Assignment - 1

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L25+L26

MAT2002 - Application of Differential and difference equations

Experiment - 1

→ Basics of Matlab (12/7/19)

- Component-wise multiplication :
  x.*y , a.*b
  Similarly for division and exponents

eg. $x = [x_1 , x_2 , x_3]$
$y = [y_1 , y_2 , y_3]$
$x./y = [x_1/y_1 \quad x_2/y_2 \quad x_3/y_3]$

* Without the dot, matrix multiplication is considered

- cosec(x) → csc(x)
  sind(x) : sin(x), where 'x' is in degrees
  asin(x) : $\sin^{-1}(x)$

- linspace (xfirst, xlast, n) :
  Creates an array of 'n' elements, starting with 'xfirst' till 'xlast'

- zeroes (m,n) : $\text{Matrix}_{m \times n}$ of zeroes
  ones (m,n) : $\text{Matrix}_{m \times n}$ of ones
  eye (m) : Unit vector of side 'm'

- Transpose of matrix : A'
  Inverse of matrix : inv(A)
  Trace of matrix : trace (A)
  Diagonal elements of a matrix : diag (A)

- A(1,:) → First row elements
  A(:,2) → Second column elements
  A(3,4) → 1 particular element at [3,4]

- To find roots of quadratic equation:

  syms x
  solve ('x^2 - 4')
  ↑
  Equation itself

  roots ([1 0 -4])
  ↑
  Coefficients of the 'x' terms

- y = f(x)
  plot (x,y) : Graph plotting function

- Creating vectors:
  i] x = 0: 0.5: pi
  ↳ Create vector with equally spaced intervals
  ii] x = linspace (0, pi, 7)
  ↳ Create vector with 'n' equally spaced intervals
  iii] x = logspace (1,2,7)
  ↳ 7 equally distributed values between 10^1 and 10^2

- stem (x,y) : Plotting discrete sequences

- Setting axis scales:
  axis ([xmin xmax ymin ymax])

eg.
```
x = [0: 0.01: 10]
y = exp(-x) .* sin( 2*x + 3)
plot (x,y) , axis ([0 10 -1 1])
```

- xlabel(' '), ylabel(' '), title(' ')
- hold on → plotting on same graph

  hold off → stop "plotting on same graph"

eg.
```
x = linspace(0, 2*pi, 100);
y = sin(x);
plot(x,y);
title('Sine function');
xlabel('x');
ylabel('f(x)');
```

- Multiple graph plots using legend

  → Screenshot 1

- subplot(m,n,p)

  m → no. of rows   p → specifies where to put

  n → no. of columns   a particular plot

  → Screenshots 2 & 3

---

1. Draw a circle centered at (1,3) with radius 2

ans → Screenshot 4
```
clc
clear all
t = linspace(0, 2*pi, 100);
x = 1+ 2*cos(t);
y = 3+ 2*sin(t);
plot(x,y,'b-');
axis equal          % Important
xlabel('x');
ylabel('y');
title('Circle');
```

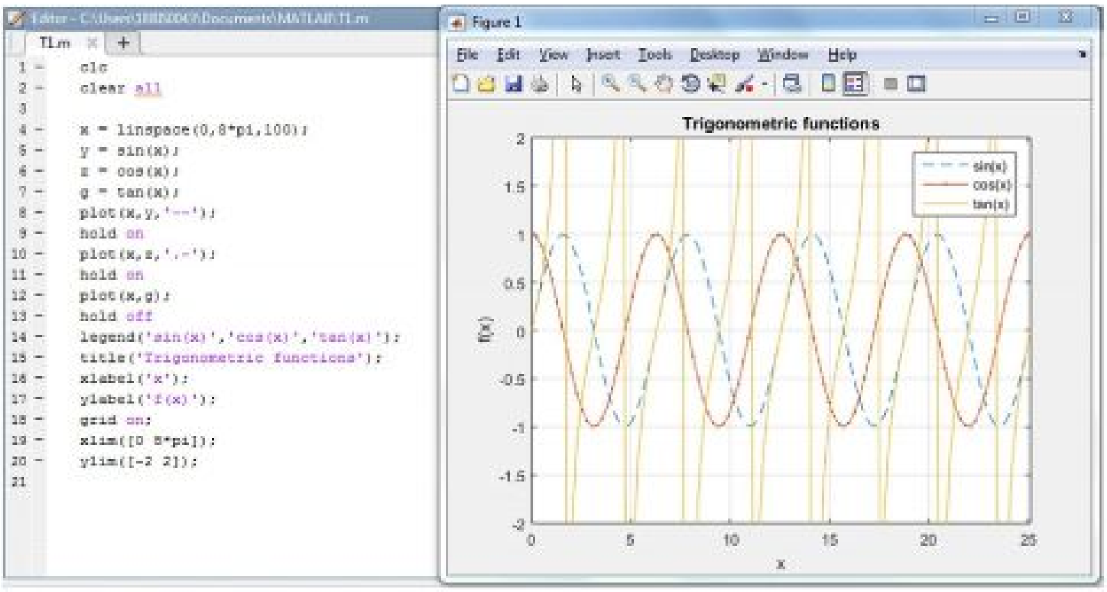
Page 4

- Plotting polynomials → <u>Screenshot 5</u>
- Plotting with ezplot() → <u>Screenshot 6</u>
- Plotting in 3D → <u>Screenshot 7</u>

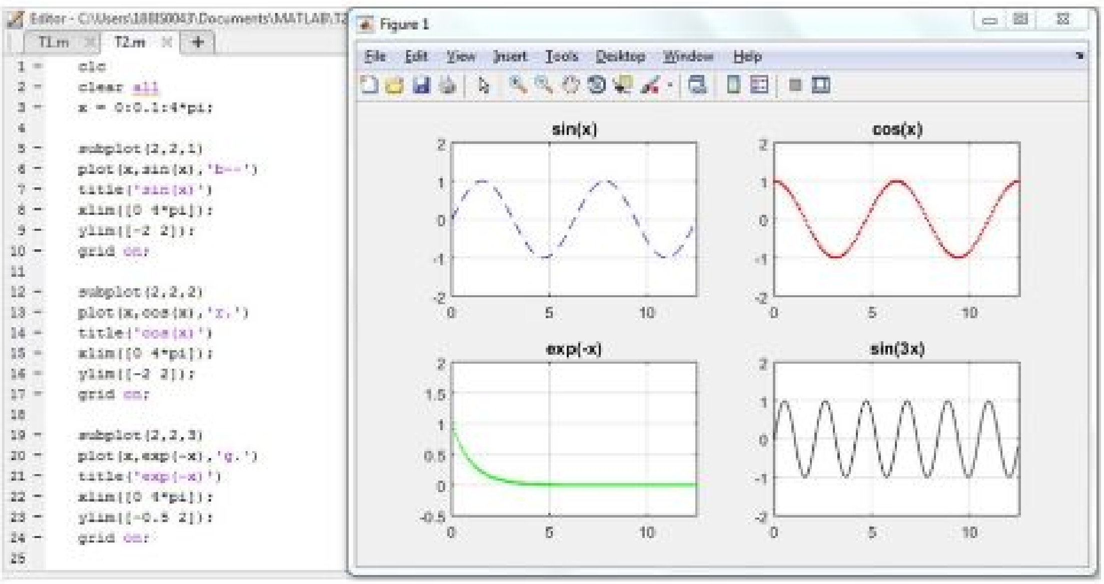
x ———————— x ———————— x

Experiment 1

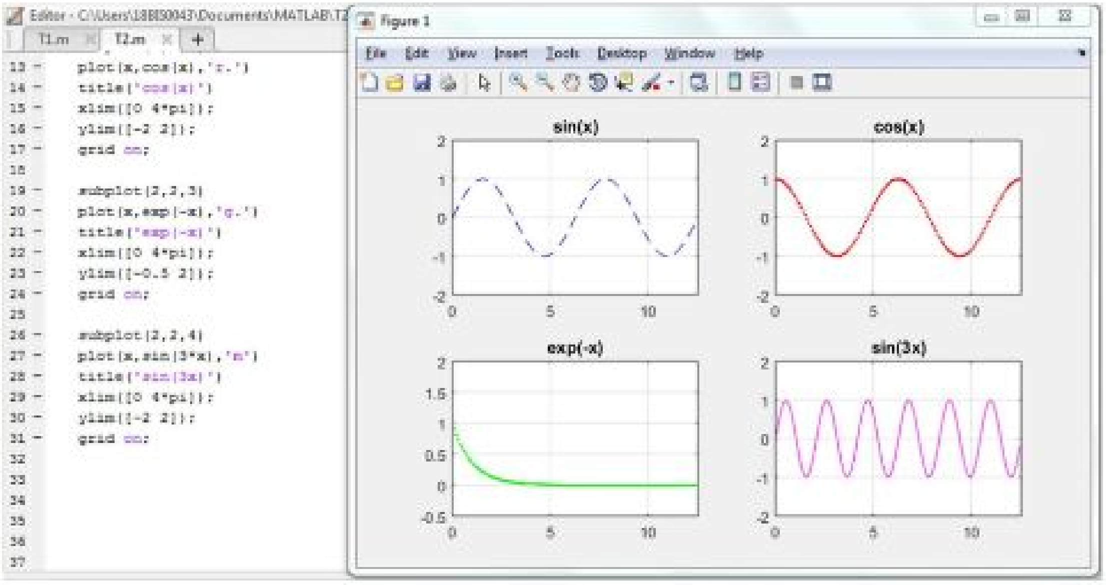
1.



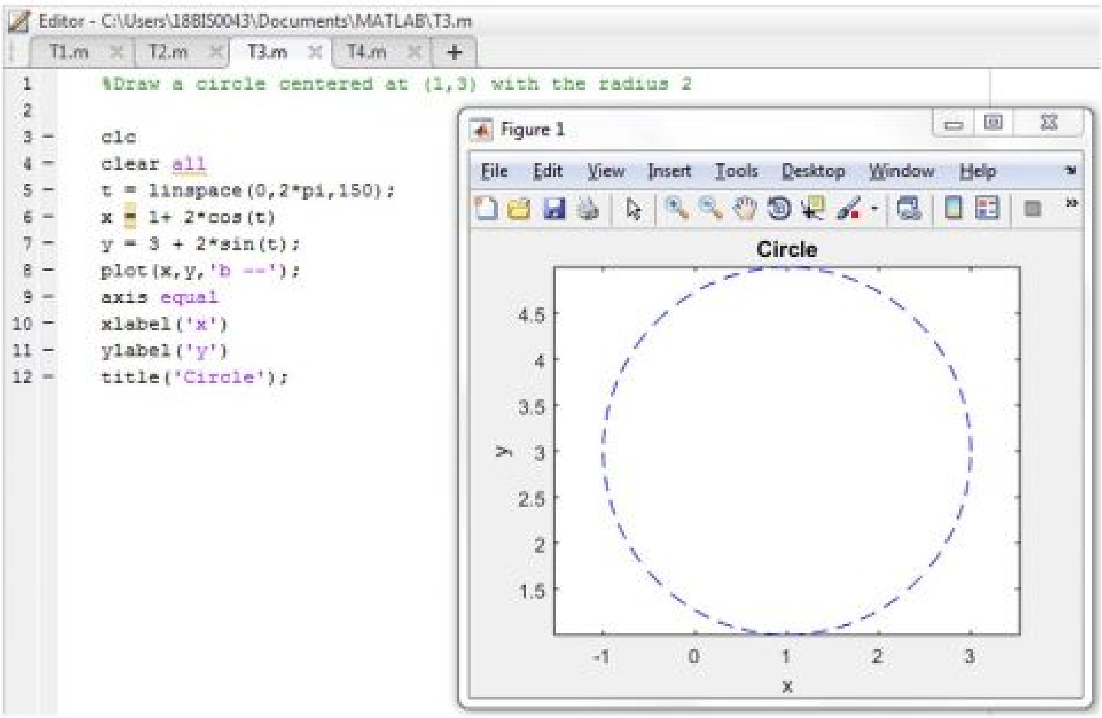
2.



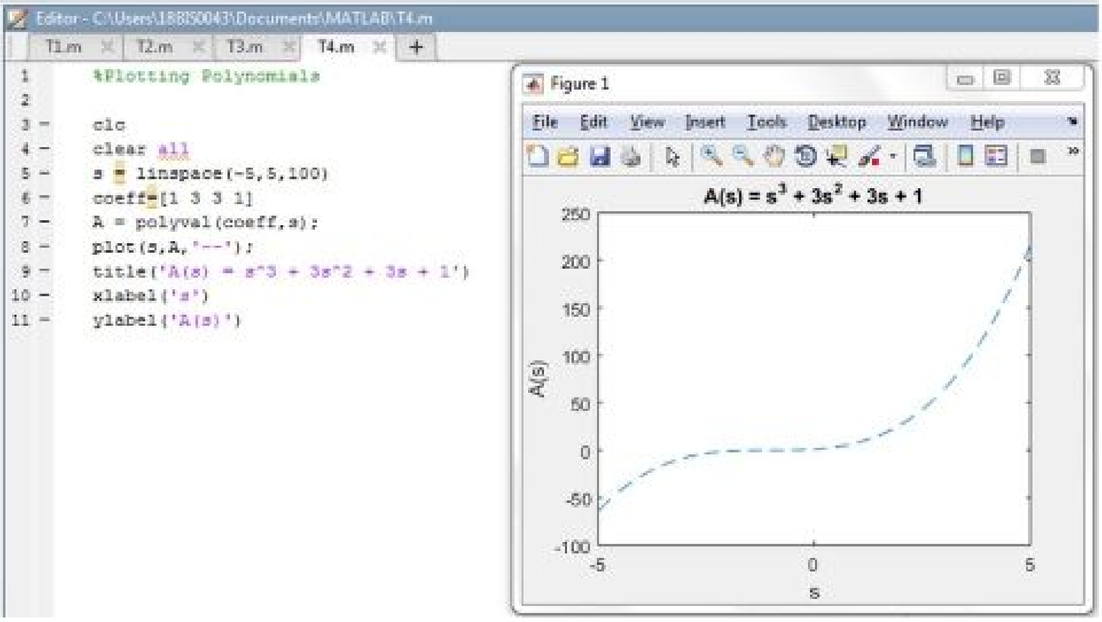
3.


Editor - C:\Users\18BIS0043\Documents\MATLAB\T2
T1.m
T2.m
plot(x,cos(x),'r.')
title('cos(x)')
xlim([0 4*pi]);
ylim([-2 2]);
grid on;
subplot(2,2,3)
plot(x,exp(-x),'g.')
title('exp(-x)')
xlim([0 4*pi]);
ylim([-0.5 2]);
grid on;
subplot(2,2,4)
plot(x,sin(3*x),'m')
title('sin(3x)')
xlim([0 4*pi]);
ylim([-2 2]);
grid on;
Figure 1
File Edit View Insert Tools Desktop Window Help
sin(x)
cos(x)
exp(-x)
sin(3x)

4.

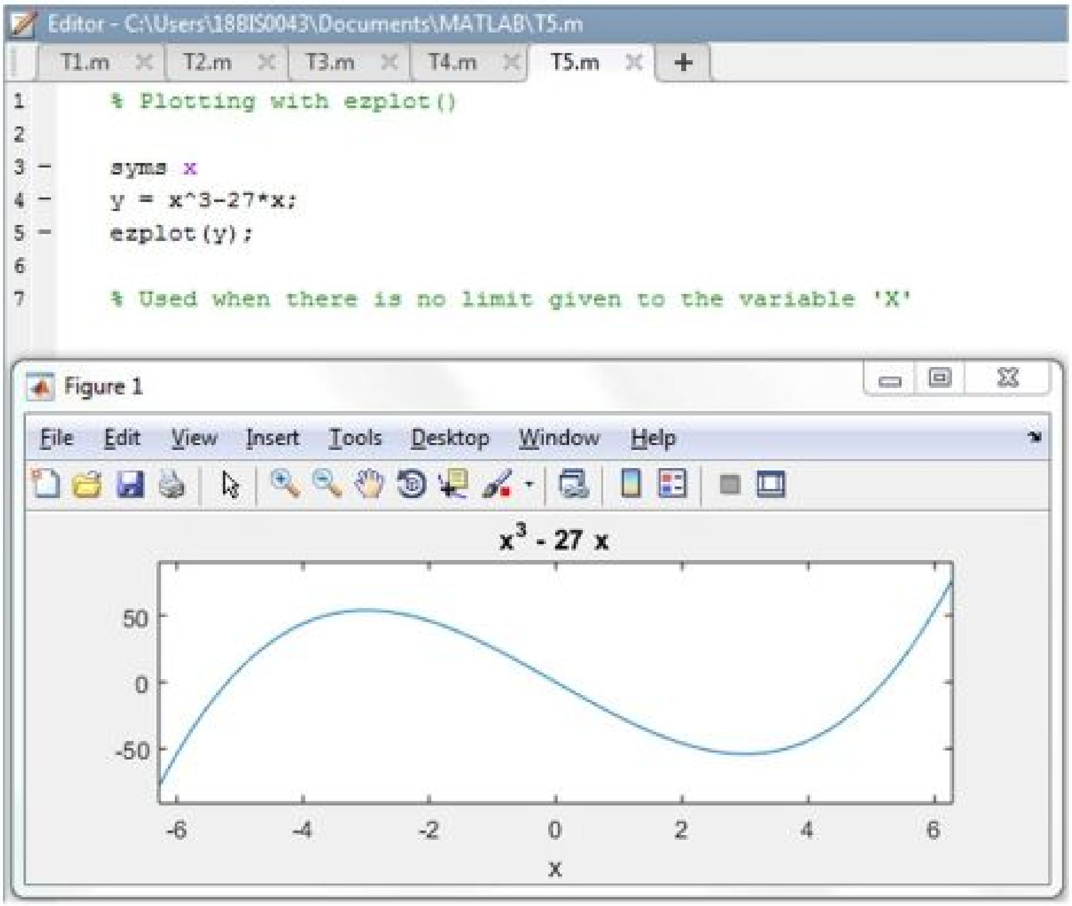

Editor - C:\Users\18BIS0043\Documents\MATLAB\T3.m
T1.m
T2.m
T3.m
T4.m
%Draw a circle centered at (1,3) with the radius 2
clc
clear all
t = linspace(0,2*pi,150);
x = 1+ 2*cos(t)
y = 3 + 2*sin(t);
plot(x,y,'b --');
axis equal
xlabel('x')
ylabel('y')
title('Circle');
Figure 1
File Edit View Insert Tools Desktop Window Help
Circle
x
y

5.



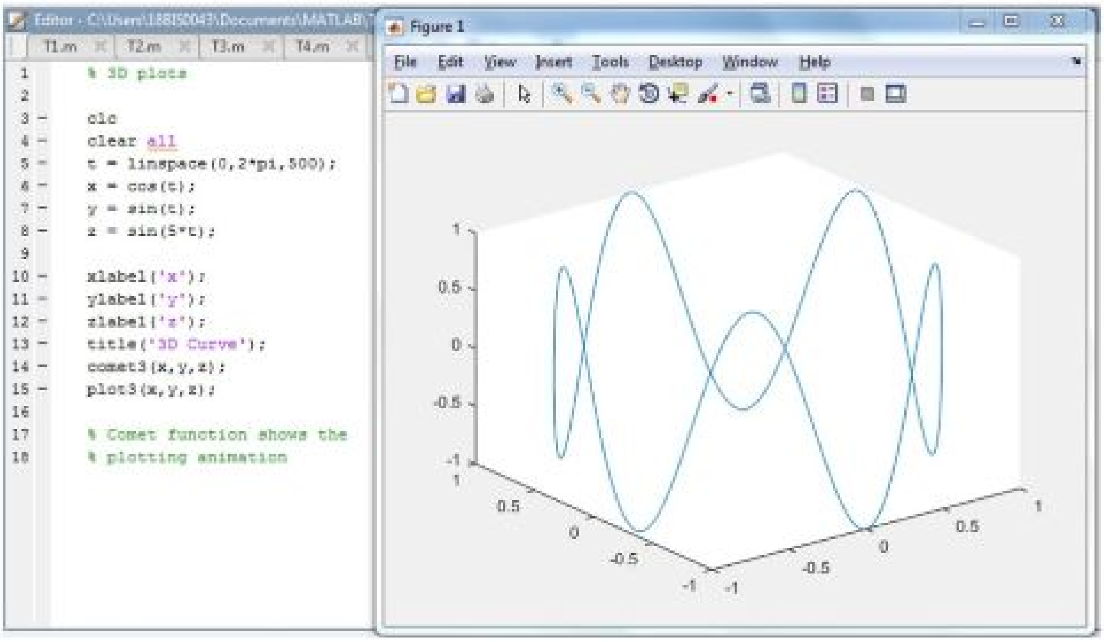
Editor - C:\Users\18BIS0043\Documents\MATLAB\T4.m
T1.m
T2.m
T3.m
T4.m
%Plotting Polynomials
clc
clear all
s = linspace(-5,5,100)
coeff=[1 3 3 1]
A = polyval(coeff,s);
plot(s,A,'--');
title('A(s) = s^3 + 3s^2 + 3s + 1')
xlabel('s')
ylabel('A(s)')
Figure 1
File Edit View Insert Tools Desktop Window Help
A(s) = s3 + 3s2 + 3s + 1
A(s)
s

6.



Editor - C:\Users\18BIS0043\Documents\MATLAB\T5.m
T1.m
T2.m
T3.m
T4.m
T5.m
% Plotting with ezplot()
syms x
y = x^3-27*x;
ezplot(y);
% Used when there is no limit given to the variable 'X'
Figure 1
File Edit View Insert Tools Desktop Window Help
x3 - 27 x
x

7.


Editor - C:\Users\188150043\Documents\MATLAB\
T1.m
T2.m
T3.m
T4.m
% 3D plots
clc
clear all
t = linspace(0,2*pi,500);
x = cos(t);
y = sin(t);
z = sin(5*t);
xlabel('x');
ylabel('y');
zlabel('z');
title('3D Curve');
comet3(x,y,z);
plot3(x,y,z);
% Comet function shows the
% plotting animation
Figure 1
File Edit View Insert Tools Desktop Window Help
1
0.5
0
-0.5
-1
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

Experiment - 2

→ Finding Eigen values and eigen vectors of the given square matrix (19/7/19)

- A non-zero vector is an eigen vector of a square matrix A if there exists a λ (scalar) such that $Ax = \lambda x$, for which λ is called the eigen value of the matrix A

$AX = \lambda x$

$\downarrow$ AX: Square matrix

λ → Eigen values

x → Eigen vectors

- Properties of eigen values and eigen vectors:

i] Sum(eig(A)) = trace(A)

ii] A matrix is singular if and only if it has a zero eigen value

iii] eig(triangular matrix) = elements on its diagonal

iv] $eig(A) = \lambda \Rightarrow eig(A^{-1}) = 1/\lambda$

v] $eig(A) = \lambda \Rightarrow eig(A^{T}) = \lambda$

vi] $eig(A) = \lambda \Rightarrow eig(kA) = k\lambda$

k → Arbitrary constant

vii] $eig(A) = \lambda \Rightarrow eig(A^{k}) = \lambda^{k}$

k → positive integer

- $p = poly(A)$:

  A is an $n \times n$ matrix returns n+1 element row vector with elements - coefficients of characteristic polynomial, $|\lambda I - A|$ - which will be stored in 'p'.

- $r = roots(p)$:

  Both commands combined will give all eigen values of the matrix

- $[V, D] = eig(A)$:

  D → diagonal matrix with eigen values in diagonal

  V → Modal matrix whose columns are the corresponding eigen vectors

- $eye(n)$:

  Returns $n \times n$ identity matrix

---

1. $$A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

Find,

a) Characteristic polynomial

b) Roots of characteristic polynomial

c) Eigen values of A

d) Eigen vectors of A

e) Eigen values of A^{-1}

f) Eigen values of A^T

g) Eigen value of $B = A^2 + 3A + 2I$

ans % In command window

a] poly(A)

1 -4 5 -2

b] roots(poly(A))

2.0000 + 0.0000i

1.0000 + 0.0000i

1.0000 - 0.0000i

[v,d] = eig(A)

c] eig(A)

2

1

1

e] eig(inv(A))

0.5000

1.0000

1.0000

f] eig(transpose(A))

2

1

1

g] eig(A^2 + 3*A + 2*eye(3))

12

6

6

Page 8

2. $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$

find,

a] Characteristic polynomial of A

b] Roots of characteristic polynomial

c] Eigen values of A

d] Eigen vectors of A

e] Eigen values of A^{-1}

f] Eigen values of A^T

g] Eigen values of $B = 7A^3 - 6A^2 + 2A - 3I$

ans

→ Screenshot 1 : a,b

→ Screenshot 2 : c,d

→ Screenshot 3 : f,g

→ Screenshot 4 : e

Since there is 1 eigen value = 0, it is a singular matrix and inverse is not possible

Answers are similar to Q1

x ——— x — x — x ——— x

2/8/24

Experiment 2

A, B

```
Command Window

A =

     1     2     1
     6    -1     0
    -1    -2    -1

>> poly(A)

ans =

    1.0000    1.0000  -12.0000    0.0000

>> roots(poly(A))

ans =

   -4.0000
    3.0000
    0.0000
```

C, D

Command Window

```
>> eig(A)

ans =

   -4.0000
    3.0000
    0.0000

>> [v,d] = eig(A)

v =

    0.4082   -0.4851   -0.0697
   -0.8165   -0.7276   -0.4180
   -0.4082    0.4851    0.9058


d =

   -4.0000         0         0
         0    3.0000         0
         0         0    0.0000
```

F, G

```
Command Window
>> B = 7*A^3 - 6*A^2 + 2*A - 3*eye(3)

B =

   -73   198    86
   516  -258   -78
    70  -198   -89

>> eig(B)

ans =

 -555.0000
  138.0000
   -3.0000

>> eig(transpose(A))

ans =

   -4.0000
    3.0000
   -0.0000
```

Inverse matrix 'e'

```
>> inv(A)
Warning: Matrix is singular to working precision.

ans =

   Inf   Inf   Inf
   Inf   Inf   Inf
   Inf   Inf   Inf

fx >>
```

Experiment - 3

→ Google's mechanism of ranking webpages (2/8/19)
To understand mathematics behind the most successful search engine, google, using the PageRank Algorithm

- Originally devised by Larry Page and Sergey Brin
- PageRank : Function that assigns a real number to each page in the web

For a web of pages A, B, C, D

$$PR(A) = \frac{1-d}{N} + d\left[\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)} \dots\right]$$

L(.) denotes outbound links
PR(.) denotes the page rank
N denotes the number of documents

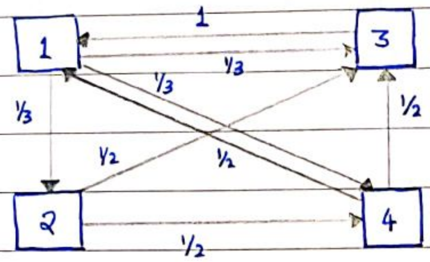
- Damping factor: The probability at any step that the person will continue at the same webpage. Generally assumed to be set around 0.85

- Outbound link will send visitors away from the current webpage he/she is currently on

- Transition/Stochastic matrix:
Matrix in which, each of its entries is a non-negative real number, representing a probability. Types :
i] Right transition matrix is a real symmetric matrix with each row summing to 1.
ii] Left transition matrix is a real symmetric matrix with each column summing to 1.
iii] Double transition matrix is a real matrix with both rows and columns summing to 1.

- The largest possible absolute value of all eigen values of a transition matrix, is 1

- Web consisting of 4 pages



$$x_1 = 0x_1 + 0x_2 + 1x_3 + \tfrac{1}{2}x_4$$
$$x_2 = \tfrac{1}{3}x_1 + 0x_2 + 0x_3 + 0x_4$$
$$x_3 = \tfrac{1}{3}x_1 + \tfrac{1}{2}x_2 + 0x_3 + \tfrac{1}{2}x_4$$
$$x_4 = \tfrac{1}{3}x_1 + \tfrac{1}{2}x_2 + 0x_3 + 0x_4$$

Using inbound links, we have constructed the equations and convert it to matrix form.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1/2 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 0 & 1/2 \\ 1/3 & 1/2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

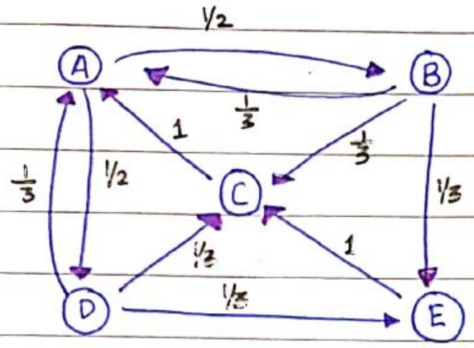
- We find the PageRank vector

$$u = \begin{bmatrix} 0.3871 \\ 0.1290 \\ 0.2903 \\ 0.1935 \end{bmatrix} \begin{matrix} \leftarrow A \\ \leftarrow B \\ \leftarrow C \\ \leftarrow D \end{matrix}$$

Each value in the matrix, shows the importance of the corresponding webpages

$\therefore$ 'A' is the most important while 'B' is the least important webpage.

1.



* $x_1 \to A$
  $x_2 \to B$
  $x_3 \to C$
  $x_4 \to D$
  $x_5 \to E$

ans Simultaneous equations :

$$x_1 = 0x_1 + \tfrac{1}{3}x_2 + 1x_3 + \tfrac{1}{3}x_4 + 0x_5$$
$$x_2 = \tfrac{1}{2}x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5$$
$$x_3 = 0x_1 + \tfrac{1}{3}x_2 + 0x_3 + \tfrac{1}{3}x_4 + 1x_5$$
$$x_4 = \tfrac{1}{2}x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5$$
$$x_5 = 0x_1 + \tfrac{1}{3}x_2 + 0x_3 + \tfrac{1}{3}x_4 + 0x_5$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 & 1/3 & 1 & 1/3 & 0 \\ 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 1 \\ 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 1/3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

Using matlab, we get the vector as

$$u = \begin{bmatrix} 0.3333 \\ 0.1667 \\ 0.2222 \\ 0.1667 \\ 0.1111 \end{bmatrix} \begin{matrix} \leftarrow A \\ \leftarrow B \\ \leftarrow C \\ \leftarrow D \\ \leftarrow E \end{matrix}$$

(Most important) — A

(Least important) — E

$\therefore$ The importance of webpages are used to rank them using this algorithm

2/8/4

x ——————— x – x ——————— x

Experiment 3

Example:

Command Window

```
>> a = [0 0 1 1/2
1/3 0 0 0
1/3 1/2 0 1/2
1/3 1/2 0 0]

a =

         0         0    1.0000    0.5000
    0.3333         0         0         0
    0.3333    0.5000         0    0.5000
    0.3333    0.5000         0         0

>> eig(a)

ans =

   1.0000 + 0.0000i
  -0.3606 + 0.4110i
  -0.3606 - 0.4110i
  -0.2788 + 0.0000i
```

Command Window

```
>> [V,D] = eigs(a)

V =

   0.5065 + 0.0000i   0.7552 + 0.0000i   0.7552 + 0.0000i   0.7210 + 0.0000i
  -0.6057 + 0.0000i  -0.3037 - 0.3461i  -0.3037 + 0.3461i   0.2403 + 0.0000i
  -0.3815 + 0.0000i  -0.0932 + 0.2747i  -0.0932 - 0.2747i   0.5408 + 0.0000i
   0.4807 + 0.0000i  -0.3584 + 0.0714i  -0.3584 - 0.0714i   0.3605 + 0.0000i


D =

  -0.2788 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.3606 + 0.4110i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3606 - 0.4110i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   1.0000 + 0.0000i
```

Command Window

```
>> u = v(:,1)

u =

    0.7210
    0.2403
    0.5408
    0.3605

>> u/sum(u) %Normalization

ans =

    0.3871
    0.1290
    0.2903
    0.1935
```

Question 1:

Command Window

```
>> A = [0 1/3 1 1/3 0
1/2 0 0 0 0
0 1/3 0 1/3 1
1/2 0 0 0 0
0 1/3 0 1/3 0]

A =

         0    0.3333    1.0000    0.3333         0
    0.5000         0         0         0         0
         0    0.3333         0    0.3333    1.0000
    0.5000         0         0         0         0
         0    0.3333         0    0.3333         0

>> eig(A)

ans =

   1.0000 + 0.0000i
  -0.7181 + 0.0000i
  -0.1410 + 0.6666i
  -0.1410 - 0.6666i
   0.0000 + 0.0000i
```

Command Window

```
>> [v,d] = eig(A)

v =

  -0.6975 + 0.0000i   0.6386 + 0.0000i  -0.0175 - 0.5057i  -0.0175 + 0.5057i   0.0000 + 0.0000i
  -0.3487 + 0.0000i  -0.4447 + 0.0000i  -0.3604 + 0.0894i  -0.3604 - 0.0894i   0.7071 + 0.0000i
  -0.4650 + 0.0000i  -0.1621 + 0.0000i   0.5798 + 0.0000i   0.5798 + 0.0000i   0.0000 + 0.0000i
  -0.3487 + 0.0000i  -0.4447 + 0.0000i  -0.3604 + 0.0894i  -0.3604 - 0.0894i  -0.7071 + 0.0000i
  -0.2325 + 0.0000i   0.4128 + 0.0000i   0.1585 + 0.3269i   0.1585 - 0.3269i   0.0000 + 0.0000i


d =

   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.7181 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.1410 + 0.6666i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.1410 - 0.6666i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
```

Command Window

```
>> u = v(:,1)

u =

   -0.6975
   -0.3487
   -0.4650
   -0.3487
   -0.2325

>> u/sum(u) % Normalization

ans =

    0.3333
    0.1667
    0.2222
    0.1667
    0.1111
```