16/8/19

Experiment - 3

→ Fourier Series: FS of a periodic function (16/8/19)
$f(x)$ of period $2l$ defined on the interval
$(\alpha, \alpha+2l)$ is given by, sine and cosine terms.

$$f(x) \approx a_0/2 + \sum_{n=1}^{\infty} \left(a_n \cos(n\pi x/l) + b_n \sin(n\pi x/l)\right)$$

- Fourier coefficients:

$$a_0 = \frac{1}{l} \int_{\alpha}^{\alpha+2l} f(x)\, dx$$

$$a_n = \frac{1}{l} \int_{\alpha}^{\alpha+2l} f(x) \cos(n\pi x/l)$$

$$b_n = \frac{1}{l} \int_{\alpha}^{\alpha+2l} f(x) \sin(n\pi x/l)$$

MATLAB Syntax:

- syms var1 : Declaring variable 1
- disp (x) : Print statement

- int (expr, var, a, b) : Evaluates integral of 'expr' with respect to var from 'a' to 'b'
- ezplot (func, [xmin, xmax]) : Plots 'func' over domain (xmin, xmax).

- No. of repetitions of the loop = No. of harmonics required

- strcat : String concatenation
- str2num : String to integer

- For a fourier series, $(a_1 \cos x + b_1 \sin x)$ is the first/fundamental harmonic and $(a_2 \cos 2x + b_2 \sin 2x)$ is the secondary harmonic and so on...

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1. Find the fourier series expansion of $f(x) = x - x^2$ from $-\pi < x < \pi$, with 3 harmonics

ans → Screenshot 1 : Code
→ Screenshot 2 : I/O
→ Screenshot 3 : Harmonic 1
→ Screenshot 4 : Harmonic 2
→ Screenshot 5 : Harmonic 3

16/08

Experiment – 3: Fourier Series

Editor - C:\Users\18BIS0043\Documents\MATLAB\E3_1.m

E3_1.m

```
clear all; clc
syms x
f = input('f(x): ');
a = input('Lower limit "a": ');
b = input('Upper limit "b": ');
m = input('No. of Harmonics: ');
L = (b-a)/2;
a0 = (1/L)*int(f,a,b);
Fx = a0/2;
for n=1:m
    figure;
    an(n) = (1/L)*int(f*cos(n*pi*x/L),a,b);
    bn(n) = (1/L)*int(f*sin(n*pi*x/L),a,b);
    Fx1 = Fx + an(n)*cos(n*pi*x/L) + bn(n)*sin(n*pi*x/L);
    Fx = vpa(Fx1,4);
    ezplot(Fx,[a,b]);
    hold on
    ezplot(f,[a,b]);
    grid on;
    title(['Fourier series with ',num2str(n),' harmonics']);
    legend('Fourier series','Function plot');
    hold off
end
disp(['Fourier series with ',num2str(n),' harmonic is: ']);
disp(char(Fx));
```

Command Window

```
f(x): x-x^2
Lower limit "a": -pi
Upper limit "b": pi
No. of Harmonics: 3
Fourier series with 3 harmonic is:
0.4444*cos(3.0*x) - 1.0*sin(2.0*x) - 1.0*cos(2.0*x) + 0.6667*sin(3.0*x) + 4.0*cos(x) + 2.0*sin(x) - 3.29
>>
```

Harmonic 1:

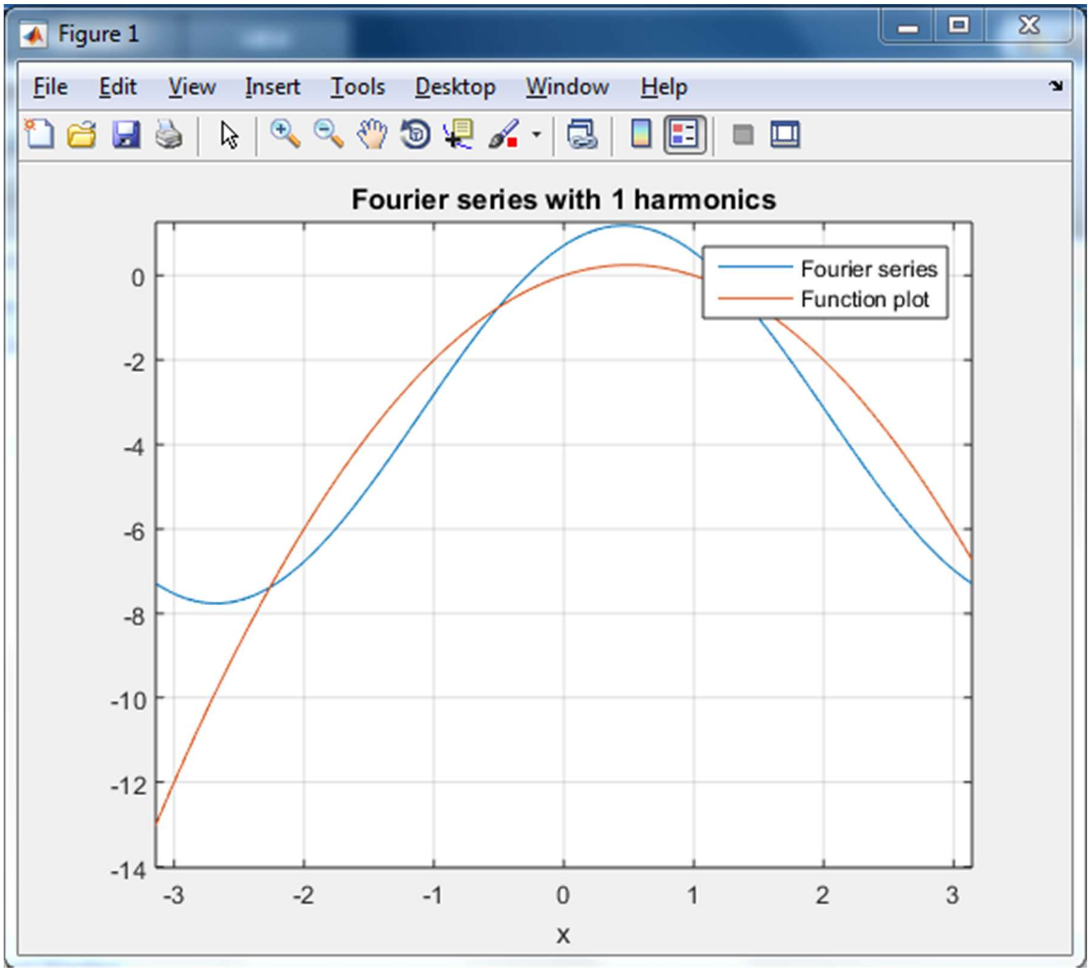

Figure 1
File
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Fourier series with 1 harmonics
Fourier series
Function plot
0
-2
-4
-6
-8
-10
-12
-14
-3
-2
-1
0
1
2
3
x

Harmonic 2:

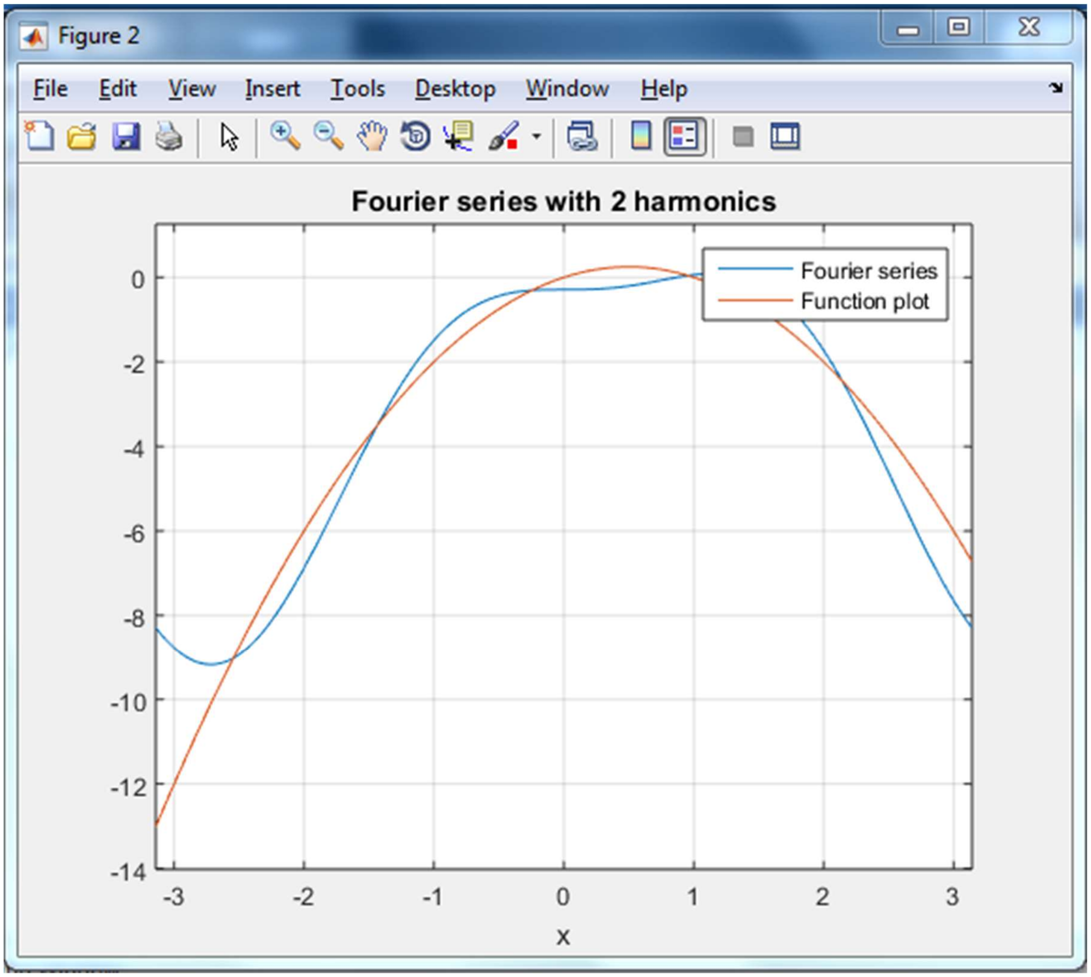

Figure 2
File Edit View Insert Tools Desktop Window Help
Fourier series with 2 harmonics
Fourier series
Function plot
0
-2
-4
-6
-8
-10
-12
-14
-3
-2
-1
0
1
2
3
x

Harmonic 3:

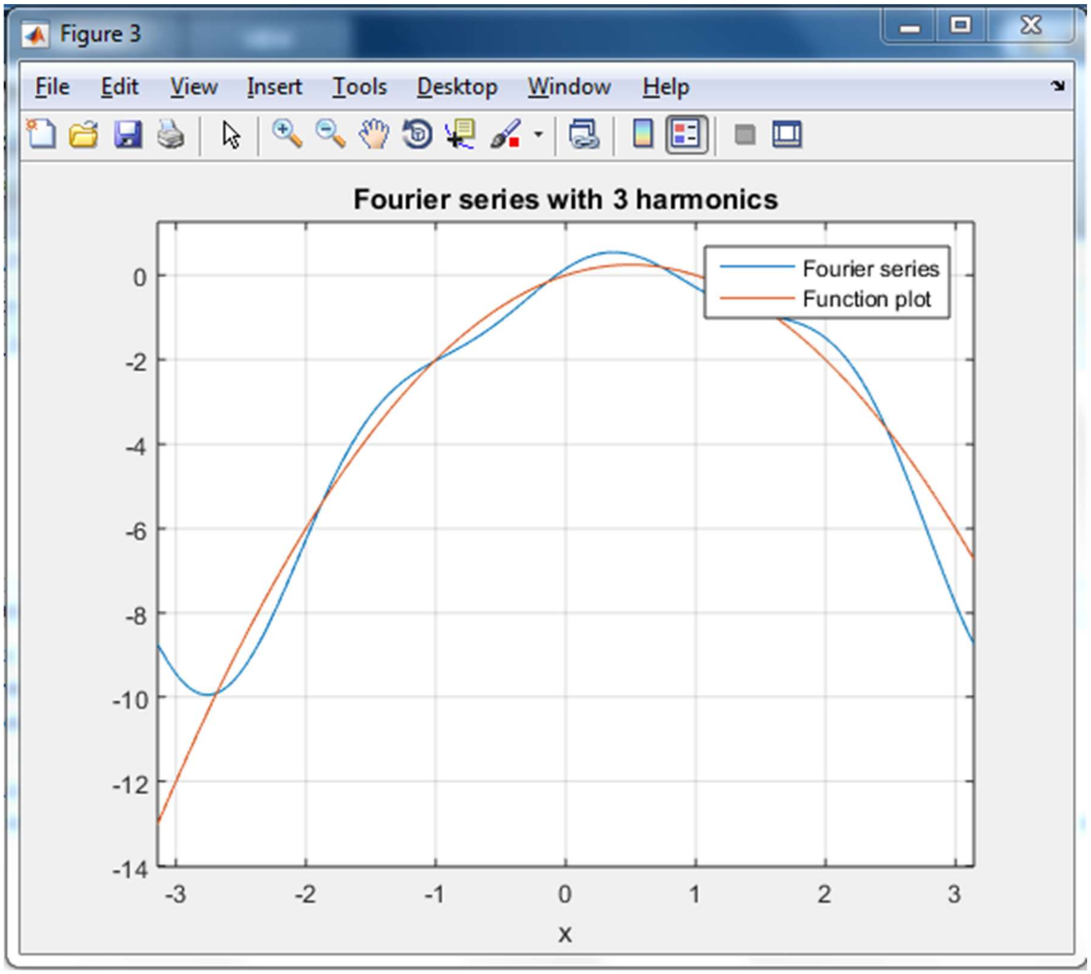
Figure 3
File Edit View Insert Tools Desktop Window Help
Fourier series with 3 harmonics
Fourier series
Function plot
0
-2
-4
-6
-8
-10
-12
-14
-3
-2
-1
0
1
2
3
x

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30/8/19 Experiment - 4

→ Harmonic analysis (30/8/19)

- Whenever the table values are given, fourier series is obtained using harmonic analysis as shown

$$f(x) = a_0/2 + a_1 \cos\theta_1 + b_1 \sin\theta_1 + a_2 \cos 2\theta_1 + b_2 \cos 2\theta_1 \ldots$$

$$\theta = \pi x / l$$

Fourier coefficients,

$$a_0 = 2 \times 1/2l \int_{\alpha}^{\alpha+2l} f(x)\,dx = 2 \times \text{Mean of } f(x) \text{ in } (\alpha, \alpha+2l)$$

$$a_n = 2 \times 1/2l \int_{\alpha}^{\alpha+2l} f(x) \cos(n\pi x/l)\,dx$$
$$= 2 \times \text{Mean of } f(x) \cos(n\pi x/l) \text{ in } (\alpha, \alpha+2l)$$

$$b_n = 2 \times 1/2l \int_{\alpha}^{\alpha+2l} f(x) \sin(n\pi x/l)\,dx$$
$$= 2 \times \text{Mean of } f(x) \sin(n\pi x/l) \text{ in } (\alpha, \alpha+2l)$$

- Given m data points, between 2π or $2l$

$$a_0 = \frac{2 \sum f(x_i)}{m} \qquad a_n = \frac{2 \sum_{i=1}^{m} f(x_i) \cos n\theta_i}{m}$$

$$\theta_i = \pi x_i / l, \quad n = 1, 2, \ldots \qquad b_n = \frac{2 \sum_{i=1}^{m} f(x_i) \sin n\theta_i}{m}$$

- Vpa → variable precision arithmetic

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1. Compute the first 4 harmonics of the fourier series given by,

x	0	1	2	3	4	5
y	4	8	15	7	6	2

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ans → screenshot 1-7

Experiment – 4: Harmonic Analysis

Editor - C:\Users\18BIS0043\Documents\MATLAB\T1.m

T1.m

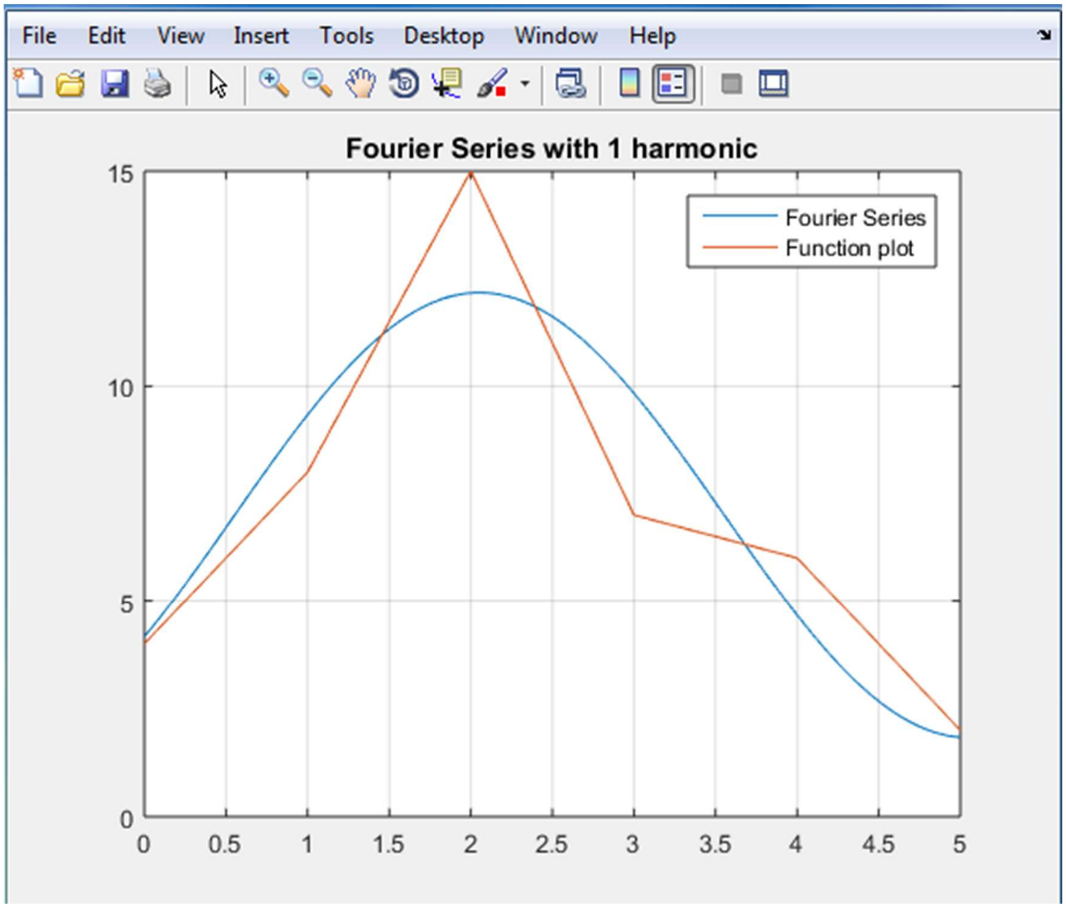
```
clc
clear all
syms t
x = input('Enter the equally spaced values of x: ');
y = input('Enter the values of y = f(x): ');
m = input('Enter the number of harmonics required: ');
n = length(x);
a = x(1);
b = x(n);
h = x(2)-x(1);
l = (b-a+h)/2;
theta = pi*x/l;
a0 = (2/n)*sum(y);
Fx = a0/2;
x1 = linspace(a,b,100);
```

```
for i=1:m
    figure
    an = (2/n)*sum(y.*cos(i*theta));
    bn = (2/n)*sum(y.*sin(i*theta));
    Fx = Fx + an*cos(i*pi*t/l) + bn*sin(i*pi*t/l) ;
    Fx = vpa(Fx,4);
    Fx1 = subs(Fx,t,x1);
    plot(x1,Fx1);
    hold on
    plot(x,y);
    title(['Fourier Series with ',num2str(i),' harmonic'])
    legend('Fourier Series','Function plot');
    grid on;
    hold off;
end
disp(strcat('Fourier Series with',num2str(i),'harmonics is: ',char(Fx)));
```

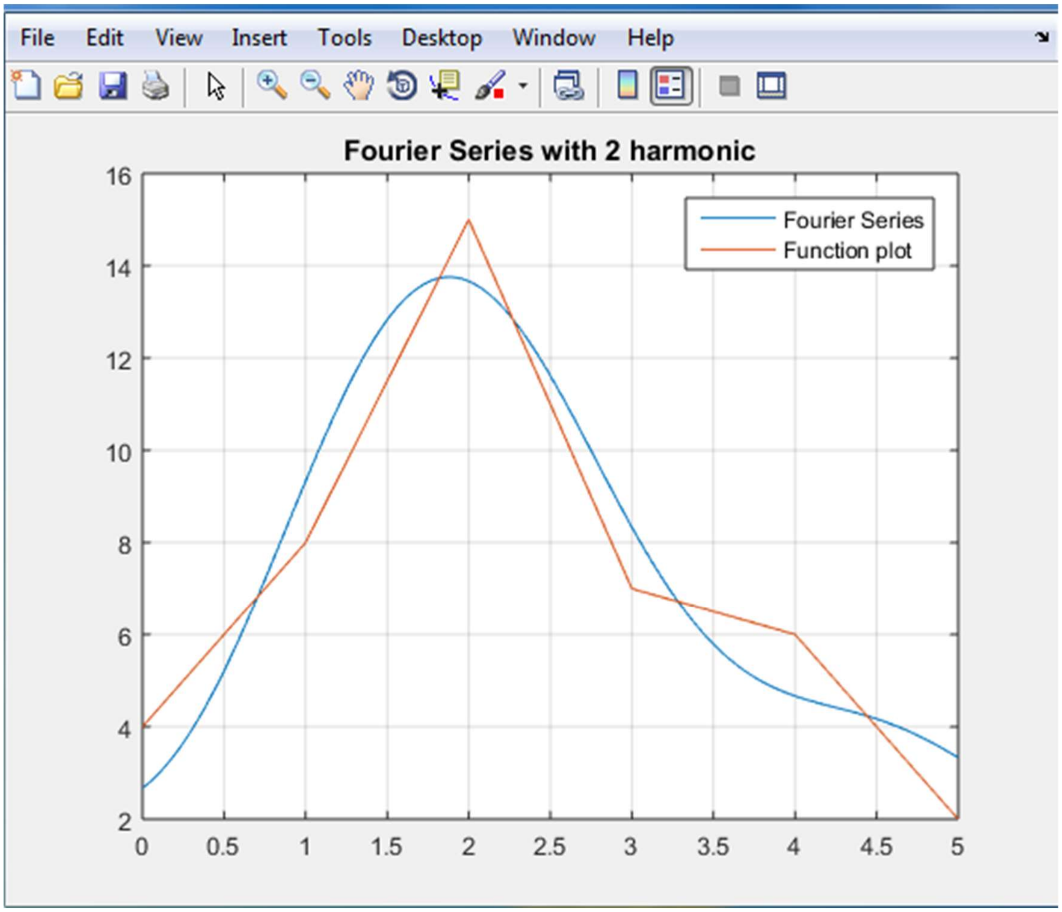
Command Window

```
Enter the equally spaced values of x: [0:5]
Enter the values of y = f(x): [4 8 15 7 6 2]
Enter the number of harmonics required: 4
```

Harmonic 1:

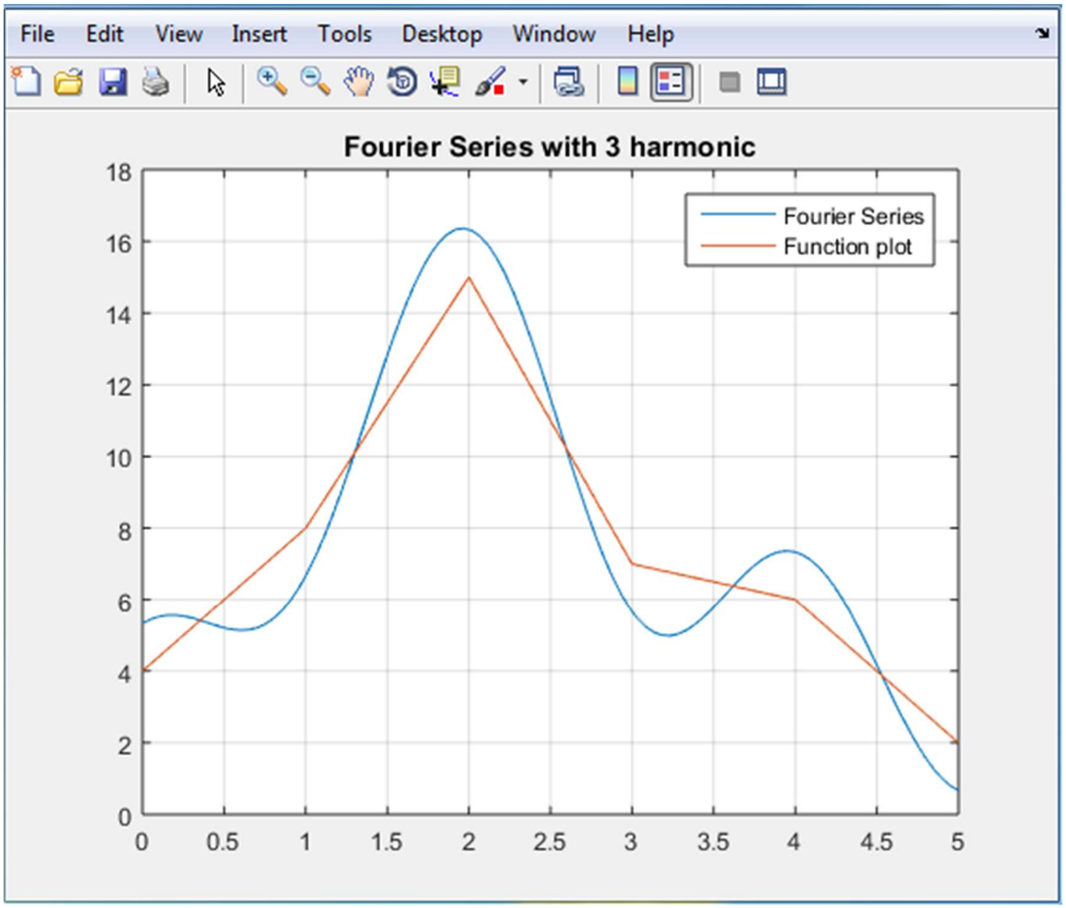

File Edit View Insert Tools Desktop Window Help
Fourier Series with 1 harmonic
Fourier Series
Function plot
15
10
5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Harmonic 2:



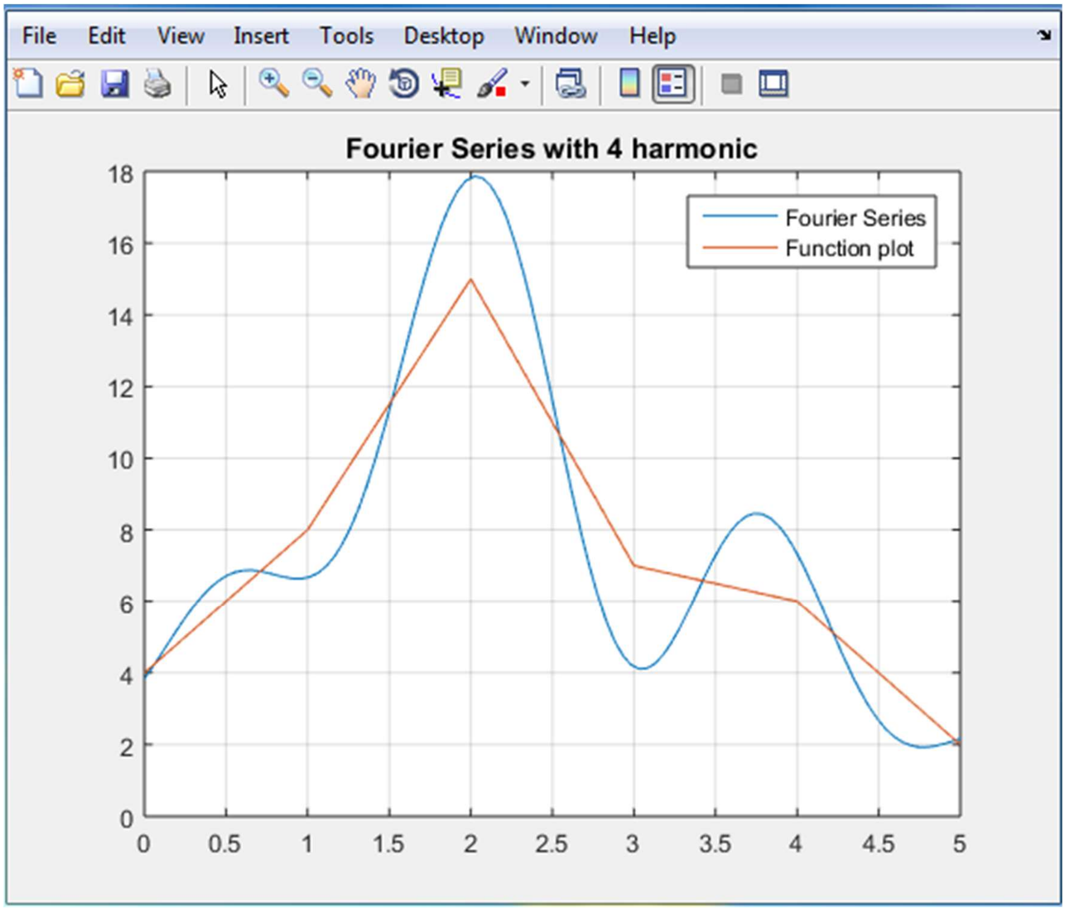
File Edit View Insert Tools Desktop Window Help
Fourier Series with 2 harmonic
Fourier Series
Function plot
16
14
12
10
8
6
4
2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Harmonic 3:



File
Edit
View
Insert
Tools
Desktop
Window
Help
Fourier Series with 3 harmonic
Fourier Series
Function plot
0
2
4
6
8
10
12
14
16
18
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Harmonic 4:


File Edit View Insert Tools Desktop Window Help
Fourier Series with 4 harmonic
Fourier Series
Function plot
18
16
14
12
10
8
6
4
2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5