

Experiment – 9

Code:

```
Editor - Z:\MAT2002\Assignment 5\Experiment 9\E9_2.m
E9_1.m  E9_2.m  +
1 -      clc
2 -      clear all
3
4 -      syms n z y(n) Y
5 -      yn = y(n);
6 -      yn1 = y(n+1);
7 -      yn2 = y(n+2);
8 -      F = input('Input coefficients [a,b,c]: ');
9 -      a = F(1);
10 -     b = F(2);
11 -     c = F(3);
12
13 -     nh = input('Enter the non-homogenous part of f(n): ');
14 -     eqn = a*yn2+b*yn1+c*yn-nh;
15 -     ZTY = ztrans(eqn);
16
17 -     IC = input('Enter the initial conditions [y0,y1]: ');
18 -     y0 = IC(1);
19 -     y1 = IC(2);
20
21 -     ZTY = subs(ZTY,{'ztrans(y(n),n,z)','y(0)','y(1)'},{Y,y0,y1});
22 -     eq = collect(ZTY, Y);
23 -     Y = simplify(solve(eq,Y));
24 -     yn = simplify(iztrans(Y));
25 -     disp('The solution of the difference equation: ');
26 -     disp(yn);
27
28 -     m = linspace(0,20,100);
29 -     y = subs(yn,n,m);
30 -     stem(y, 'r--.');
31 -     title('Difference equation');
32 -     xlabel('n');
33 -     ylabel('y(n)');
34 -     grid on;
35
```

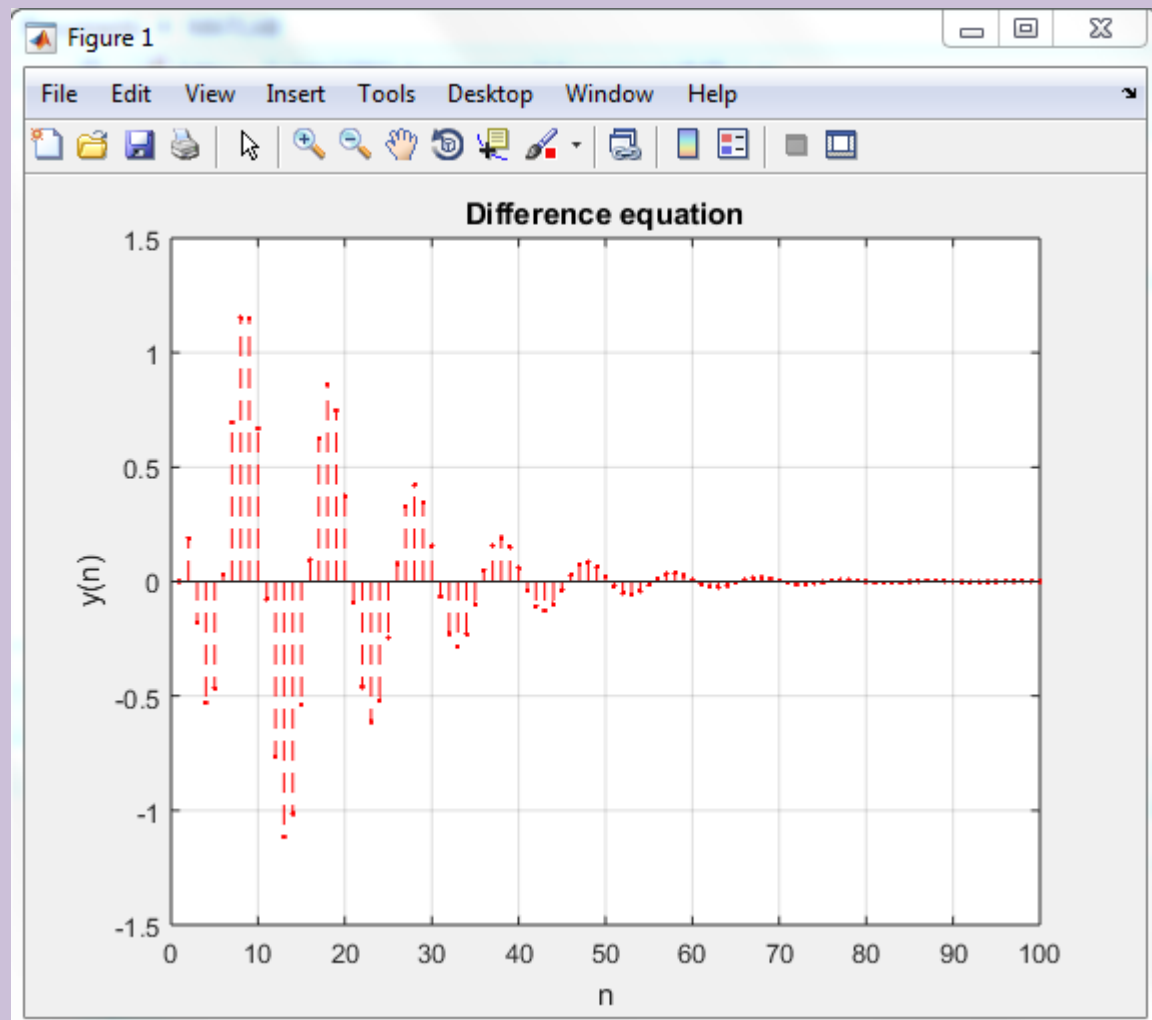
I/O - 1:

```
Command Window
New to MATLAB? See resources for Getting Started.

Input coefficients [a,b,c]: [9 9 2]
Enter the non-homogenous part of f(n): 0
Enter the initial conditions [y0,y1]: [1 1]
The solution of the difference equation:
5*(-1/3)^n - 4*(-2/3)^n
```

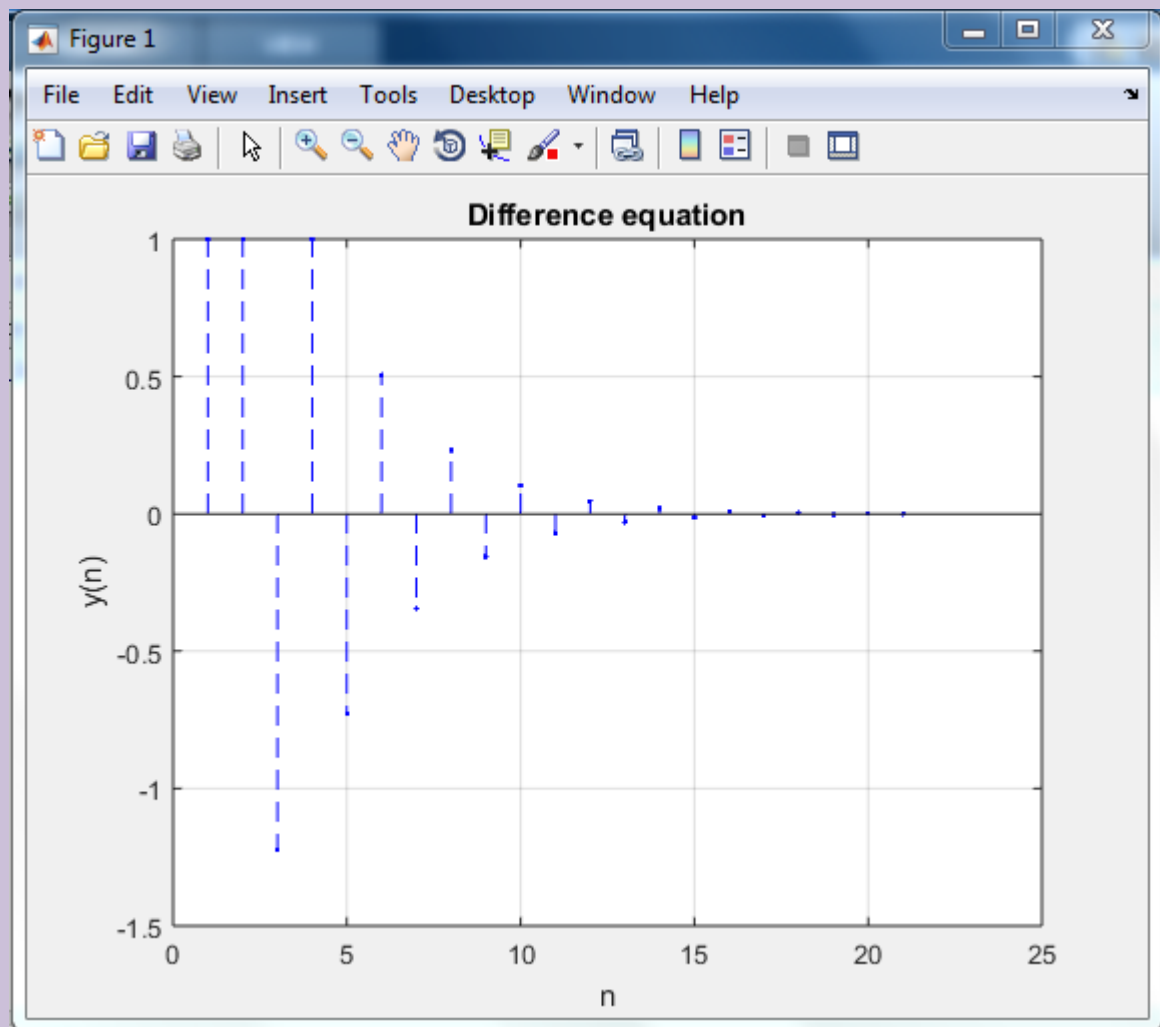
Graph - 1:

$m = \text{linspace}(0,20,100)$



Graph - 2:

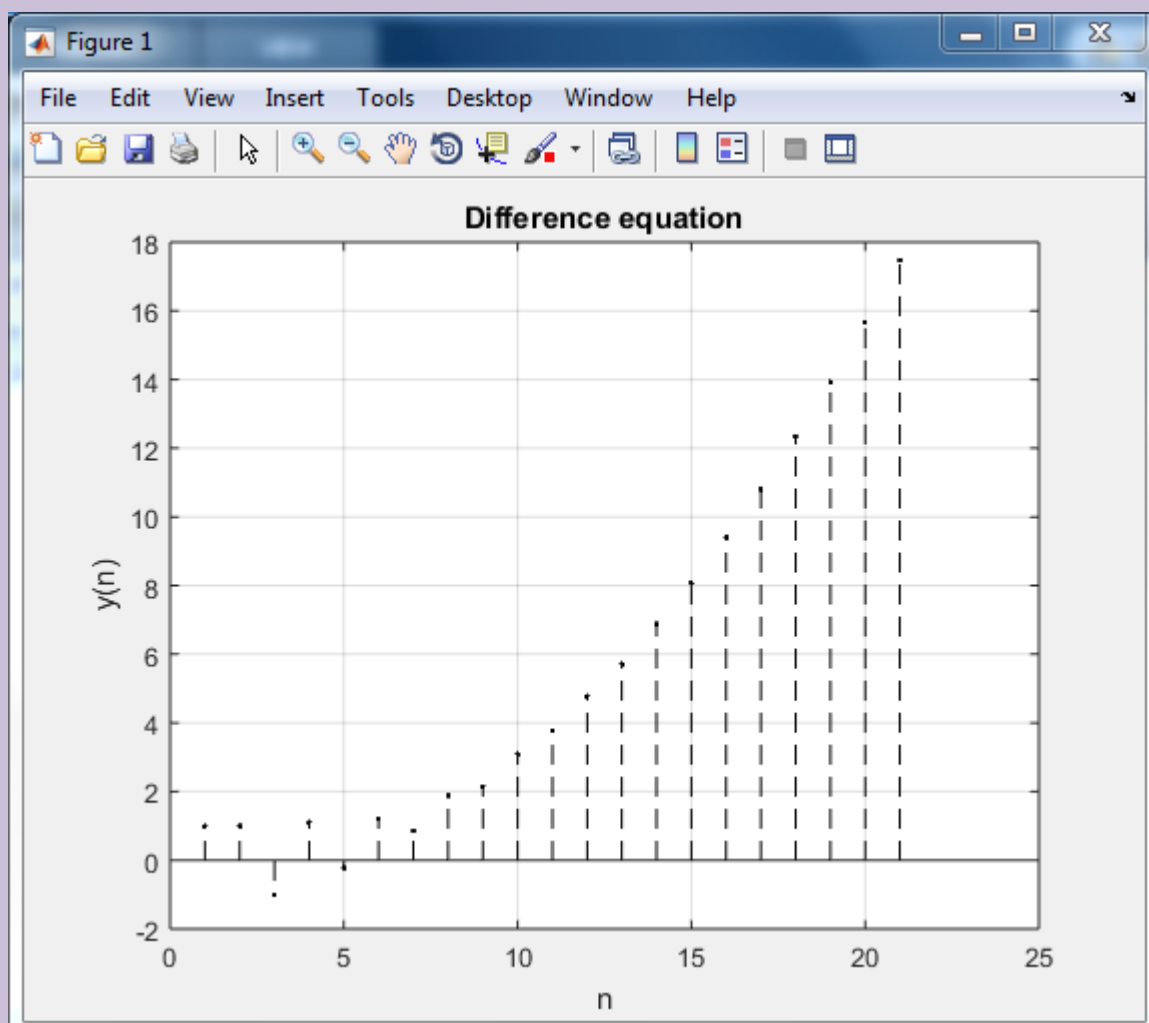
$$m = 0:20$$



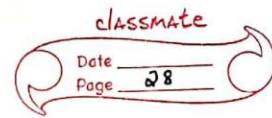
I/O - 2:

```
Command Window
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Input coefficients [a,b,c]: [9 9 2]
Enter the non-homogenous part of f(n): n^2 + 2
Enter the initial conditions [y0,y1]: [1 1]
The solution of the difference equation:
(3*n)/200 + nchoosek(n - 1, 2)/10 + (141*(-1/3)^n)/32 - (447*(-2/3)^n)/125 + 279/4000
```

Graph - 1:

18/10/19

Experiment - 9

→ Z-transform and their applications for solving difference equations

- If function u_n is defined for discrete values ($n = 0, 1, 2, \dots$) and $u_n = 0$ for $n < 0$

$$\bar{U}(z) = Z\{u_n\} = \sum_{n=0}^{\infty} u_n / z^n$$

Inverse Z-transform:

$$u_n = Z^{-1}\{\bar{U}(z)\}$$

- $ztrans(f)$: Z-transform of the scalar symbol f with the default independent variable n .
Default return is a function of z

$ztrans(f, \omega)$: Makes z a function of ω , not z

$ztrans(f, k, \omega)$: Takes f to be a function of the symbolic variable k

- $iztrans(F)$: Inverse Z-transform
 $iztrans(F, k)$
 $iztrans(F, k, \omega)$

- $collect(P, var)$: Rewrites P in terms of power variable 'var' given as the second parameter

- $stem(y)$: Plots the data sequence 'y' as stems that extend from equally spaced automatically generated values of x-axis



1. Z-transform of $y_n = \frac{1}{4}^n, n \geq 0$

>> syms z n;

>> ztrans(1/4^n)

ans =

$$z(z - \frac{1}{4})$$

2. Inverse z-transform of $y(z) = \frac{2z}{2z-1}$

>> syms z n;

>> iztrans(2*z/(2*z-1))

ans =

$$(\frac{1}{2})^n$$

• Solution of linear difference equations by z-transform:

$$ay_{n+2} + by_{n+1} + cy = f(n)$$

subject to initial conditions:

$$y_0 = \alpha \quad y_1 = \beta$$

• Working procedure:

i] Input difference equation coefficients and RHS of equation

ii] Input initial conditions

iii] Apply z-transform, find $Y(z)$

iv] Apply inverse, find y_n

3. $ay_{n+2} + by_{n+1} + cy = 0$

$$y_0 = 1 \quad y_1 = 1$$