Kotlin∇: A Shape-safe DSL for Differentiable Programming

Breandan Considine, Michalis Famelis, Liam Paull

Main Idea

- We create an embedded DSL for differentiable programming in Kotlin
- Supports shape checking and inference for multi-dimensional arrays
- Implementable in any language with first-class functions and generics

Shape errors

There are three broad strategies for handling array shape errors:

- Perform type coercion by implicitly broadcasting or reshaping arrays
- Raise a runtime error (e.g. `tf.errors.InvalidArgumentError`)
- Do not allow programs which can result in a shape error to compile

In Kotlin∇, we prefer the last strategy. Consider the following scenario:

```
a = np.array('0 1 2 3 4 5')
b = np.array([6, 7, 8])
c = a + b
```

```
val a = Vector(0, 1, 2, 3, 4, 5)
val b = Vector4(arrayOf(6, 7, 8))
val c = a + b
```

Similarly, when the inner dimensions of two matrices do not match:

```
d = np.matrix('0 1 2; 3 4 5')
e = np.matrix('6 7; 8 9')
f = d @ e
```

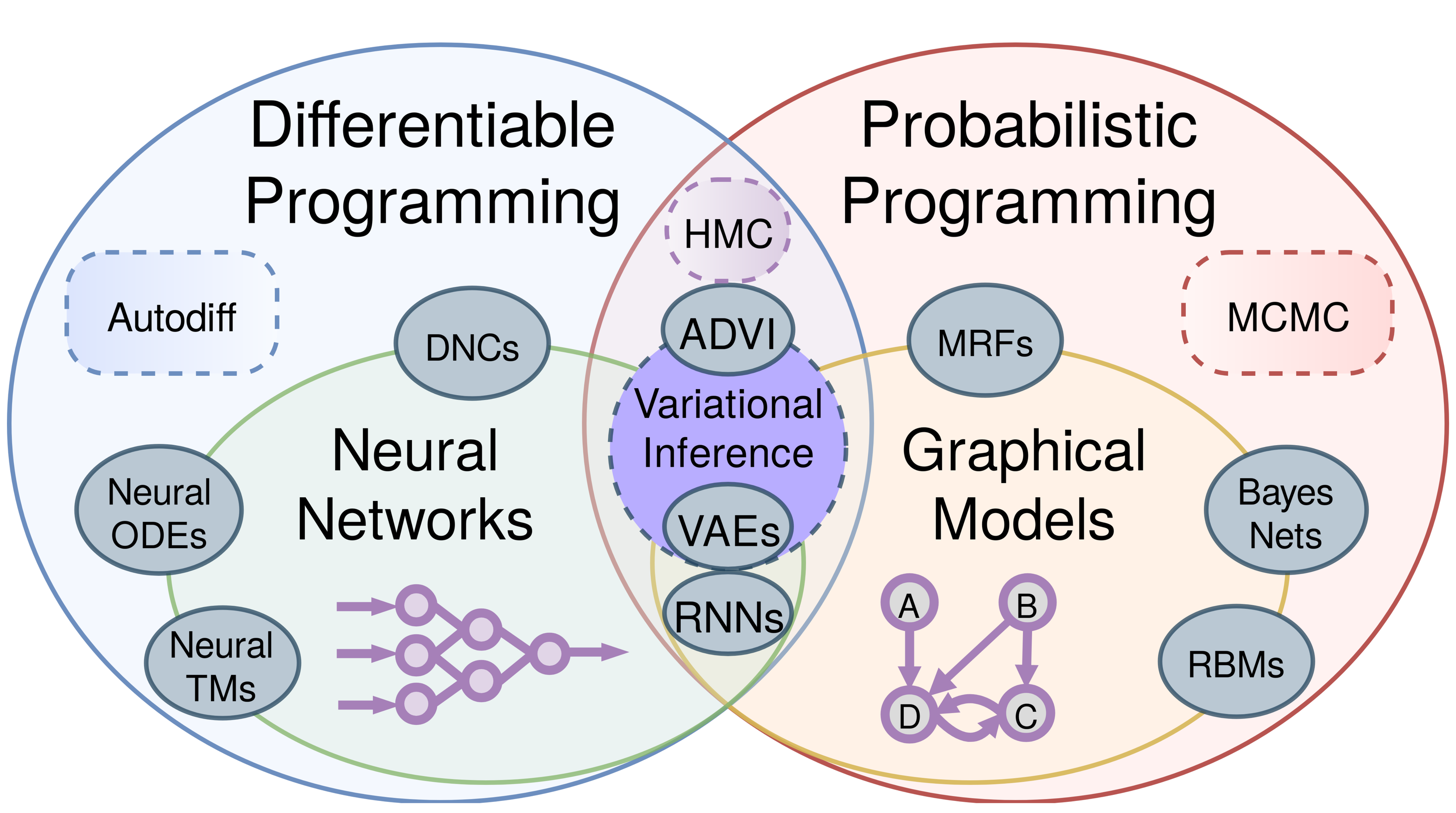
```
val d = Matrix2x3(0, 1, 2, 3, 4, 5)
val e = Matrix2x2(6, 7, 8, 9)
val f = d * e
```

We can detect the presence and location of the error within the editor.

Type system

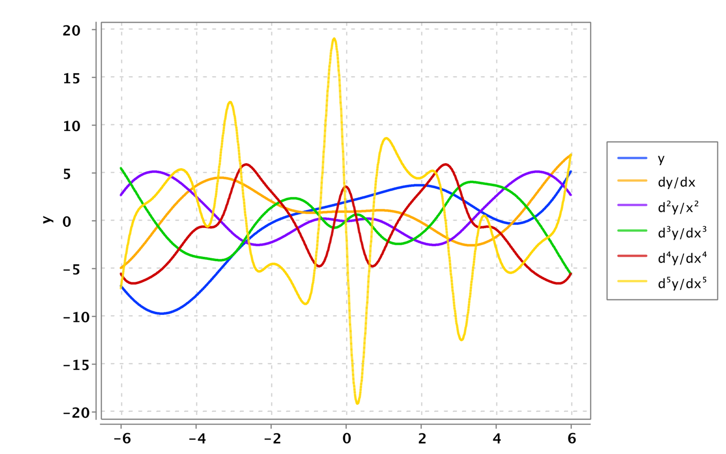
Math	Infix	Prefix	Postfix	Operator Type Signature
$\mathbf{A}(\mathbf{B})$ $\mathbf{A} \circ \mathbf{B}$	`a(b)`			$(\mathtt{a}\colon \mathbb{R}^\tau \to \mathbb{R}^\pi,\ \mathtt{b}\colon \mathbb{R}^\lambda \to \mathbb{R}^\tau) \to (\mathbb{R}^\lambda \to \mathbb{R}^\pi)$
$\mathbf{A} \pm \mathbf{B}$	`a + b` `a - b`	`plus(a, b)` `minus(a, b)`		$(\mathtt{a}\colon \mathbb{R}^\tau \to \mathbb{R}^\pi,\ \mathtt{b}\colon \mathbb{R}^\lambda \to \mathbb{R}^\pi) \to (\mathbb{R}^? \to \mathbb{R}^\pi)$
$\mathbf{AB}$	`a * b` `a.times(b)`	`times(a, b)`		$(\mathtt{a}\colon \mathbb{R}^\tau \to \mathbb{R}^{m\times n},\ \mathtt{b}\colon \mathbb{R}^\lambda \to \mathbb{R}^{n\times p}) \to (\mathbb{R}^? \to \mathbb{R}^{m\times p})$
$\frac{\mathbf{A}}{\mathbf{B}}$ $\mathbf{AB}^{-1}$	`a / b` `a.div(b)`	`div(a, b)`		$(\mathtt{a}\colon \mathbb{R}^\tau \to \mathbb{R}^{m\times n},\ \mathtt{b}\colon \mathbb{R}^\lambda \to \mathbb{R}^{p\times n}) \to (\mathbb{R}^? \to \mathbb{R}^{m\times p})$
$\log_b \mathbf{A}$	`a.log(b)`	`log(a, b)`		$(\mathtt{a}\colon \mathbb{R}^\tau \to \mathbb{R}^{m\times m},\ \mathtt{b}\colon \mathbb{R}^\lambda \to \mathbb{R}^{m\times m}) \to (\mathbb{R}^? \to \mathbb{R})$
$\mathbf{A}^b$	`a.pow(b)`	`pow(a, b)`		$(\mathtt{a}\colon \mathbb{R}^\tau \to \mathbb{R}^{m\times m},\ \mathtt{b}\colon \mathbb{R}^\lambda \to \mathbb{R}) \to (\mathbb{R}^? \to \mathbb{R}^{m\times m})$
$\frac{da}{db}, \frac{\partial a}{\partial b}$ $D_b a$	`a.d(b)` `d(a)/d(b)`	`grad(a)[b]`		$\big(\mathtt{a}\colon C(\mathbb{R}^\tau \to \mathbb{R}),\ \mathtt{b}\colon C(\mathbb{R}^\lambda \to \mathbb{R})\big) \to (\mathbb{R}^? \to \mathbb{R})$
∇a		`grad(a)`	`a.grad()`	$\big(\mathtt{a}\colon C(\mathbb{R}^\tau \to \mathbb{R})\big) \to (\mathbb{R}^\tau \to \mathbb{R}^\tau)$
$\nabla_{\mathbf{B}} a$	`a.d(b)` `a.grad(b)`	`grad(a, b)`		$\big(\mathtt{a}\colon C(\mathbb{R}^\tau \to \mathbb{R}),\ \mathtt{b}\colon C(\mathbb{R}^\lambda \to \mathbb{R}^n)\big) \to (\mathbb{R}^? \to \mathbb{R}^n)$
$\nabla \cdot \mathbf{A}$		`divg(a)`	`a.divg()`	$\big(\mathtt{a}\colon C(\mathbb{R}^\tau \to \mathbb{R}^m)\big) \to (\mathbb{R}^\tau \to \mathbb{R})$
$\nabla \times \mathbf{A}$		`curl(a)`	`a.curl()`	$\big(\mathtt{a}\colon C(\mathbb{R}^3 \to \mathbb{R}^3)\big) \to (\mathbb{R}^3 \to \mathbb{R}^3)$

Differentiable Programming

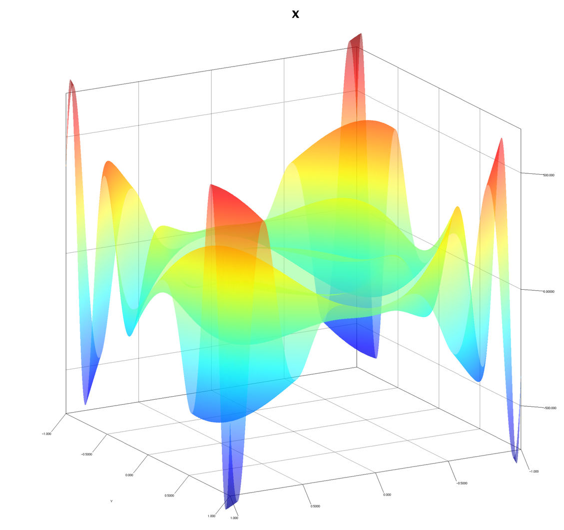


Visualization

```
val y = sin(sin(sin(x))) / x
val z = y + sin(x) * x + cos(x) + x
val d1 = d(z) / d(x)
val d2 = d(d1) / d(x)
val d3 = d(d2) / d(x)
val d4 = d(d3) / d(x)
val d5 = d(d4) / d(x)
plot2D(-6..6, z, d1, d2, d3, d4, d5)
```



```
val Z = x * x + pow(y, 2)
val Z10 = Z * 10
val sinZ = sin(Z10)
val sinZ_10 = sinZ / 10
val dZ_dx = d(sinZ_10) / d(x)
val d2Z_dxdy = d(dZ_dx) / d(y)
val d3Z_d2xdy = d(d2Z_dxdy) / d(x)
plot3D(-1..1, d3Z_d2xdy)
```



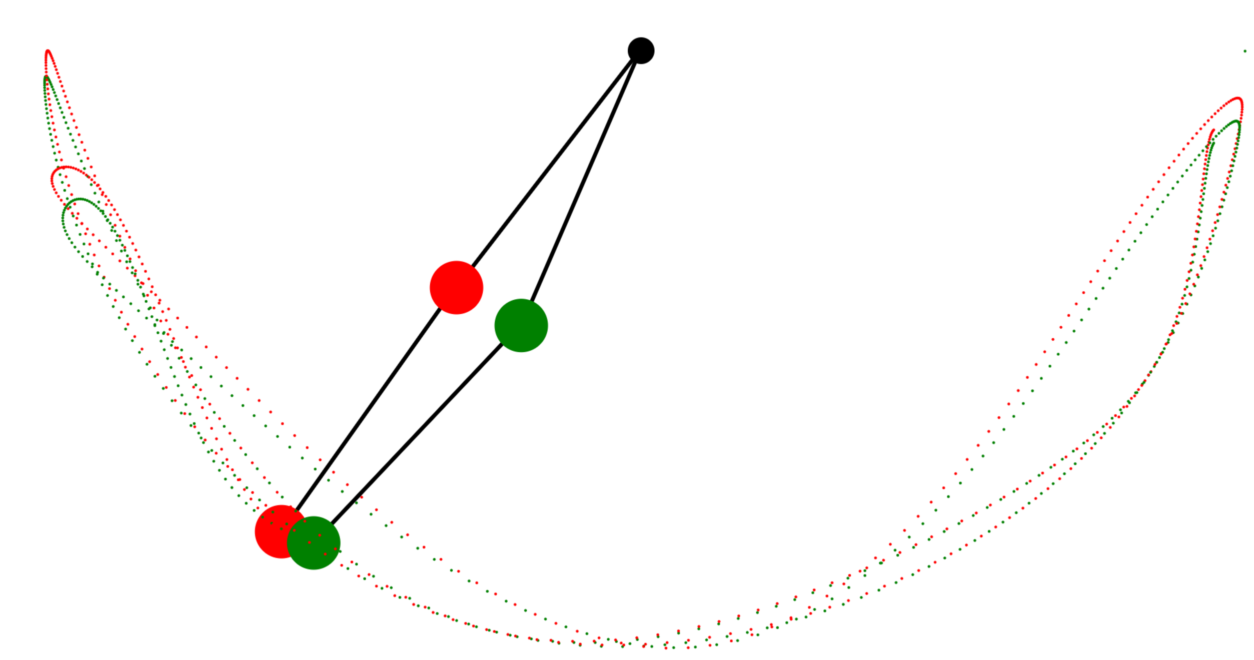
Physical Simulation

$$L = \tfrac{1}{6}ml^2\left({\omega_2}^2 + 4{\omega_1}^2 + 3\omega_1\omega_2\cos(\theta_1 - \theta_2)\right) + \tfrac{1}{2}mgl\,(3\cos\theta_1 + \cos\theta_2)$$

$$\omega_1 = \frac{6}{ml^2}\frac{2p_{\theta_1} - 3\cos(\theta_1 - \theta_2)p_{\theta_2}}{16 - 9\cos^2(\theta_1 - \theta_2)}$$

$$\omega_2 = \frac{6}{ml^2}\frac{8p_{\theta_2} - 3\cos(\theta_1 - \theta_2)p_{\theta_1}}{16 - 9\cos^2(\theta_1 - \theta_2)}$$

$$p_{\theta_1} = \frac{\partial L}{\partial \omega_1}, p_{\theta_2} = \frac{\partial L}{\partial \omega_2}, \dot{p}_{\theta_1} = \frac{\partial L}{\partial \theta_1}, \dot{p}_{\theta_2} = \frac{\partial L}{\partial \theta_2}$$



Numerical Precision

Log errors evaluating $f(x) = \frac{\sin\sin\sin x}{x} + x\sin x + \cos x + x$