

```
mpo.solver_interface.cvxopt_interface.solve_lp_cvxopt (c:          numpy.ndarray,  
A:          numpy.ndarray,  b:  
numpy.ndarray,  equal-  
ity_constraints=None,  ver-  
bose=False, get_duals=True,  
cvx_solver='glpk') → Op-  
tional[mpo.solver_interface.solver_utils.SolverOutput]
```

mpo.solver_interface.gurobi_solver_interface module

```
mpo.solver_interface.gurobi_solver_interface.solve_lp_gurobi (c:  numpy.ndarray,  
A:  numpy.ndarray,  
b:  numpy.ndarray,  
equality_constraints=None,  
verbose=False,  
get_duals=True)  
→ Op-  
tional[mpo.solver_interface.solver_utils.Sol
```

This is the breakout for solving mixed integer linear programs with gurobi, This is feed directly into the MIQP solver that is defined in the same file.

The Mixed Integer Linear program programming problem $\min_{\{xy\}} c^T[xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

xy is the parameter vector of mixed real and binary inputs

Parameters

- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. `output['sol']` = primal variables, `output['dual']` = dual variables, `output['obj']` = objective value, `output['const']` = slacks, `output['active']` = active constraints.

```

mpo.solver_interface.gurobi_solver_interface.solve_milp_gurobi(c:
                                                                numpy.ndarray,
                                                                A:
                                                                numpy.ndarray,
                                                                b:
                                                                numpy.ndarray,
                                                                equality_
                                                                constraints:
                                                                Optional[Iterable[int]]
                                                                =
                                                                None,
                                                                bin_vars:
                                                                Optional[Iterable[int]]
                                                                =
                                                                None,
                                                                verbose=False,
                                                                get_duals=True)
                                                                →
                                                                Optional[mpo.solver_interface.solver_utils.

```

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The Mixed Integer Linear program programming problem $\min_{\{xy\}} c^T[xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

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Parameters

- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **bin_vars** – List of binary variable indices
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True (false for all mixed integer models)

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. $output['sol']$ = primal variables, $output['dual']$ = dual variables, $output['obj']$ = objective value, $output['const']$ = slacks, $output['active']$ = active constraints.

```

mpo.solver_interface.gurobi_solver_interface.solve_miqp_gurobi(Q:
                                                                    numpy.ndarray,
                                                                    c:
                                                                    numpy.ndarray,
                                                                    A:
                                                                    numpy.ndarray,
                                                                    b:
                                                                    numpy.ndarray,
                                                                    equality_constraints:
                                                                    Optional[Iterable[int]]
                                                                    = None,
                                                                    bin_vars: Optional[Iterable[int]]
                                                                    = None,
                                                                    verbose: bool
                                                                    = False,
                                                                    get_duals: bool
                                                                    = True) → Optional[mpo.solver_interface.solver_utils.

```

This is the breakout for solving mixed integer quadratic programs with gurobi

The Mixed Integer Quadratic program programming problem $\min_{\{xy\}} \frac{1}{2} [xy]^T Q [xy] + c^T [xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

xy is the parameter vector of mixed real and binary inputs

Parameters

- **Q** – Square matrix, can be None
- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **bin_vars** – List of binary variable indices
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True (false for all mixed integer models)

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. n output[‘sol’] = primal

variables, output[‘dual’] = dual variables, output[‘obj’] = objective value, output[‘const’] = slacks, output[‘active’] = active constraints.

```

mpo.solver_interface.gurobi_solver_interface.solve_qp_gurobi (Q: numpy.ndarray,
                                                             c: numpy.ndarray,
                                                             A: numpy.ndarray,
                                                             b: numpy.ndarray,
                                                             equality_constraints:
                                                             Optional[Iterable[int]]
                                                             = None, verbose=False,
                                                             get_duals=True)
                                                             → Optional[mpo.solver_interface.solver_utils.Sol

```

This is the breakout for solving mixed integer quadratic programs with gurobi

The Mixed Integer Quadratic program programming problem $\min_{\{xy\}} \frac{1}{2} [xy]^T Q [xy] + c^T [xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

xy is the parameter vector of mixed real and binary inputs

Parameters

- **Q** – Square matrix, can be None
- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True (false for all mixed integer models)

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. n output['sol'] = primal variables, output['dual'] = dual variables, output['obj'] = objective value, output['const'] = slacks, output['active'] = active constraints.

mpo.solver_interface.solver_interface module

```

mpo.solver_interface.solver_interface.solve_lp (c: Optional[numpy.ndarray],
                                                  A: Optional[numpy.ndarray],
                                                  b: Optional[numpy.ndarray],
                                                  equality_constraints=None, verbose=False,
                                                  get_duals=True, deterministic_solver='glpk')
                                                  → Optional[mpo.solver_interface.solver_utils.SolverOutput]

```

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The Mixed Integer Linear program programming problem $\min_{\{xy\}} c^T [xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

xy is the parameter vector of mixed real and binary inputs

Parameters

- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True
- **deterministic_solver** – The underlying solver to use, eg. gurobi, ect

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. output['sol'] = primal variables, output['dual'] = dual variables, output['obj'] = objective value, output['const'] = slacks, output['active'] = active constraints.

```
mpo.solver_interface.solver_interface.solve_milp(c: Optional[numpy.ndarray],
A: Optional[numpy.ndarray],
b: Optional[numpy.ndarray],
equality_constraints: Optional[Iterable[int]] = None,
bin_vars: Optional[Iterable[int]] = None, verbose=False,
get_duals=True, deterministic_solver='gurobi') → Optional[mpo.solver_interface.solver_utils.SolverOutput]
```

This is the breakout for solving mixed integer linear programs with gurobi, This is feed directly into the MIQP solver that is defined in the same file.

The Mixed Integer Linear program programming problem $\min_{\{xy\}} c^T[xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

xy is the parameter vector of mixed real and binary inputs

Parameters

- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **bin_vars** – List of binary variable indices
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True (false for all mixed integer models)
- **deterministic_solver** – The underlying solver to use, eg. gurobi, ect

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. output['sol'] = primal variables, output['dual'] = dual variables, output['obj'] = objective value, output['const'] = slacks, output['active'] = active constraints.

```
mpo.solver_interface.solver_interface.solve_miqp(Q: Optional[numpy.ndarray],
c: Optional[numpy.ndarray],
A: Optional[numpy.ndarray], b: Optional[numpy.ndarray], equality_constraints: Iterable[int]
= (), bin_vars: Iterable[int] = (), verbose: bool = False,
get_duals: bool = True, deterministic_solver='gurobi') → Optional[mpo.solver_interface.solver_utils.SolverOutput]
```

This is the breakout for solving mixed integer quadratic programs with gurobi

The Mixed Integer Quadratic program programming problem $\min_{\{xy\}} \frac{1}{2} [xy]^T Q [xy] + c^T [xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

xy is the parameter vector of mixed real and binary inputs

Parameters

- **Q** – Square matrix, can be None
- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **bin_vars** – List of binary variable indices
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True (false for all mixed integer models)
- **deterministic_solver** – The underlying solver to use, eg. gurobi, ect

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. n output['sol'] = primal

variables, output['dual'] = dual variables, output['obj'] = objective value, output['const'] = slacks, output['active'] = active constraints.

```
mpo.solver_interface.solver_interface.solve_qp(Q: Optional[numpy.ndarray],
c: Optional[numpy.ndarray],
A: Optional[numpy.ndarray], b: Optional[numpy.ndarray],
equality_constraints: Optional[Iterable[int]] = None,
verbose=False, get_duals=True, deterministic_solver='gurobi') → Optional[mpo.solver_interface.solver_utils.SolverOutput]
```

This is the breakout for solving mixed integer quadratic programs with gurobi

The Mixed Integer Quadratic program programming problem $\min_{\{xy\}} \frac{1}{2} [xy]^T Q [xy] + c^T [xy]$

s.t. $A[xy] \leq b$ $Aeq*[xy] = beq$

xy is the parameter vector of mixed real and binary inputs

Parameters

- **Q** – Square matrix, can be None
- **c** – Column Vector, can be None
- **A** – Constraint LHS matrix, can be None
- **b** – Constraint RHS matrix, can be None
- **equality_constraints** – List of Equality constraints
- **verbose** – Flag for output of underlying solver, default False
- **get_duals** – Flag for returning dual variable of problem, default True (false for all mixed integer models)
- **deterministic_solver** – The underlying solver to use, eg. gurobi, ect

Returns A dictionary of the solver outputs, or none if infeasible or unbounded. `output['sol']` = primal variables, `output['dual']` = dual variables, `output['obj']` = objective value, `output['const']` = slacks, `output['active']` = active constraints.

```
mpo.solver_interface.solver_interface.solver_not_supported(solver_name: str) →  
None
```

This is an internal method that throws an error and prompts the user when they use an unsupported solver

mpo.solver_interface.solver_utils module

```
class mpo.solver_interface.solver_utils.SolverOutput (obj: float, sol:  
numpy.ndarray, slack: Op-  
tional[numpy.ndarray],  
active_set: Op-  
tional[numpy.ndarray], dual:  
Optional[numpy.ndarray])
```

Bases: object

Solver information object

Members: obj: objective value of the optimal solution

sol: x*, numpy.ndarray

Optional Parameters -> None or numpy.ndarray type

slack: the slacks associated with every constraint

active_set: the active set of the solution, including strongly and weakly active constraints

dual: the lagrange multipliers associated with the problem

active_set: Optional[numpy.ndarray]

dual: Optional[numpy.ndarray]

obj: float

slack: Optional[numpy.ndarray]

sol: numpy.ndarray


```
mpo.solver_interface.solver_utils.get_program_parameters (Q: Optional[numpy.ndarray],
                                                         c: Optional[numpy.ndarray],
                                                         A: Optional[numpy.ndarray],
                                                         b: Optional[numpy.ndarray])
```

Given a set of possibly None optimization parameters determine the number of variables and constraints

Module contents

mpo.upop package

Submodules

mpo.upop.language_generation module

```
mpo.upop.language_generation.gen_array (data: list, name: str, vartype: str, options=('const'),
                                         lang='cpp') → str
mpo.upop.language_generation.gen_cpp_array (data: list, name: str, vartype: str, options: list
                                             = ('const')) → str
mpo.upop.language_generation.gen_cpp_variable (data, name: str, vartype: str, options: list
                                                = ('const')) → str
mpo.upop.language_generation.gen_js_array (data: list, name: str, vartype: str, options: list
                                            = ('const')) → str
mpo.upop.language_generation.gen_js_variable (data, name: str, vartype: str, options: list
                                                = ('const')) → str
mpo.upop.language_generation.gen_python_array (data: list, name: str, vartype: str, options:
                                                list = ('const')) → str
mpo.upop.language_generation.gen_python_variable (data, name: str, vartype: str, options:
                                                    list = ('const')) → str
mpo.upop.language_generation.gen_variable (data, name: str, vartype: str, options=('const'),
                                            lang='cpp') → str
```

mpo.upop.linear_code_gen module

mpo.upop.point_location module

```
class mpo.upop.point_location.PointLocation (solution: mpo.solution.Solution)
```

Bases: object

```
evaluate (theta: numpy.ndarray) → Optional[numpy.ndarray]
```

Evaluates the value of $x(\theta)$, of the

Parameters *theta* –

Returns

```
is_inside (theta: numpy.ndarray) → bool
```

Determines if the *theta* point in inside of the feasible space

param theta A point in the theta space

return True, if theta in region

False, if theta not in region

locate (*theta: numpy.ndarray*) → int

Finds the index of the critical region that theta is inside

Parameters theta –

Returns

mpo.upop.ucontroller module

class mpo.upop.ucontroller.**BVH** (*parent, fundamental_list, region_list, depth, index*)

Bases: object

mpo.upop.ucontroller.**classify_polytope** (*region: mpo.critical_region.CriticalRegion, hyper_plane: numpy.ndarray*) → int

We are going to classify the polytopic critical region by solving 2 LPS

max $\|<x, A>\|_d$ for x in Critical region

min $\|<x, A>\|_d$ for x in Critical region

The result of the objective function will tell us the side of the hyper plane the point is on

Parameters

- **region** – Critical region
- **hyper_plane** – A fundamental hyperplane

Returns -1 if completely not in support, 0 if intersected, 1 if completely in support

mpo.upop.ucontroller.**determine_hyperplane** (*regions: List[mpo.critical_region.CriticalRegion], hyper_planes: numpy.ndarray*)

Finds the ‘best’ splitting hyper plane for this task

In this case best means minimizing the number of intersected regions while also maximizing the difference between supported and not supported regions

Parameters

- **regions** –
- **hyper_planes** –

Returns []

mpo.upop.ucontroller.**generate_code** (*solution: mpo.solution.Solution*) → List[str]

Generates C++17 code for point location and function evaluation on microcontrollers

This forms a BVH to accelerate solution times

WARNING: This breaks down at high dimensions

Parameters solution – a solution to a MPLP or MPQP solution

Returns List of the strings of the C++17 datafiles that integrate with uPOP

mpo.upop.upop_utils module

`mpo.upop.upop_utils.find_unique_hyperplanes` (*overall*: `numpy.ndarray`) → `Tuple[List[int], List[int], List[int]]`

Generates the list of indices of the fundamental hyperplanes of the solution, as well as the indices of the associated hyperplanes from the original solution and the parity of the constraint

This is linear w.r.t. number of hyper planes and is quite quick ~25 ns per constraint in the solution

It first creates approximate(near exact) integer representations for each constraint for each region in the solution

This approximation step is justified in that it will find equality between 2 constraints if the L2 norm of the difference is below 10E-12

Then the positive and negative versions of these constraints [$-x < -1$, $x < 1$ are on different sides of the same hyperplane] are made into a format that can be hashed (tuples of ints)

With this is relatively strait forward to check for uniqueness with the set

The first loop scans thru all of the constraints and if the constraint contains a unique hyperplane

- 1) if it is a unique hyper plane store the index, add the integer representation to the set, then index the integer representation to the index
- 2) if it is not a unique hyperplane do nothing

The second loop scans thru the constraints again and assigns them unique hyper plane indices and the parity(what side of the hyper plane that they are on)

Parameters overall – The solution of a multiparametric programming problem

Returns returns indices of fundamental hyperplanes, indices of constraints back to fundamental hyperplane, parity of constraint

`mpo.upop.upop_utils.find_unique_region_functions` (*solution*: `mpo.solution.Solution`) → `Tuple[List[int], List[int], List[int]]`

`mpo.upop.upop_utils.find_unique_region_hyperplanes` (*solution*: `mpo.solution.Solution`) → `Tuple[List[int], List[int], List[int]]`

This is an overload of the find_unique_hyperplane function

Parameters solution –

Returns

`mpo.upop.upop_utils.get_chebychev_centers` (*solution*: `mpo.solution.Solution`) → `List[numpy.ndarray]`

Calculates and returns a list of all of the theta chebychev centers for the critical regions in the solution

Parameters solution – An mp programming Solution

Returns A list of all of the chebychev centers of the regions in the solutions

`mpo.upop.upop_utils.get_descriptions` (*solution*: `mpo.solution.Solution`) → `dict`

`mpo.upop.upop_utils.get_outer_boundaries` (*indices*: `List[int]`, *parity*: `List[int]`)

Takes in the global constraint indices to the fundamental hyperplanes and their parity finds all planes with only one parity version aka only one verity of them appears in the original set.

This method is linear w.r.t. number of indices, by the use of sets and hash maps

Parameters

- **indices** – list of indices that maps the solution constraints into the fundamental hyperplanes

- **parity** – the side of the hyperplane that the constraint represents

Returns

`mpo.upop.upop_utils.verify_outer_boundary` (*solution*: `mpo.solution.Solution`, *hyper_indices*: `List[int]`, *outer_indices*: `List[int]`, *chebychev_centers*: `Optional[List[numpy.ndarray]]` = `None`) → `List[int]`

This checks all of the possible outer boundary indices for errors, failures to solve for the minimal set of fundamental hyperplanes in the solution

Parameters

- **solution** – An mp programming solution
- **hyper_indices** – The list of all fundamental hyperplane indices
- **outer_indices** – The list of identified exterior hyperplane indices
- **chebychev_centers** – the list of chebychev centers in the theta space for every critical region {Optional}

Returns List of verified outer boundary constraints

Module contents**mpo.utils package****Submodules****mpo.utils.chebyshev_ball module**

`mpo.utils.chebyshev_ball.chebyshev_ball` (*A*: `numpy.ndarray`, *b*: `numpy.ndarray`, *equality_constraints*: `Optional[Iterable[int]]` = `None`, *bin_vars*: `Iterable[int]` = `()`, *deterministic_solver*=`'glpk'`)

Chebyshev ball finds the largest ball inside of a polytope defined by $Ax \leq b$ This is solved by the following LP

$\min\{x, r\} -r$

st: $Ax + \|A_{eq}\|r \leq b$

$A_{eq} \cdot x = b_{eq}$

$r \geq 0$

Returns a List with [pos, r] where pos is a numpy array r is a real number

Parameters

- **A** – LHS Constraint Matrix
- **b** – RHS Constraint column vector
- **equality_constraints** – indices of

rows that have strict equality $A[eq] \cdot x = b[eq]$:param *bin_vars*: indices of binary variables :param *deterministic_solver*: The underlying solver to use, eg. gurobi, ect :return: the optimization output of the LP problem, the coordinates can be found in `output['sol']`, with `output['sol'][-1]` giving the chebyshev radius

mpo.utils.constraint_utilities module

`mpo.utils.constraint_utilities.calculate_redundant_constraints(A, b)`

Removes weakly redundant constraints, method is from the appendix of the Oberdieck paper

url: <https://www.sciencedirect.com/science/article/pii/S0005109816303971>

Parameters

- **A** – LHS constraint matrix
- **b** – RHS constraint column vector

Returns The processes constraint pair [A, b]

`mpo.utils.constraint_utilities.cheap_remove_redundant_constraints(A:`

`numpy.ndarray,`

`b:`

`numpy.ndarray)`

`→`

`List[numpy.ndarray]`

Removes zero rows, normalizes the constraint rows to $\|A_{\text{ill}}\|_{L_2} = 1$, and removes duplicate rows

Parameters

- **A** – LHS constraint matrix
- **b** – RHS constraint column vector

Returns The processes constraint pair [A, b]

`mpo.utils.constraint_utilities.constraint_norm(A: numpy.ndarray) → numpy.ndarray`

Finds the L2 norm of each row of a matrix

Parameters **A** – numpy matrix

Returns A column vector of the row norms

`mpo.utils.constraint_utilities.facet_ball_elimination(A: numpy.ndarray, b: numpy.ndarray) →`

`List[numpy.ndarray]`

`→`

Removes weakly redundant constraints, method is from the appendix of the Oberdieck paper

url: <https://www.sciencedirect.com/science/article/pii/S0005109816303971>

Parameters

- **A** – LHS constraint matrix
- **b** – RHS constraint column vector

Returns The processes constraint pair [A, b]

`mpo.utils.constraint_utilities.is_full_rank(A: numpy.ndarray, indices: Optional[List[int]] = None) → bool`

Op-

Tests if the matrix A[indices] is full rank Empty matrices e.g. A[[]] will default to be full rank

Parameters

- **A** – Matrix
- **indices** – indices to consider in rank

Returns if the matrix is full rank or not

`mpo.utils.constraint_utilities.process_region_constraints` (*A*: *numpy.ndarray*, *b*:
numpy.ndarray) →
List[*numpy.ndarray*]

Removes all strongly and weakly redundant constraints

Parameters

- **A** – LHS constraint matrix
- **b** – RHS constraint column vector

Returns The processes constraint pair [A, b]

`mpo.utils.constraint_utilities.remove_duplicate_rows` (*A*: *numpy.ndarray*, *b*:
numpy.ndarray) →
List[*numpy.ndarray*]

Finds and removes duplicate rows in the constraints A @ x <= b

`mpo.utils.constraint_utilities.remove_strongly_redundant_constraints` (*A*:
numpy.ndarray,
b:
numpy.ndarray)
→
List[*numpy.ndarray*]

Removes strongly redundant constraints by testing the feasibility of each constraint if activated

`mpo.utils.constraint_utilities.remove_zero_rows` (*A*: *numpy.ndarray*, *b*: *numpy.ndarray*)
→ List[*numpy.ndarray*]

Finds rows equal to zero in A and then removes them from A and b

Parameters

- **A** – LHS Matrix constraint
- **b** – RHS Column vector

Returns a list[A_cleaned, b_cleaned] of filtered constraints

`mpo.utils.constraint_utilities.row_equality` (*row_1*: *numpy.ndarray*, *row_2*:
numpy.ndarray, *tol*=1e-16) → bool

Tests if 2 row vectors are approximately equal

Parameters

- **row_1** –
- **row_2** –
- **tol** – tolerable L2 norm of the difference

Returns True if rows are equal

`mpo.utils.constraint_utilities.scale_constraint` (*A*: *numpy.ndarray*, *b*: *numpy.ndarray*)
→ List[*numpy.ndarray*]

Normalizes constraints

Parameters

- **A** – LHS Matrix constraint
- **b** – RHS column vector constraint

Returns a list [A_scaled, b_scaled] of normalized constraints

mpo.utils.general_utils module

`mpo.utils.general_utils.latex_matrix` (*A: Union[List[str], numpy.ndarray]*) → str
Creates a latex string for a given numpy array

Parameters **A** – A numpy array

Returns A latex string for the matrix A

`mpo.utils.general_utils.make_column` (*x: Union[List, numpy.ndarray]*) → numpy.ndarray
Makes x into a column vector :param x: a list or a numpy array :return: a numpy array that is a column vector

`mpo.utils.general_utils.make_row` (*x: Union[List, numpy.ndarray]*) → numpy.ndarray
Makes x into a row vector :param x: a list or a numpy array :return: a numpy array that is a row column

`mpo.utils.general_utils.mpo_block` (*mat_list*)

`mpo.utils.general_utils.num_cpu_cores` ()
Finds the number of allocated cores, with different behavior in windows and linux.

In Windows, returns number of physical cpu cores

In Linux, returns number of available cores for processing (this is for running on cluster or managed environment)

Returns number of cores

`mpo.utils.general_utils.remove_size_zero_matrices` (*list_matrices: List[numpy.ndarray]*) → List[numpy.ndarray]
Removes size zero matrices from a list

Parameters **list_matrices** – A list of numpy arrays

Returns returns all matrices from the list that do not have a dimension of 0 in any index

`mpo.utils.general_utils.select_not_in_list` (*A: numpy.ndarray, list_: Iterable[int]*) → numpy.ndarray
Filters a numpy array to select all rows that are not in a list

Parameters

- **A** – a numpy array
- **list** – a list of indices that you want to remove

Returns return a numpy array of A[not in **list**]

mpo.utils.geometric module

`mpo.utils.geometric.gen_tess_points_simplex` (*simplex*)

Parameters **simplex** –

Returns

`mpo.utils.geometric.make_domain_subdivision` (*A_t, b_t*)

`mpo.utils.geometric.make_simplex` (*n: int*)

`mpo.utils.geometric.make_subdomains` (*points*)

`mpo.utils.geometric.revised_tess_simplex` (*simplex, half_split=False*)

mpo.utils.mppq_utils module

`mpo.utils.mppq_utils.calculate_control_law` (*program*: `mpo.mp_program.MPQP_Program`,
active_set: `List[int]`) → `Tuple`

`mpo.utils.mppq_utils.check_feasibility` (*program*: `mpo.mp_program.MPQP_Program`, *active_set*)

`mpo.utils.mppq_utils.check_optimality` (*program*: `mpo.mp_program.MPQP_Program`, *active_set*: `list`)

Tests if the active set is optimal for the provided mppq program

x | theta | lambda | slack | t max t

- 1) $Q_u + (A_{Ai})^T \lambda_{Ai} + c = 0$
- 2) $A_{Ai} * u - b_{ai} - F_{ai} * \theta = 0$
- 3) $A_{Aj} * u - b_{aj} - F_{aj} * \theta + s_{jk} = 0$
- 4) $t * e_1 \leq \lambda_{Ai}$,
- 5) $t * e_2 \leq s_{Ji}$
- 6) $t \geq 0$,
- 7) $\lambda_{Ai} \geq 0$,
- 8) $s_{Ji} \geq 0$
- 9) $A_t * \theta \leq b_t$

Parameters

- **program** – an mppq program
- **active_set** – active set being considered in the optimality test

Returns dictionary of parameters, or None if active set is not optimal

`mpo.utils.mppq_utils.gen_cr_from_active_set` (*program*: `mpo.mp_program.MPQP_Program`,
active_set: `List[int]`,
check_full_dim=True) → `Optional[mpo.critical_region.CriticalRegion]`

Builds the critical region of the given mppq from the active set.

Parameters

- **program** – the MQMP_Program to be solved
- **active_set** – the active set combination to build this critical region from
- **check_full_dim** – Keyword Arg, if true will return null if the region has lower dimensionality

Returns Returns the associated critical region if fully dimensional else returns None

`mpo.utils.mppq_utils.get_boundary_types` (*region*: `numpy.ndarray`, *omega*: `numpy.ndarray`,
lagrange: `numpy.ndarray`, *regular*:
`numpy.ndarray`) → `List`

Classifies the boundaries of a polytope into Omega constraints, Lagrange multiplier = 0 constraints, and Activated program constraints :param region: :param omega: :param lagrange: :param regular: :return:

`mpo.utils.mppq_utils.get_feasible_theta` (*program*: `mpo.mp_program.MPQP_Program`) → `Union[None, numpy.ndarray]`


```
mpo.utils.mqp_utils.get_feasible_theta_2 (program: mpo.mp_program.MQP_Program)
→ Union[None, numpy.ndarray]
```

Finds a feasible theta constraint of the multi-parametric problem

Pseudo-Code Steps:

- 1) Find and calculate the Theta Ball of the theta feasible space
- 2) See if the center of the ball is a valid theta point
- 3) if not retry up to {100} times to find a feasible point in the theta ball of the problem
- 4) If one can not be found in the theta ball returns None

Parameters program – MP Program

Returns feasible theta point or None[illegible]

```
mpo.utils.mppp_utils.is_full_dimensional(A, b)
```

```
mpo.utils.mppq_utils.theta_ball(program: mpo.mp_program.MPPQ_Program) → Op-  
tional[mpo.solver_interface.solver_utils.SolverOutput]
```

Finds the chebychev ball in the theta feasible space of a problem.

Parameters `program` – MPQP_Program

Returns basic result from the LP

```
mpo.utils.mppp_utils.zeros(x: int, y: int) → numpy.ndarray
```

Auxiliary function returns a numpy array of zeros of dimensions x by y (rows, columns)

Parameters

- **x** – Number of rows
- **y** – Number of Columns

Returns Numpy array of zeros

mpo.utils.solver_utils module

```
mpo.utils.solver_utils.get_active_set (soln: mpo.solver_interface.solver_utils.SolverOutput)
                                         → numpy.array
```

finds the active set of a problem output

Parameters `soln` – dict results from the solver interface

Returns the active set of a solution

Module contents

3.1.2 Submodules

3.1.3 mpo.critical_region module

```
class mpo.critical_region.CriticalRegion(A: numpy.ndarray, b: numpy.ndarray, C:
                                         numpy.ndarray, d: numpy.ndarray, E:
                                         numpy.ndarray, f: numpy.ndarray, active_set:
                                         Union[List[int], numpy.ndarray], omega_set:
                                         Union[List[int], numpy.ndarray] = <factory>,
                                         lambda_set: Union[List[int], numpy.ndarray]
                                         = <factory>, regular_set: Union[List[int],
                                         numpy.ndarray] = <factory>)
```

Bases: object

Critical region is a polytope that defines a region in the uncertainty space with an associated optimal value, active set, lagrange multipliers and constraints

$x() = A + b$

$() = C + d$

$CR := \{ : E \leq f \}$

active_set: numpy array of indices

constraint_set: if this is a $A @ x = b + F @ \theta$ boundary

lambda_set: if this is a $a = 0$ boundary

boundary_set: if this is a $E \leq f$ boundary

A: numpy.ndarray

C: numpy.ndarray

E: numpy.ndarray

active_set: Union[List[int], numpy.ndarray]

b: numpy.ndarray

d: numpy.ndarray

evaluate (*theta: numpy.ndarray*) \rightarrow numpy.ndarray

Evaluates $x() = A + b$

f: numpy.ndarray

get_constraints ()

is_full_dimension () \rightarrow bool

Tests dimensionality of critical region

is_inside (*theta: numpy.ndarray*) \rightarrow numpy.ndarray

Tests if point is inside of the critical region

lagrange_multipliers (*theta: numpy.ndarray*) \rightarrow numpy.ndarray

Evaluates $() = C + d$

lambda_set: Union[List[int], numpy.ndarray]

omega_set: Union[List[int], numpy.ndarray]

```
regular_set: Union[List[int], numpy.ndarray]
```

3.1.4 mpo.mp_program module

```
class mpo.mp_program.MPLP_Program(A: numpy.ndarray, b: numpy.ndarray, c: numpy.ndarray,
                                   A_t: numpy.ndarray, b_t: numpy.ndarray, F:
                                   numpy.ndarray, active_set: Union[List[int],
                                   numpy.ndarray])
```

Bases: object

The standard class for linear multiparametric programming

A: numpy.ndarray

A_t: numpy.ndarray

F: numpy.ndarray

active_set: Union[List[int], numpy.ndarray]

b: numpy.ndarray

b_t: numpy.ndarray

c: numpy.ndarray

display_latex() → None

Displaces Latex text of the multiparametric problem

display_warnings() → None

Displaces warnings

latex() → List[str]

Generates latex of the multiparametric problem

Returns returns latex of the

num_constraints() → int

Returns number of constraints

num_t() → int

Returns number of uncertain variables

num_x() → int

Returns number of parameters

process_constraints() → None

Removes redundant constraints from the multiparametric programming problem

scale_constraints() → None

Rescales the constraints of the multiparametric problem to $\|A-F\|_i = 1$, in the L2 sense

solve_theta(*theta_point*: numpy.ndarray, *deterministic_solver*='glpk') → Optional[*mpo.solver_interface.solver_utils.SolverOutput*]

Substitutes theta into the multiparametric problem and solves

Parameters *theta_point* – an uncertainty realization

Returns the solver output of the substituted problem, returns None if not solvable

warnings() → List[str]

Checks the dimensions of the matrices to ensure consistency

```
class mpo.mp_program.MPOP_Program(A: numpy.ndarray, b: numpy.ndarray, c: numpy.ndarray, Q:
                                   numpy.ndarray, A_t: numpy.ndarray, b_t: numpy.ndarray,
                                   F: numpy.ndarray, active_set=None)
```

Bases: `mpo.mp_program.MPLP_Program`

The standard class for quadratic multiparametric programming, inherits from `MPLP_Program`

A: `numpy.ndarray`

A_t: `numpy.ndarray`

F: `numpy.ndarray`

active_set: `Union[List[int], numpy.ndarray]`

b: `numpy.ndarray`

b_t: `numpy.ndarray`

c: `numpy.ndarray`

latex() \rightarrow `List[str]`

Creates a latex output for the multiparametric problem

process_constraints() \rightarrow `None`

Removes redundant constraints from the multiparametric programming problem

solve_theta(*theta_point*: `numpy.ndarray`) \rightarrow `Optional[mpo.solver_interface.solver_utils.SolverOutput]`

Substitutes theta into the multiparametric problem and solves

Parameters *theta_point* – an uncertainty realization

Returns the solver output of the substituted problem, returns `None` if not solvable

warnings() \rightarrow `List[str]`

Checks the dimensions of the matrices to ensure consistency

3.1.5 mpo.plot module

`mpo.plot.gen_vertices`(*solution*: `mpo.solution.Solution`)

Generates the vertices associated with the mixed

Parameters *solution* – a multiparametric region

Returns a list of a collection of vertices sorted counterclockwise that correspond to the specific region

`mpo.plot.parametric_plot`(*solution*: `mpo.solution.Solution`, *save_path*: `Optional[str] = None`,
show=`True`) \rightarrow `None`

Makes a simple plot from a solution

Parameters

- **solution** – a multiparametric solution
- **save_path** – if specified saves the plot in the directory
- **show** – Keyword argument, if `True` displays the plot otherwise does not display

Returns no return, creates graph of solution

`mpo.plot.plotly_plot`(*solution*: `mpo.solution.Solution`, *save_path*: `Optional[str] = None`,
show=`True`) \rightarrow `None`

Makes a plot via the `plotly` library, this is good for interactive figures that you can embed into webpages and handle interactively

Parameters

- **solution** –
- **save_path** – Keyword argument, if a directory path is specified it will save a html copy and a png to that directory
- **show** – Keyword argument, if True displays the plot otherwise does not display

Returns

`mpo.plot.sort_clockwise(vertices: List[numpy.ndarray]) → List[numpy.ndarray]`
 Sorts the vertices in clockwise order, fixes crossed polytopes in rendering

Parameters vertices –**Returns**

3.1.6 mpo.problem_generator module

`mpo.problem_generator.generate_mplp(x: int = 2, t: int = 2, m: int = 10) → mpo.mp_program.MPLP_Program`

Parameters

- **x** – number of parameters
- **t** – number of uncertain variables
- **m** – number of constraints

Returns

`mpo.problem_generator.generate_mqp(x: int = 2, t: int = 2, m: int = 10) → mpo.mp_program.MQP_Program`

Generates a random mqp problem with of the following characteristics

Parameters

- **x** – number of x dimensions
- **t** – number of theta dimensions
- **m** – number of constraints

Returns A random mqp problem of the specified type

3.1.7 mpo.settings module

3.1.8 mpo.solution module

`class mpo.solution.Solution(program: Union[mpo.mp_program.MPLP_Program, mpo.mp_program.MQP_Program], critical_regions: List[mpo.critical_region.CriticalRegion])`

Bases: object

The Solution object is the output of multiparametric solvers, it contains all of the critical regions as well as holds a copy of the original problem that was solved

`add_region(region: mpo.critical_region.CriticalRegion) → None`
 Adds a region to the solution

Parameters region – region to add to the solution

Returns None

critical_regions: List [*mpo.critical_region.CriticalRegion*]

evaluate (*theta_point: numpy.ndarray*) → Union[None, numpy.ndarray]
returns the optimal x^* from the solution

Parameters *theta_point* – an uncertainty realization

Returns the calculated x^* from *theta*

get_region (*theta_point: numpy.ndarray*) → Union[None, *mpo.critical_region.CriticalRegion*]
Find the critical region in the solution that corresponds to the *theta* provided

Parameters *theta_point* – an uncertainty realization

Returns the region that contains *theta*

program: Union [*mpo.mp_program.MPLP_Program*, *mpo.mp_program.MQP_Program*]

verify_solution ()

verify_theta (*theta_point: numpy.ndarray*) → bool
Checks that the result of the solution is consistent with *theta* substituted multiparametric problem

Parameters *theta_point* – an uncertainty realization

Returns True if they are the same, False if they are different

3.1.9 Module contents

INDICES AND TABLES

- `genindex`
- `modindex`
- `search`

PYTHON MODULE INDEX

m

- [mpo](#), [34](#)
- [mpo.critical_region](#), [30](#)
- [mpo.geometry](#), [9](#)
- [mpo.geometry.polytope_operations](#), [9](#)
- [mpo.mp_program](#), [31](#)
- [mpo.mp_solvers](#), [13](#)
- [mpo.mp_solvers.mppqp_combinatorial](#), [10](#)
- [mpo.mp_solvers.mppqp_geometric](#), [11](#)
- [mpo.mp_solvers.mppqp_graph](#), [11](#)
- [mpo.mp_solvers.mppqp_parallel_combinatorial](#),
[11](#)
- [mpo.mp_solvers.mppqp_parallel_graph](#), [12](#)
- [mpo.mp_solvers.solve_mplp](#), [12](#)
- [mpo.mp_solvers.solve_mppqp](#), [13](#)
- [mpo.plot](#), [32](#)
- [mpo.problem_generator](#), [33](#)
- [mpo.settings](#), [33](#)
- [mpo.solution](#), [33](#)
- [mpo.solver_interface](#), [21](#)
- [mpo.solver_interface.cvxopt_interface](#),
[13](#)
- [mpo.solver_interface.gurobi_solver_interface](#),
[14](#)
- [mpo.solver_interface.solver_interface](#),
[17](#)
- [mpo.solver_interface.solver_utils](#), [20](#)
- [mpo.upop](#), [24](#)
- [mpo.upop.language_generation](#), [21](#)
- [mpo.upop.point_location](#), [21](#)
- [mpo.upop.ucontroller](#), [22](#)
- [mpo.upop.upop_utils](#), [23](#)
- [mpo.utils](#), [30](#)
- [mpo.utils.chebyshev_ball](#), [24](#)
- [mpo.utils.constraint_utilities](#), [25](#)
- [mpo.utils.general_utils](#), [27](#)
- [mpo.utils.geometric](#), [27](#)
- [mpo.utils.mppqp_utils](#), [28](#)
- [mpo.utils.solver_utils](#), [29](#)

A

A (*mpo.critical_region.CriticalRegion* attribute), 30
A (*mpo.mp_program.MPLP_Program* attribute), 31
A (*mpo.mp_program.MPQP_Program* attribute), 32
A_t (*mpo.mp_program.MPLP_Program* attribute), 31
A_t (*mpo.mp_program.MPQP_Program* attribute), 32
active_set (*mpo.critical_region.CriticalRegion* attribute), 30
active_set (*mpo.mp_program.MPLP_Program* attribute), 31
active_set (*mpo.mp_program.MPQP_Program* attribute), 32
active_set (*mpo.solver_interface.solver_utils.SolverOutput* attribute), 20
add () (*mpo.mp_solvers.mqpq_combinatorial.CombinationTester* method), 10
add_combos () (*mpo.mp_solvers.mqpq_parallel_combinatorial.CombinationTester* method), 11
add_region () (*mpo.solution.Solution* method), 33

B

b (*mpo.critical_region.CriticalRegion* attribute), 30
b (*mpo.mp_program.MPLP_Program* attribute), 31
b (*mpo.mp_program.MPQP_Program* attribute), 32
b_t (*mpo.mp_program.MPLP_Program* attribute), 31
b_t (*mpo.mp_program.MPQP_Program* attribute), 32
BVH (class in *mpo.upop.ucontroller*), 22

C

C (*mpo.critical_region.CriticalRegion* attribute), 30
c (*mpo.mp_program.MPLP_Program* attribute), 31
c (*mpo.mp_program.MPQP_Program* attribute), 32
calculate_control_law () (in module *mpo.utils.mqpq_utils*), 28
calculate_redundant_constraints () (in module *mpo.utils.constraint_utilities*), 25
cheap_remove_redundant_constraints () (in module *mpo.utils.constraint_utilities*), 25
chebyshev_ball () (in module *mpo.utils.chebyshev_ball*), 24
check () (*mpo.mp_solvers.mqpq_combinatorial.CombinationTester* method), 10

check () (*mpo.mp_solvers.mqpq_parallel_combinatorial.CombinationTester* method), 11
check_child_feasibility () (in module *mpo.mp_solvers.mqpq_combinatorial*), 10
check_feasibility () (in module *mpo.utils.mqpq_utils*), 28
check_optimality () (in module *mpo.utils.mqpq_utils*), 28
classify_polytope () (in module *mpo.upop.ucontroller*), 22
CombinationTester (class in *mpo.mp_solvers.mqpq_combinatorial*), 10
CombinationTester (class in *mpo.mp_solvers.mqpq_parallel_combinatorial*), 11
combos (*mpo.mp_solvers.mqpq_combinatorial.CombinationTester* attribute), 10
constraint_norm () (in module *mpo.utils.constraint_utilities*), 25
critical_regions (*mpo.solution.Solution* attribute), 34
CriticalRegion (class in *mpo.critical_region*), 30

D

d (*mpo.critical_region.CriticalRegion* attribute), 30
determine_hyperplane () (in module *mpo.upop.ucontroller*), 22
display_latex () (*mpo.mp_program.MPLP_Program* method), 31
display_warnings () (*mpo.mp_program.MPLP_Program* method), 31
dual (*mpo.solver_interface.solver_utils.SolverOutput* attribute), 20
Dustin (*mpo.mp_solvers.solve_mplp.mplp_solver* attribute), 12

E

E (*mpo.critical_region.CriticalRegion* attribute), 30
evaluate () (*mpo.critical_region.CriticalRegion* method), 30
evaluate () (*mpo.solution.Solution* method), 34

`evaluate()` (*mpo.upop.point_location.PointLocation* method), 21

F

`f` (*mpo.critical_region.CriticalRegion* attribute), 30

`F` (*mpo.mp_program.MPLP_Program* attribute), 31

`F` (*mpo.mp_program.MPQP_Program* attribute), 32

`facet_ball_elimination()` (in module *mpo.utils.constraint_utilities*), 25

`filter_solution()` (in module *mpo.mp_solvers.solve_mpqp*), 13

`find_unique_hyperplanes()` (in module *mpo.upop.upop_utils*), 23

`find_unique_region_functions()` (in module *mpo.upop.upop_utils*), 23

`find_unique_region_hyperplanes()` (in module *mpo.upop.upop_utils*), 23

`full_process()` (in module *mpo.mp_solvers.mpqp_parallel_combinatorial*), 11

G

`gen_array()` (in module *mpo.upop.language_generation*), 21

`gen_cpp_array()` (in module *mpo.upop.language_generation*), 21

`gen_cpp_variable()` (in module *mpo.upop.language_generation*), 21

`gen_cr_from_active_set()` (in module *mpo.utils.mpqp_utils*), 28

`gen_js_array()` (in module *mpo.upop.language_generation*), 21

`gen_js_variable()` (in module *mpo.upop.language_generation*), 21

`gen_python_array()` (in module *mpo.upop.language_generation*), 21

`gen_python_variable()` (in module *mpo.upop.language_generation*), 21

`gen_tess_points_simplex()` (in module *mpo.utils.geometric*), 27

`gen_variable()` (in module *mpo.upop.language_generation*), 21

`gen_vertices()` (in module *mpo.plot*), 32

`generate_children()` (in module *mpo.mp_solvers.mpqp_combinatorial*), 10

`generate_children()` (in module *mpo.mp_solvers.mpqp_parallel_combinatorial*), 11

`generate_code()` (in module *mpo.upop.ucontroller*), 22

`generate_extra()` (in module *mpo.mp_solvers.mpqp_graph*), 11

`generate_mplp()` (in module *mpo.problem_generator*), 33

`generate_mpqp()` (in module *mpo.problem_generator*), 33

`generate_reduce()` (in module *mpo.mp_solvers.mpqp_graph*), 11

`get_active_set()` (in module *mpo.utils.solver_utils*), 29

`get_boundary_types()` (in module *mpo.utils.mpqp_utils*), 28

`get_chebychev_centers()` (in module *mpo.upop.upop_utils*), 23

`get_chebyshev_information()` (in module *mpo.geometry.polytope_operations*), 9

`get_constraints()` (*mpo.critical_region.CriticalRegion* method), 30

`get_descriptions()` (in module *mpo.upop.upop_utils*), 23

`get_feasible_theta()` (in module *mpo.utils.mpqp_utils*), 28

`get_feasible_theta_2()` (in module *mpo.utils.mpqp_utils*), 28

`get_outer_boundaries()` (in module *mpo.upop.upop_utils*), 23

`get_program_parameters()` (in module *mpo.solver_interface.solver_utils*), 20

`get_region()` (in module *mpo.utils.mpqp_utils*), 29

`get_region()` (*mpo.solution.Solution* method), 34

`get_vertices()` (in module *mpo.geometry.polytope_operations*), 9

`graph` (*mpo.mp_solvers.solve_mpqp.mpqp_algorithm* attribute), 13

`gupta` (*mpo.mp_solvers.solve_mpqp.mpqp_algorithm* attribute), 13

`gupta_parallel` (*mpo.mp_solvers.solve_mpqp.mpqp_algorithm* attribute), 13

I

`is_feasible()` (in module *mpo.mp_solvers.mpqp_parallel_combinatorial*), 12

`is_full_dimension()` (*mpo.critical_region.CriticalRegion* method), 30

`is_full_dimensional()` (in module *mpo.utils.mpqp_utils*), 29

`is_full_rank()` (in module *mpo.utils.constraint_utilities*), 25

`is_inside()` (*mpo.critical_region.CriticalRegion* method), 30

`is_inside()` (*mpo.upop.point_location.PointLocation* method), 21

`is_optimal()` (in module *mpo.mp_solvers.mpqp_parallel_combinatorial*), 12

L

`lagrange_multipliers()`
 (*mpo.critical_region.CriticalRegion* method),
 30
`lambda_set` (*mpo.critical_region.CriticalRegion* at-
 tribute), 30
`latex()` (*mpo.mp_program.MPLP_Program* method),
 31
`latex()` (*mpo.mp_program.MPQP_Program* method),
 32
`latex_matrix()` (in module *mpo.utils.general_utils*),
 27
`locate()` (*mpo.upop.point_location.PointLocation*
 method), 22

M

`make_column()` (in module *mpo.utils.general_utils*),
 27
`make_domain_subdivision()` (in module
mpo.utils.geometric), 27
`make_row()` (in module *mpo.utils.general_utils*), 27
`make_simplex()` (in module *mpo.utils.geometric*), 27
`make_subdomains()` (in module
mpo.utils.geometric), 27
module
 `mpo`, 34
 `mpo.critical_region`, 30
 `mpo.geometry`, 9
 `mpo.geometry.polytope_operations`, 9
 `mpo.mp_program`, 31
 `mpo.mp_solvers`, 13
 `mpo.mp_solvers.mpqp_combinatorial`,
 10
 `mpo.mp_solvers.mpqp_geometric`, 11
 `mpo.mp_solvers.mpqp_graph`, 11
 `mpo.mp_solvers.mpqp_parallel_combinatorial`,
 11
 `mpo.mp_solvers.mpqp_parallel_graph`,
 12
 `mpo.mp_solvers.solve_mplp`, 12
 `mpo.mp_solvers.solve_mpqp`, 13
 `mpo.plot`, 32
 `mpo.problem_generator`, 33
 `mpo.settings`, 33
 `mpo.solution`, 33
 `mpo.solver_interface`, 21
 `mpo.solver_interface.cvxopt_interface`,
 13
 `mpo.solver_interface.gurobi_solver_interface`,
 14
 `mpo.solver_interface.solver_interface`,
 17
 `mpo.solver_interface.solver_utils`,
 20
 `mpo.upop`, 24
 `mpo.upop.language_generation`, 21
 `mpo.upop.point_location`, 21
 `mpo.upop.ucontroller`, 22
 `mpo.upop.upop_utils`, 23
 `mpo.utils`, 30
 `mpo.utils.chebyshev_ball`, 24
 `mpo.utils.constraint_utilities`, 25
 `mpo.utils.general_utils`, 27
 `mpo.utils.geometric`, 27
 `mpo.utils.mpqp_utils`, 28
 `mpo.utils.solver_utils`, 29
`MPLP_Program` (class in *mpo.mp_program*), 31
`mplp_solver` (class in *mpo.mp_solvers.solve_mplp*),
 12
`mpo`
 module, 34
`mpo.critical_region`
 module, 30
`mpo.geometry`
 module, 9
`mpo.geometry.polytope_operations`
 module, 9
`mpo.mp_program`
 module, 31
`mpo.mp_solvers`
 module, 13
`mpo.mp_solvers.mpqp_combinatorial`
 module, 10
`mpo.mp_solvers.mpqp_geometric`
 module, 11
`mpo.mp_solvers.mpqp_graph`
 module, 11
`mpo.mp_solvers.mpqp_parallel_combinatorial`
 module, 11
`mpo.mp_solvers.mpqp_parallel_graph`
 module, 12
`mpo.mp_solvers.solve_mplp`
 module, 12
`mpo.mp_solvers.solve_mpqp`
 module, 13
`mpo.plot`
 module, 32
`mpo.problem_generator`
 module, 33
`mpo.settings`
 module, 33
`mpo.solution`
 module, 33
`mpo.solver_interface`
 module, 21
`mpo.solver_interface.cvxopt_interface`
 module, 13
`mpo.solver_interface.gurobi_solver_interface`

module, 14
 mpo.solver_interface.solver_interface
 module, 17
 mpo.solver_interface.solver_utils
 module, 20
 mpo.upop
 module, 24
 mpo.upop.language_generation
 module, 21
 mpo.upop.point_location
 module, 21
 mpo.upop.ucontroller
 module, 22
 mpo.upop.upop_utils
 module, 23
 mpo.utils
 module, 30
 mpo.utils.chebyshev_ball
 module, 24
 mpo.utils.constraint_utilities
 module, 25
 mpo.utils.general_utils
 module, 27
 mpo.utils.geometric
 module, 27
 mpo.utils.mppq_utils
 module, 28
 mpo.utils.solver_utils
 module, 29
 mpo_block() (in module mpo.utils.general_utils), 27
 mppq_algorithm (class in mpo.mp_solvers.solve_mppq), 13
 MPQP_Program (class in mpo.mp_program), 31

N

num_constraints() (mpo.mp_program.MPLP_Program method), 31
 num_cpu_cores() (in module mpo.utils.general_utils), 27
 num_t() (mpo.mp_program.MPLP_Program method), 31
 num_x() (mpo.mp_program.MPLP_Program method), 31

O

obj (mpo.solver_interface.solver_utils.SolverOutput attribute), 20
 omega_set (mpo.critical_region.CriticalRegion attribute), 30

P

parametric_plot() (in module mpo.plot), 32
 plotly_plot() (in module mpo.plot), 32

PointLocation (class in mpo.upop.point_location), 21
 process_constraints() (mpo.mp_program.MPLP_Program method), 31
 process_constraints() (mpo.mp_program.MPPQ_Program method), 32
 process_region_constraints() (in module mpo.utils.constraint_utilities), 25
 program (mpo.solution.Solution attribute), 34

R

regular_set (mpo.critical_region.CriticalRegion attribute), 30
 remove_duplicate_rows() (in module mpo.utils.constraint_utilities), 26
 remove_size_zero_matrices() (in module mpo.utils.general_utils), 27
 remove_strongly_redundant_constraints() (in module mpo.utils.constraint_utilities), 26
 remove_zero_rows() (in module mpo.utils.constraint_utilities), 26
 revised_tess_simplex() (in module mpo.utils.geometric), 27
 row_equality() (in module mpo.utils.constraint_utilities), 26

S

sample_region_convex_combination() (in module mpo.geometry.polytope_operations), 9
 scale_constraint() (in module mpo.utils.constraint_utilities), 26
 scale_constraints() (mpo.mp_program.MPLP_Program method), 31
 select_not_in_list() (in module mpo.utils.general_utils), 27
 slack (mpo.solver_interface.solver_utils.SolverOutput attribute), 20
 sol (mpo.solver_interface.solver_utils.SolverOutput attribute), 20
 Solution (class in mpo.solution), 33
 solve() (in module mpo.mp_solvers.mppq_combinatorial), 10
 solve() (in module mpo.mp_solvers.mppq_geometric), 11
 solve() (in module mpo.mp_solvers.mppq_graph), 11
 solve() (in module mpo.mp_solvers.mppq_parallel_combinatorial), 12
 solve() (in module mpo.mp_solvers.mppq_parallel_graph), 12

`solve_fully_constraints()` (in module `mpo.solver_interface.cvxopt_interface`), 13
`solve_lp()` (in module `mpo.solver_interface.solver_interface`), 17
`solve_lp_cvxopt()` (in module `mpo.solver_interface.cvxopt_interface`), 14
`solve_lp_gurobi()` (in module `mpo.solver_interface.gurobi_solver_interface`), 14
`solve_milp()` (in module `mpo.solver_interface.solver_interface`), 18
`solve_milp_gurobi()` (in module `mpo.solver_interface.gurobi_solver_interface`), 14
`solve_miqp()` (in module `mpo.solver_interface.solver_interface`), 18
`solve_miqp_gurobi()` (in module `mpo.solver_interface.gurobi_solver_interface`), 15
`solve_mplp()` (in module `mpo.mp_solvers.solve_mplp`), 12
`solve_mqpq()` (in module `mpo.mp_solvers.solve_mqpq`), 13
`solve_qp()` (in module `mpo.solver_interface.solver_interface`), 19
`solve_qp_gurobi()` (in module `mpo.solver_interface.gurobi_solver_interface`), 16
`solve_theta()` (`mpo.mp_program.MPLP_Program` method), 31
`solve_theta()` (`mpo.mp_program.MPQP_Program` method), 32
`solver_not_supported()` (in module `mpo.solver_interface.solver_interface`), 20
`SolverOutput` (class in `mpo.solver_interface.solver_utils`), 20
`sort_clockwise()` (in module `mpo.plot`), 33
`space` (`mpo.mp_solvers.solve_mqpq.mqpq_algorithm` attribute), 13
`step` (`mpo.mp_solvers.solve_mqpq.mqpq_algorithm` attribute), 13

T

`theta_ball()` (in module `mpo.utils.mqpq_utils`), 29

V

`verify_outer_boundary()` (in module `mpo.upop.upop_utils`), 24
`verify_solution()` (`mpo.solution.Solution` method), 34
`verify_theta()` (`mpo.solution.Solution` method), 34

W

`warnings()` (`mpo.mp_program.MPLP_Program`

`method`), 31
`warnings()` (`mpo.mp_program.MPQP_Program` method), 32

Z

`zeros()` (in module `mpo.utils.mqpq_utils`), 29