

identification of seakeeping models from freq-response data with structure and parameter constraints

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In 2007, we started a new
programme:

motivation for this seminar

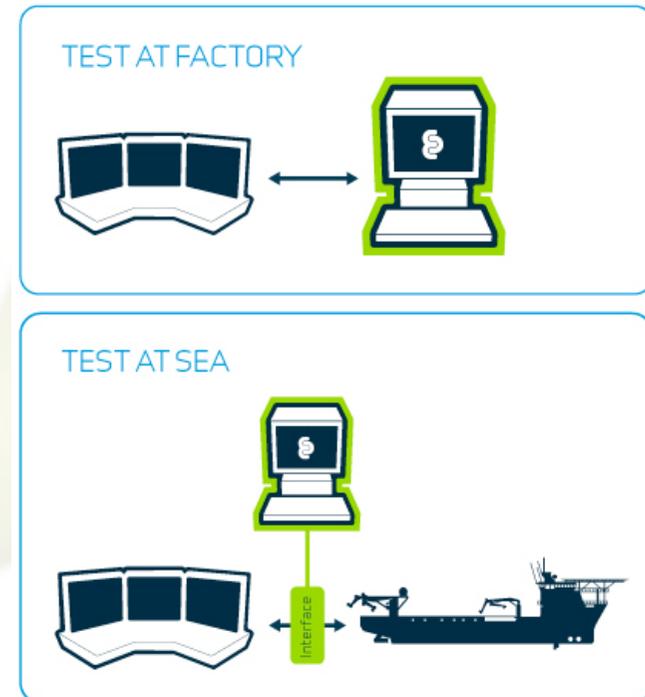
Linear time-domain models of marine structures are the basis of

- Training simulators.
- Hardware-in-the-loop (HIL) simulators.
- Motion control system design.
- Model-based fault detection and diagnosis.

Improvements in accuracy and speed can be achieved by appropriate model representations.



HIL Simulator



motivation for this seminar

One simple way to obtain such models is to use potential theory codes to compute **non-parametric models** hydrodynamic (coefficients and frequency responses) and then

- make a direct implementation of the Cummins Equation, or
- use system identification to approximate the Cummins equation by a Linear-time-invariant **parametric models**.

Approximations by LTI models in terms of state-space equations result in much faster simulations.

Depending on the complexity of the model simulations can be up to 80 times faster (Taghipour, Perez, and Moan, 2008).

outline for this presentation

- linear dynamic models of marine structures
 - equations of motion
 - Cummins equation
 - non-parametric models
 - model properties derived from hydrodynamics

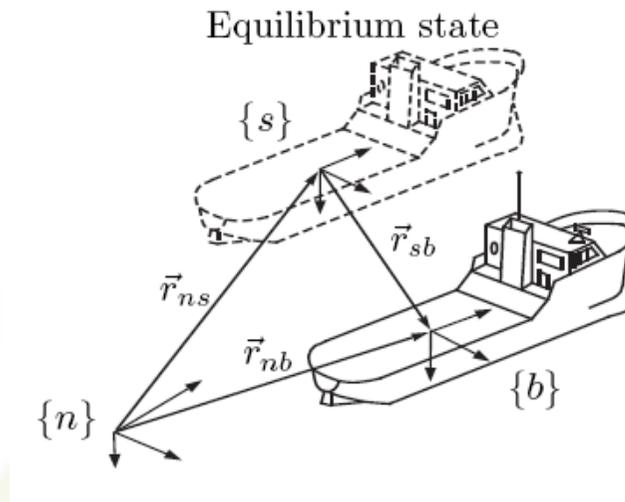
- parametric approximations
 - consequence of the properties
 - time-domain Identification
 - frequency-domain identification
 - dealing with 2D hydrodynamic data

- examples: container, semi-submersible, fpso
- Discussion

equations of motion

Equations of Motion (rigid body):

$$\dot{\xi} = \mathbf{J}(\xi)\nu$$
$$\mathbf{M}_{RB}\dot{\nu} + \mathbf{C}_{RB}(\nu)\nu = \tau$$



$$\xi \triangleq [x, y, z, \phi, \theta, \psi]^T$$

← generalised displ. wrt equilibrium

$$\nu \triangleq [u, v, w, p, q, r]^T$$

← body-fixed generalised velocities

$$\tau \triangleq [X, Y, Z, K, M, N]^T$$

← body-fixed generalised forces

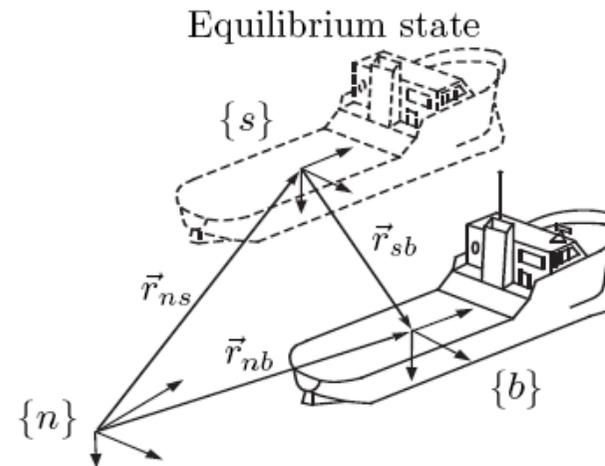
linear equations

If we consider small deviations about the equilibrium:

$$\dot{\xi} = \mathbf{J}(\xi)\nu$$
$$\mathbf{M}_{RB}\dot{\nu} + \mathbf{C}_{RB}(\nu)\nu = \tau$$



$$\mathbf{M}_{RB}\ddot{\xi} = \tau$$



Superposition of forces (radiation, restoring excitation):

$$\tau = \tau_{rad} + \tau_{res} + \tau_{exc}$$

Cummins's equation

Cummins studied the radiation potential problem in the time-domain, within the linear assumption and found that:

$$\tau_{rad} = -\mathbf{A}_{\infty} \ddot{\boldsymbol{\xi}} - \int_0^t \mathbf{K}(t - t') \dot{\boldsymbol{\xi}}(t') dt'$$

Combining terms,

$$(\mathbf{M}_{RB} + \mathbf{A}_{\infty}) \ddot{\boldsymbol{\xi}} + \int_0^t \mathbf{K}(t - t') \dot{\boldsymbol{\xi}}(t') dt' + \mathbf{G} \boldsymbol{\xi} = \tau_{exc}$$

This equation form the basis of most time-domain simulators.

Ogilvie's relations

When considered in the frequency-domain, the radiation forces and the response of the marine structure can be expressed as:

$$\tau_{rad}(j\omega) = -\mathbf{A}(\omega)\ddot{\boldsymbol{\xi}}(j\omega) - \mathbf{B}(\omega)\dot{\boldsymbol{\xi}}(j\omega)$$

$$[-\omega^2[\mathbf{M} + \mathbf{A}(\omega)] + j\omega\mathbf{B}(\omega) + \mathbf{G}]\boldsymbol{\xi}(j\omega) = \tau_{exc}(j\omega)$$

Ogilvie found the relation between the Cummins parameters and the above:

$$\mathbf{A}(\omega) = \mathbf{A}_{\infty} - \frac{1}{\omega} \int_0^{\infty} \mathbf{K}(t) \sin(\omega t) dt.$$

$$\mathbf{B}(\omega) = \int_0^{\infty} \mathbf{K}(t) \cos(\omega t) dt.$$

$$\mathbf{A}_{\infty} = \lim_{\omega \rightarrow \infty} \mathbf{A}(\omega)$$

non-parametric models

Time-domain:

$$\mathbf{K}(t) = \frac{2}{\pi} \int_0^{\infty} \mathbf{B}(\omega) \cos(\omega t) d\omega.$$

Frequency-domain:

$$\mathbf{K}(j\omega) = \int_0^{\infty} \mathbf{K}(t) e^{-j\omega t} dt = \mathbf{B}(\omega) + j\omega[\mathbf{A}(\omega) - \mathbf{A}_{\infty}]$$

These relationships are key since they are the starting point for the identification process by which parametric models are obtained.

convolution replacement

The convolution term is inconvenient to implement simulation tools, and also to analyse and design motion control systems.

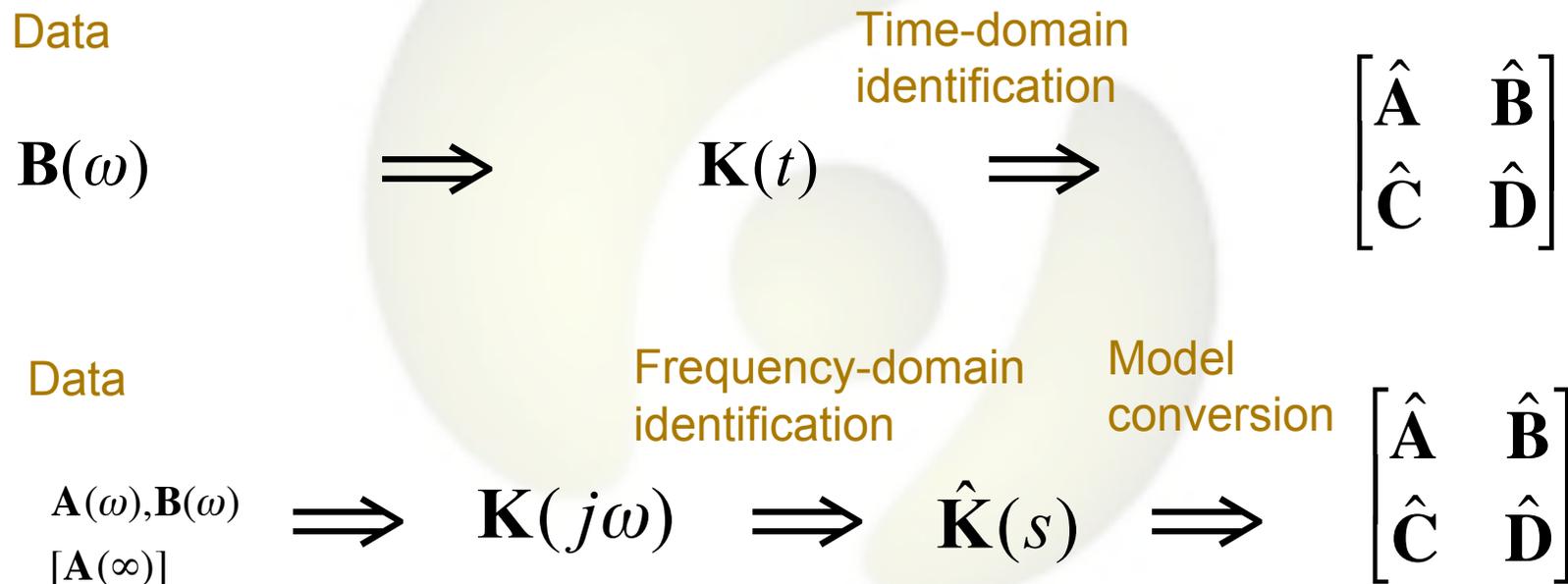
It is more convenient to seek a replacement by a state-space model:

$$\mu = \int_0^t \mathbf{K}(t - t') \dot{\xi}(t') dt' \approx \begin{aligned} \dot{\mathbf{x}} &= \hat{\mathbf{A}} \mathbf{x} + \hat{\mathbf{B}} \dot{\xi} \\ \hat{\mu} &= \hat{\mathbf{C}} \mathbf{x} + \hat{\mathbf{D}} \dot{\xi} \end{aligned}$$

- Different methods have been reported in the literature over the past 20 years.
- Due to Markovian properties of the SS-model significant gains in simulation speed can be obtained.

convolution replacement

The convolution replacement can be posed in different ways:



In practice one method can be more favourable than the other.

identification

Different proposals have appeared in the literature:

Time-domain identification:

- LS-fitting of the impulse response (Yu & Falnes, 1998)
- Realization theory (Kristiansen & Egeland, 2003)

Frequency-domain identification:

- LS-fitting of the frequency response (Jeffreys, 1984), (Damaren 2000).
- LS-fitting of added mass and damping (Soding 1982), (Xia et. al 1998), (Sutulo & Guedes-Soares 2006).

properties of retardation functions

The following properties derive from the hydrodynamics, and have implications on the parametric models:

$$\hat{K}_{ik}(s) = \frac{P_{ik}(s)}{Q_{ik}(s)} = \frac{p_r s^r + p_{r-1} s^{r-1} + \dots + p_0}{s^n + q_{n-1} s^{n-1} + \dots + q_0}$$

Property	Implication on Parametric Models $K_{ik}(s) = P(s)/Q(s)$
1) $\lim_{\omega \rightarrow 0} \mathbf{K}(j\omega) = \mathbf{0}$	There are zeros at $s = 0$.
2) $\lim_{\omega \rightarrow \infty} \mathbf{K}(j\omega) = \mathbf{0}$	Strictly proper.
3) $\lim_{t \rightarrow 0^+} \mathbf{K}(t) \neq \mathbf{0}$	Relative degree 1.
4) $\lim_{t \rightarrow \infty} \mathbf{K}(t) = \mathbf{0}$	BIBO stable.
5) The mapping $\dot{\xi} \mapsto \mu$ is Passive	$\mathbf{K}(j\omega)$ is positive real (diagonal entries $K_{ii}(j\omega)$ positive real).

This prior knowledge, and should be used in the identification process to refine the search for approximating models.

time-domain methods

identification from the impulse-response

time-domain methods

- Impulse response curve fitting (Yu & Falnes, 1995, 1998)
- Realization theory (Kristiansen & Egeland, 2003)

Both these methods use the frequency domain data to compute the retardation functions in the time domain and then perform the system identification.

$$\mathbf{K}(t) = \frac{2}{\pi} \int_0^{\infty} \mathbf{B}(\omega) \cos(\omega t) d\omega.$$

distortion of non-param. models

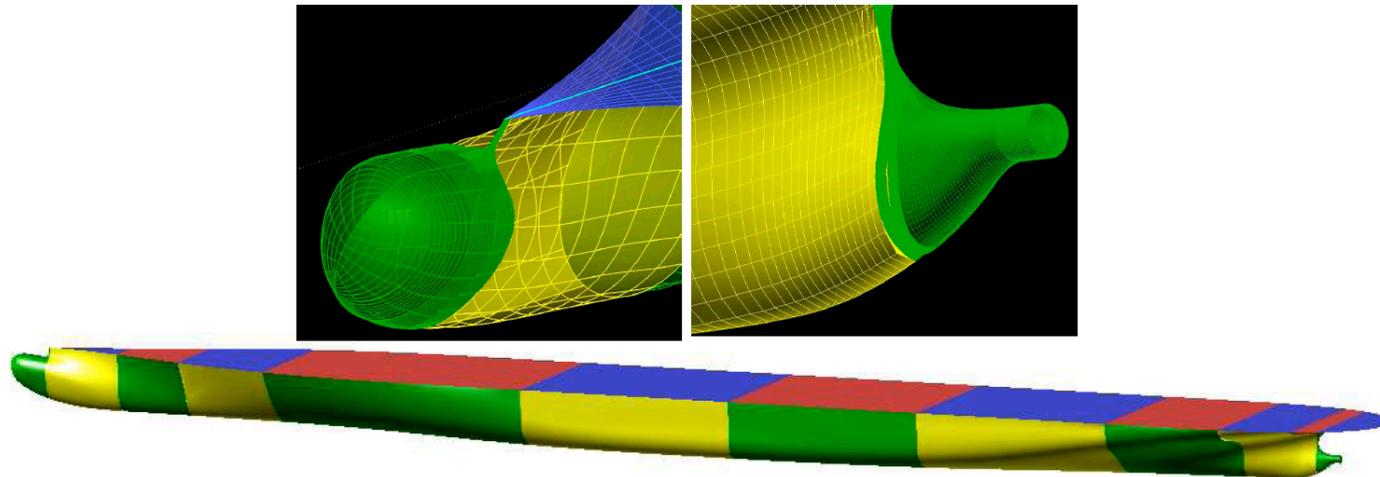
The computation of the retardation function from the damping introduces **distortion**, which can affect the identification:

$$\mathbf{K}(t) \approx \bar{\mathbf{K}}(t) = \frac{2}{\pi} \int_0^{\Omega} \mathbf{B}(\omega) \cos(\omega t) d\omega$$

The maximum frequency is related to the size of the panels. One can use asymptotic tails to extend the computations:

$$\text{as } \omega \rightarrow \infty, \quad B_{ik}(\omega) \rightarrow \frac{\beta_1}{\omega^4} + \frac{\beta_2}{\omega^2}$$

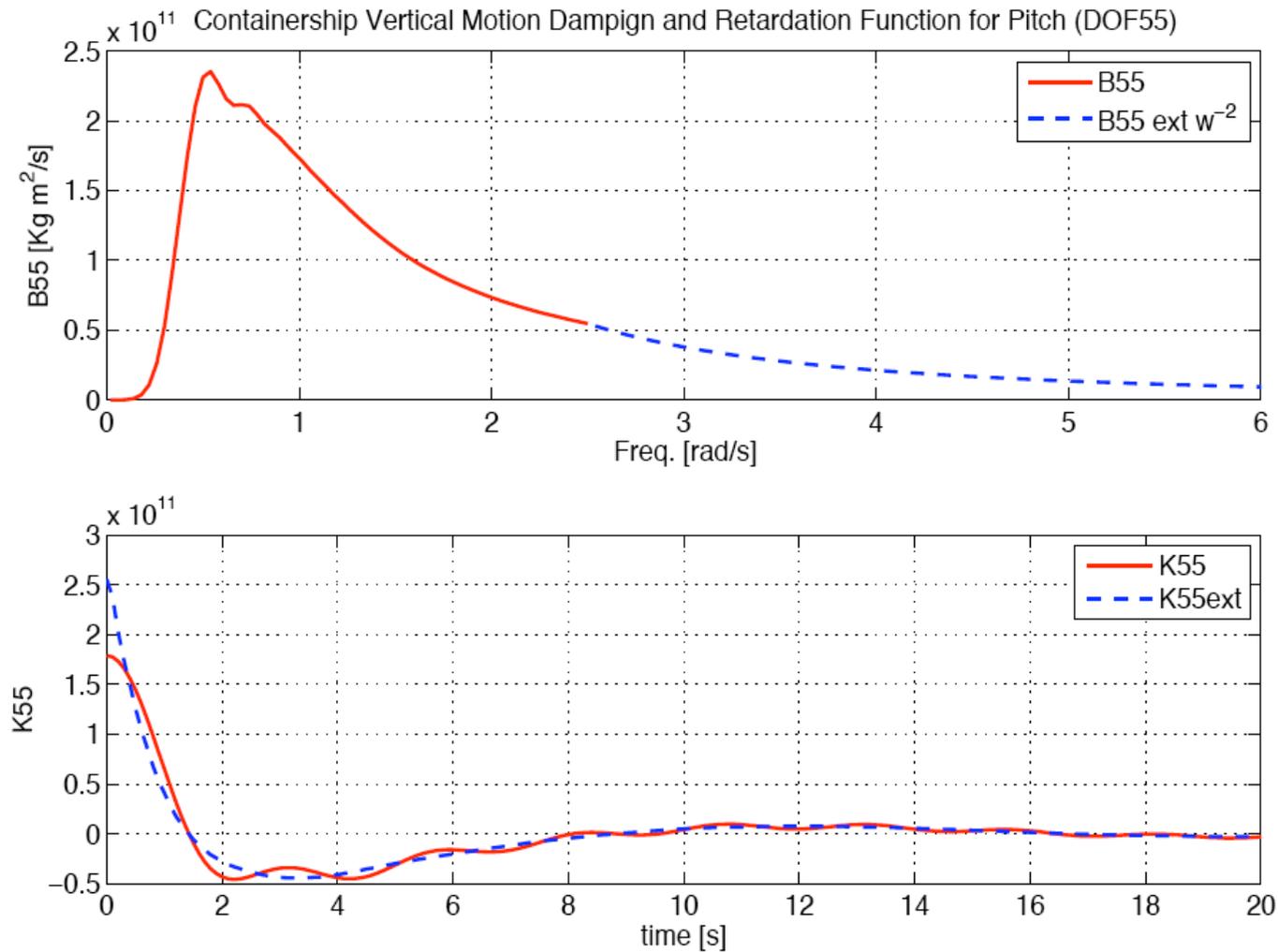
example containership (Taghipour et al., 2007a)



The panel sizing was done to be able to compute frequencies up to 2.5 rad/s.

From experience characteristic panel length $< 1/8$ - $1/10$ min wave length for low order panel methods (Faltinsen, 1993).

distortion of non-param. models



distortion of non-param. models

We can think the limited freq. damping as a product of the real damping with a rectangular window:

$$\bar{\mathbf{K}}(t) = \frac{2}{\pi} \int_0^{\infty} \mathbf{W}(\omega) \mathbf{B}(\omega) \cos(\omega t) d\omega \quad W_{ik}(\omega) = \begin{cases} 1 & \text{if } \omega \leq \Omega, \\ 0 & \text{if } \omega > \Omega. \end{cases}$$

Product in the freq.-domain \Leftrightarrow convolution in the time-domain (FT property):

$$\bar{\mathbf{K}}(t) = \int_0^{\infty} \mathbf{W}(t - t') \mathbf{K}(t) dt' \quad W_{ik}(t) = 2\Omega \frac{\sin(\Omega t)}{\Omega t}$$

The distortion can be expressed as the convolution of the true impulse response and the IFT of an ideal low-pass filter.

This is a disadvantage for time-domain methods.

impulse response curve fitting

$$\theta_{ik}^* = \arg \min_{\theta} \sum_l [\bar{K}_{ik}(t_l) - \hat{K}_{ik}(t_l, \theta)]^2$$

$$\hat{K}_{ik}(t, \theta) = \hat{C}_{ik}(\theta) \exp(\hat{A}_{ik}(\theta)t) \hat{B}_{ik}(\theta) + \hat{D}_{ik}(\theta)$$

- Optimisation problem non-linear in the parameters. Algorithms can be trapped in local minima. A good guess of the initial parameters is crucial for the success of the optimisation.
- The structure of the state-space model adopted plays an important role--there are infinite ways of doing this.
- The order of the model and the initial parameters are not easy to guess from the impulse response.
- Make no use of the prior knowledge.

realization theory

Discrete-time approximation:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{\Phi} \mathbf{x}_k + \mathbf{\Gamma} u_k \\ y_k &= \mathbf{C} \mathbf{x}_k + \mathbf{D} u_k \end{aligned} \quad \Rightarrow \quad K_k = \mathbf{C} \mathbf{\Phi}^{k-1} \mathbf{\Gamma} + \mathbf{D}$$

Steps:

- 1) Form a Hankel matrix with the impulse response samples.
- 2) Do a singular value decomposition (SVD).
- 3) Obtain the order from the number of non-zero singular values.
- 4) Obtain the model matrices via factorisation.
- 5) Convert the model to continuous time.

$$\mathcal{H}_k = \begin{bmatrix} K_1 & K_2 & \dots & K_k \\ K_2 & K_3 & \dots & K_{k+1} \\ \vdots & \vdots & \dots & \vdots \\ K_k & K_{k+1} & \dots & K_{2k-1} \end{bmatrix}$$

$$\mathcal{H}_k = [\mathbf{U}_1 \mathbf{U}_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} [\mathbf{V}_1^* \mathbf{V}_2^*] = \mathbf{U}_1 \Sigma_1 \mathbf{V}_1^*$$

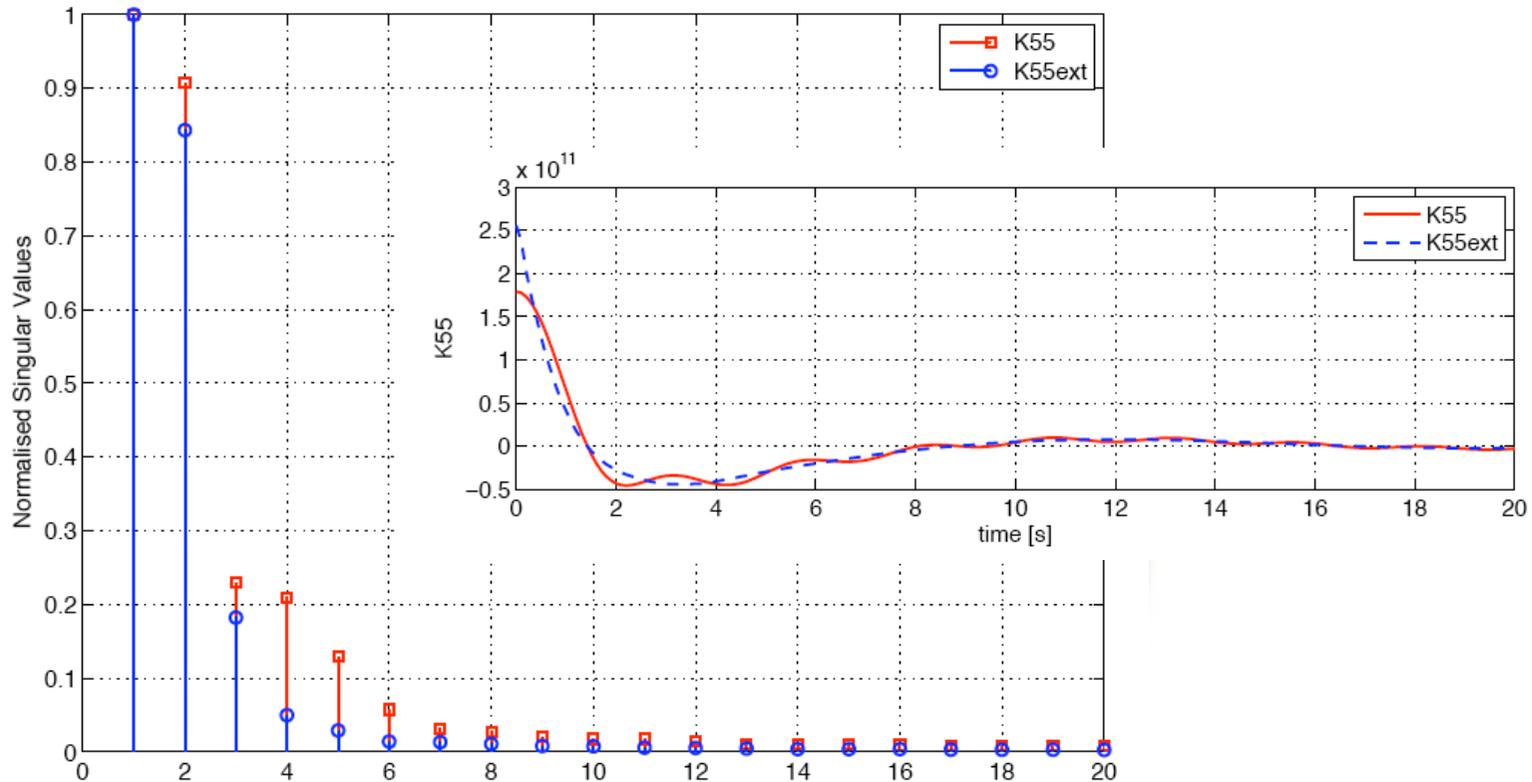
$$\mathbf{\Phi} = \Sigma_1^{-1/2} \begin{bmatrix} \mathbf{U}_{11} \\ \mathbf{U}_{12} \end{bmatrix}^T \begin{bmatrix} \mathbf{U}_{12} \\ \mathbf{U}_{13} \end{bmatrix} \Sigma_1^{1/2}$$

$$\mathbf{\Gamma} = \Sigma_1^{-1/2} \mathbf{V}_{11}^*$$

$$\mathbf{C} = \mathbf{U}_{11} \Sigma_1^{1/2}$$

$$\mathbf{D} = h(0),$$

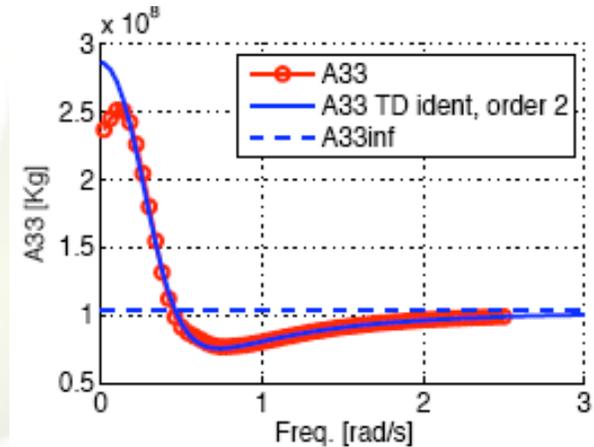
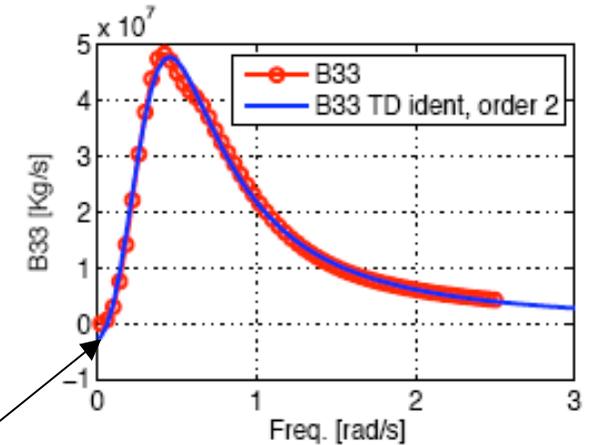
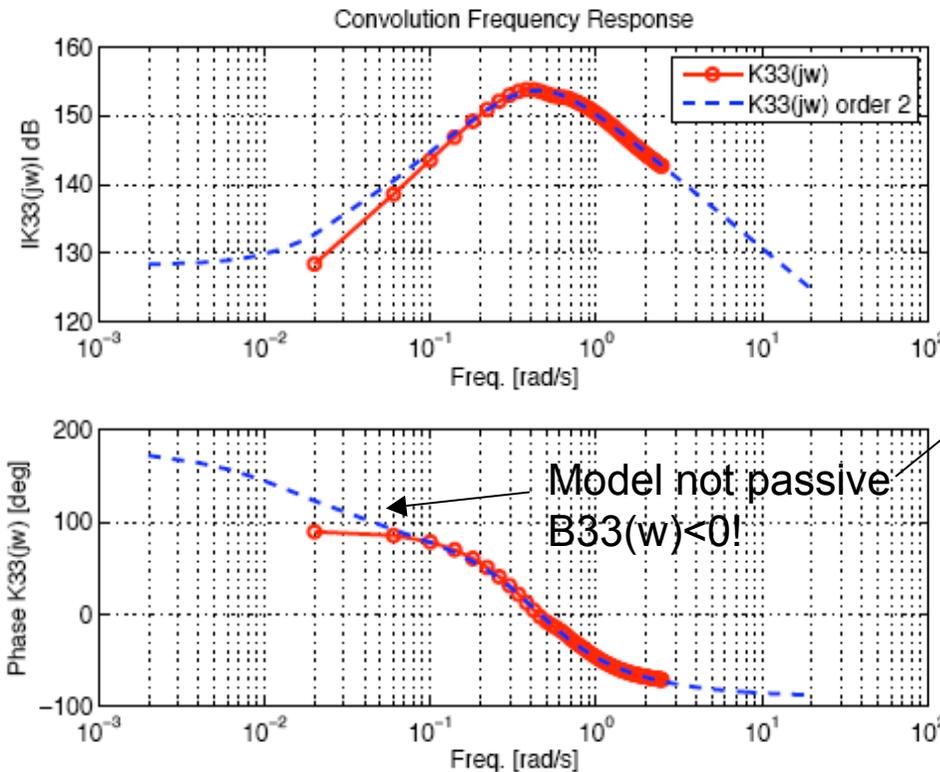
order detection via SVD



The quality of the impulse response affects the order selection.

The containership example suggests K55(s) order 2 to 5

example containership (DOF33)



Reconstruction of damping and added mass from the parametric approximation:

$$\hat{A}(\omega) = \text{Im}\{\omega^{-1} \hat{K}(j\omega)\} + A_{\infty}$$

$$\hat{B}(\omega) = \text{Re}\{\hat{K}(j\omega)\},$$

realization theory

- relatively easy to implement.
- do not require initial parameter estimates.
- allows order detection.
- requires conversion to continuous time (distortion).
- poor model quality.
- make no use of prior knowledge.

frequency-domain methods

identification from the frequency-response

frequency-response LS-fitting

The i,k entry of $\mathbf{K}(s)$ can be approximated by a rational transfer function:

$$\hat{K}(s, \theta) = \frac{P(s, \theta)}{Q(s, \theta)} = \frac{p_m s^m + p_{m-1} s^{m-1} + \dots + p_0}{s^n + q_{n-1} s^{n-1} + \dots + q_0} \quad \theta = [p_m, \dots, p_0, q_{n-1}, \dots, q_0]^T$$

Then we can estimate the parameters via LS optimization using the frequency response computed using the data generated by hydrodynamic code:

$$\theta^* = \arg \min_{\theta} \sum_l w_l (\epsilon_l^* \epsilon_l) \quad \epsilon_l = K(j\omega_l) - \frac{P(j\omega_l, \theta)}{Q(j\omega_l, \theta)}$$

$$\mathbf{K}(j\omega) = \mathbf{B}(\omega) + j\omega[\mathbf{A}(\omega) - \mathbf{A}]$$

This problem is non-linear in the parameters, but it can be linearised.

quasi-linear regression

for the non-linear problem

$$\theta^* = \arg \min_{\theta} \sum_l w_l (\epsilon_l^* \epsilon_l) \quad \epsilon_l = K(j\omega_l) - \frac{P(j\omega_l, \theta)}{Q(j\omega_l, \theta)}$$

Levi (1959) proposed a linearisation by choosing

$$\theta^* = \arg \min_{\theta} \sum_l w'_l (\epsilon'_l{}^* \epsilon'_l), \quad \epsilon'_l = Q(j\omega_l, \theta) \epsilon_l = Q(j\omega_l, \theta) K(j\omega_l) - P(j\omega_l, \theta).$$

This is equivalent to choose the following weights in the original problem above:

$$w_l = w'_l |Q(j\omega_l, \theta)|^2$$

- This problem is linear in the parameters, a standard linear LS problem.
- It does not always give a good fit if data spans a large range of frequencies.

iterative quasi-linear regression

Sanathanan and Koerner (1963), proposed an iterative solution via a sequence of linear LS problems:

1. Set $\mathbf{W}_0 = \mathbf{I}$. $\epsilon' = [\epsilon'_1, \dots, \epsilon'_N]^T$
2. Solve $\theta_k^* = \arg \min_{\theta} \epsilon'^* \mathbf{W}_k \epsilon'$, $\mathbf{W} = \text{diag}(w'_1, w'_2, \dots, w'_n)$
3. Set $\mathbf{W}_{k+1} = \text{diag}(|Q(j\omega_l, \theta_k)|^{-2})$ go to 2 until convergence.

This results in the following problem at each iteration $k > 1$:

$$\theta_k^* = \arg \min_{\theta} \sum_l \left| \frac{Q(j\omega_l, \theta)K(j\omega_l)}{Q(j\omega_l, \theta_{k-1}^*)} - \frac{P(j\omega_l, \theta)}{Q(j\omega_l, \theta_{k-1}^*)} \right|^2$$

After a few iterations $\theta_k^* \approx \theta_{k-1}^*$ and we recover the original nonlinear problem.

prior knowledge: constraints

Adding prior knowledge is important to refine the search for models, and thus obtain better quality models.

Prior knowledge usually derives from the physics of the underlying problem; in this case, from the hydrodynamics.

prior knowledge: constraints

For the transfer functions related to the convolution terms, we know

1. Relative degree 1
2. $H_{ik}(s)=0$ for $s=0$
3. Stable
4. Passive
5. Minimum order approximation is 2

Some of these properties can be enforced in the structure of the model and its parameters without complicating the optimisation.

prior knowledge: constraints

Model:

$$\hat{K}_{ik}(s) = \frac{P_{ik}(s)}{Q_{ik}(s)} = \frac{p_r s^r + p_{r-1} s^{r-1} + \dots + p_0}{s^n + q_{n-1} s^{n-1} + \dots + q_0}, \quad i, k = 1, \dots, 6.$$

- Relative degree = 1 \rightarrow Constrain $r = n-1$
- Zero at $s=0$

$$\begin{aligned} P_{ik}(s) &= s P'_{ik}(s) \\ \deg(P') &= n - 2 \end{aligned} \quad \rightarrow \quad \hat{K}_{ik}(j\omega) = \frac{(j\omega) P'_{ik}(j\omega, \theta_{ik})}{Q_{ik}(j\omega, \theta_{ik})}$$

Redefine the problem:

$$\theta_{ik}^* = \arg \min_{\theta} \sum_l \left| \frac{K_{ik}(j\omega_l)}{(j\omega_l)} - \frac{P'_{ik}(j\omega_l, \theta)}{Q_{ik}(j\omega_l, \theta)} \right|^2$$

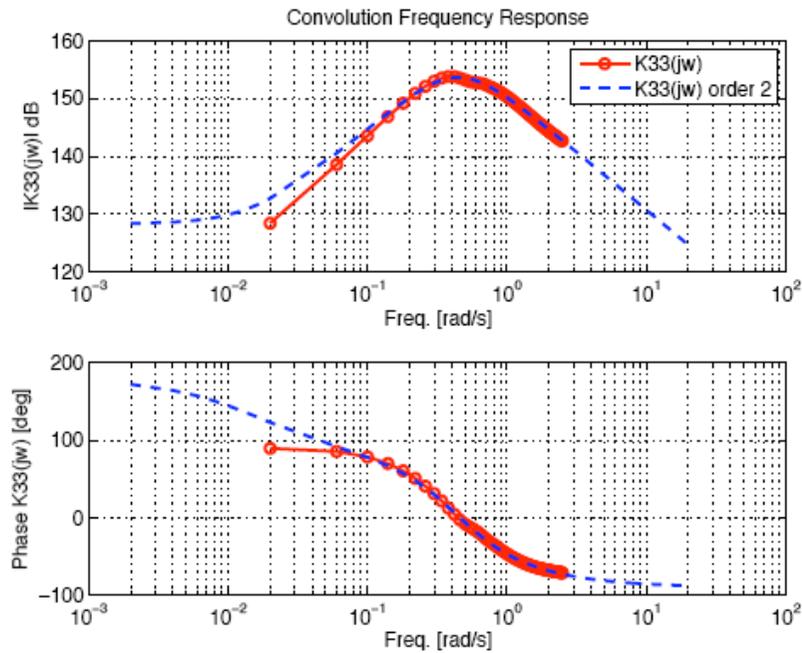
prior knowledge: constraints

- **Stability:** The LS optimisation does not ensure stability, this is one way to force it after identification:
 - (i) Compute the roots of $\lambda_1, \dots, \lambda_n$ of $Q_{ik}(s, \hat{\theta}_{ik})$.
 - (ii) If $\text{Re}\{\lambda_i\} > 0$, then set $\text{Re}\{\lambda_i\} = -\text{Re}\{\lambda_i\}$,
 - (iii) Reconstruct the polynomial: $Q_{ik}(s) = (s - \lambda_1)(s - \lambda_1) \cdots (s - \lambda_n)$
- **Passivity:** Also not enforced by LS. Low-order models are usually passive, hence if not passive try reducing the order.
- **Minimum order:** The 2nd order approximation is the lowest order approximation that can satisfy all the properties of the retardation functions:

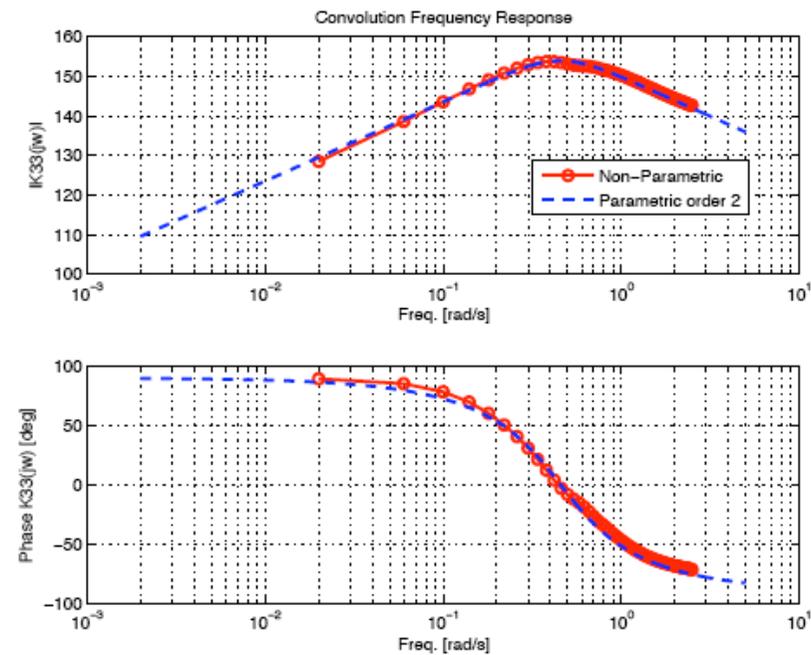
$$\hat{K}_{ik}^{min}(s) = \frac{p_0 s}{s^2 + q_1 s + q_0}$$

example container (dof 33)

Realization Theory

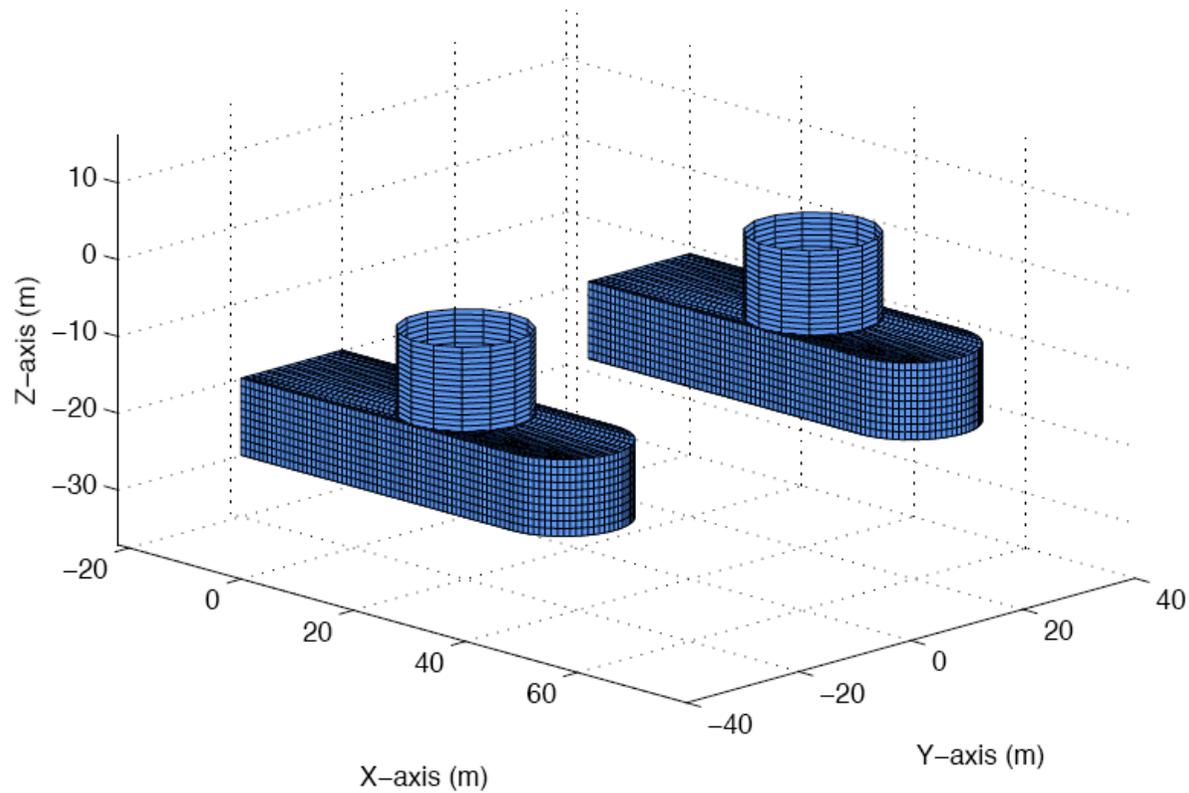


FD-Id with constraints



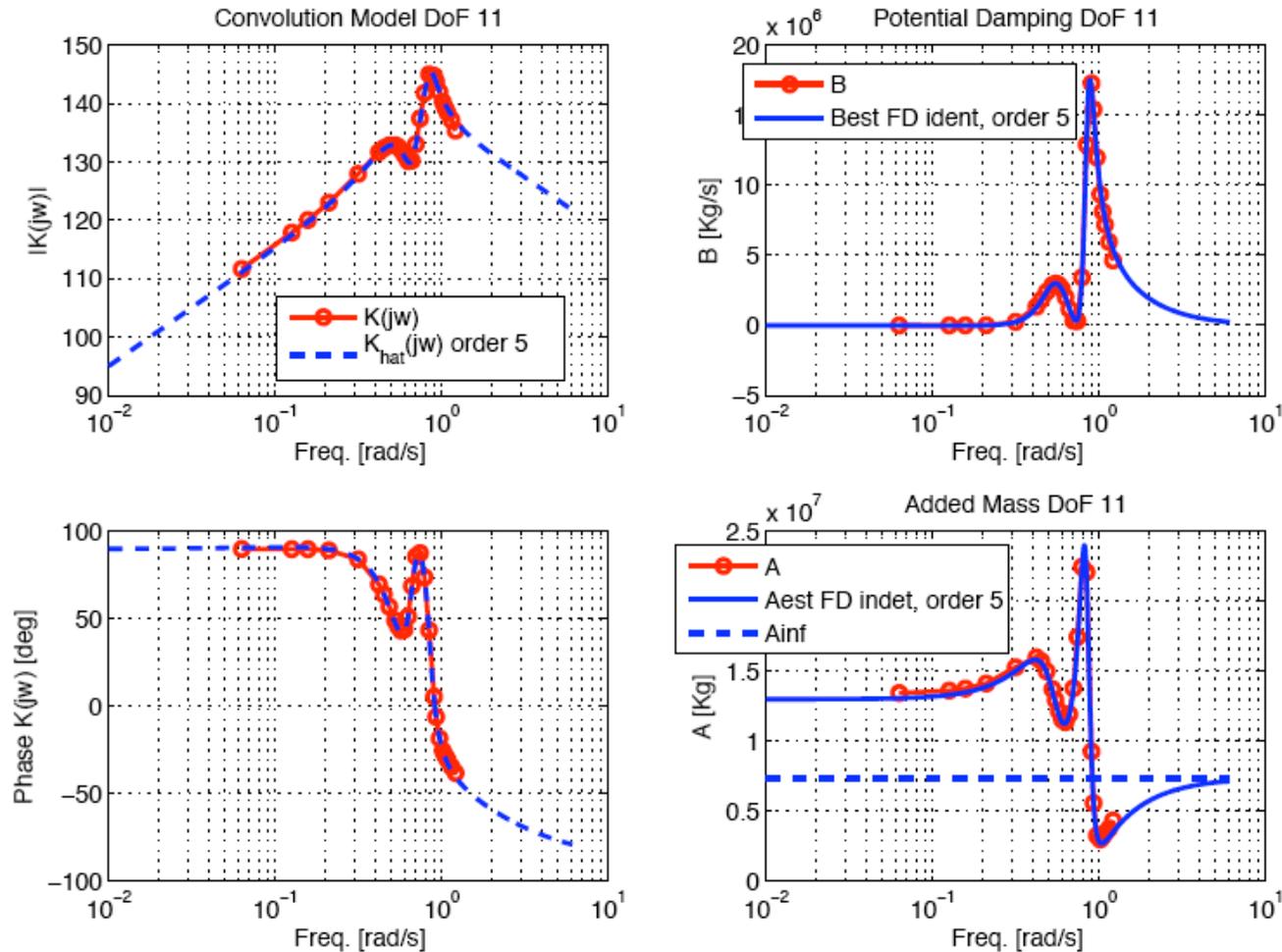
By imposing constraints on the model structure and parameters, we obtain a model that satisfy all the properties of the retardation functions and have a better quality.

example semi-sub

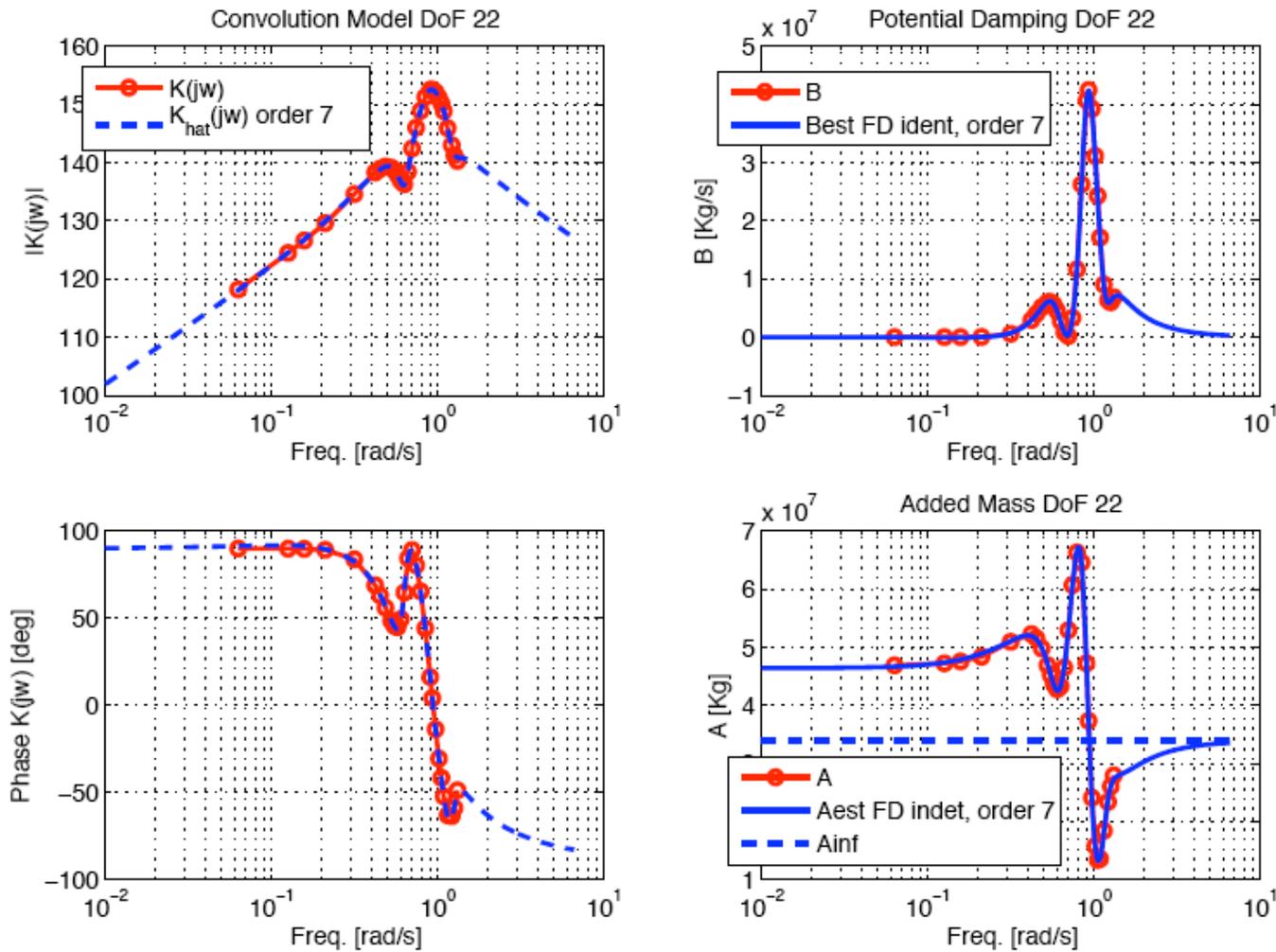


Data from www.marinecontrol.org

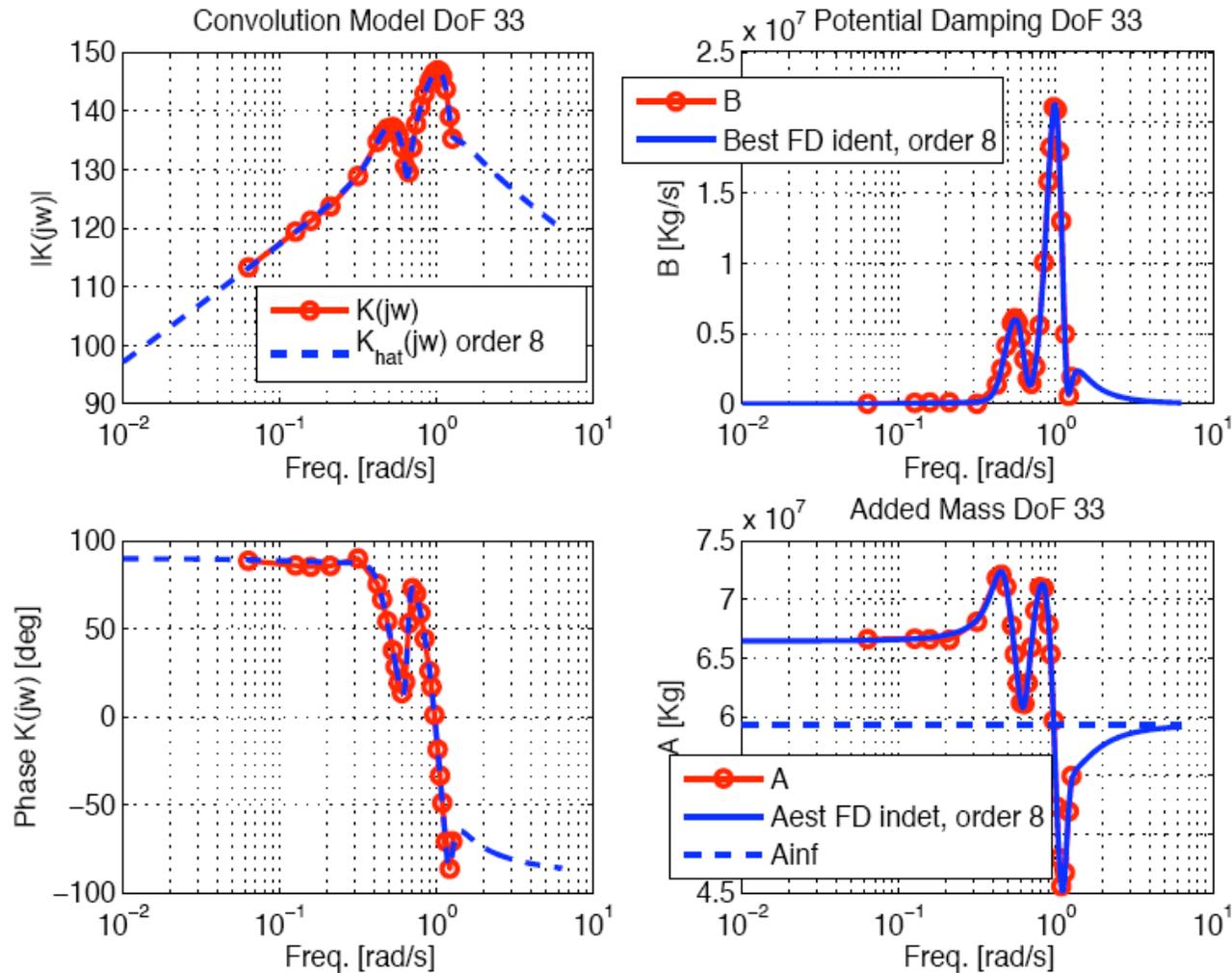
example semi-sub: surge (ord. 5)



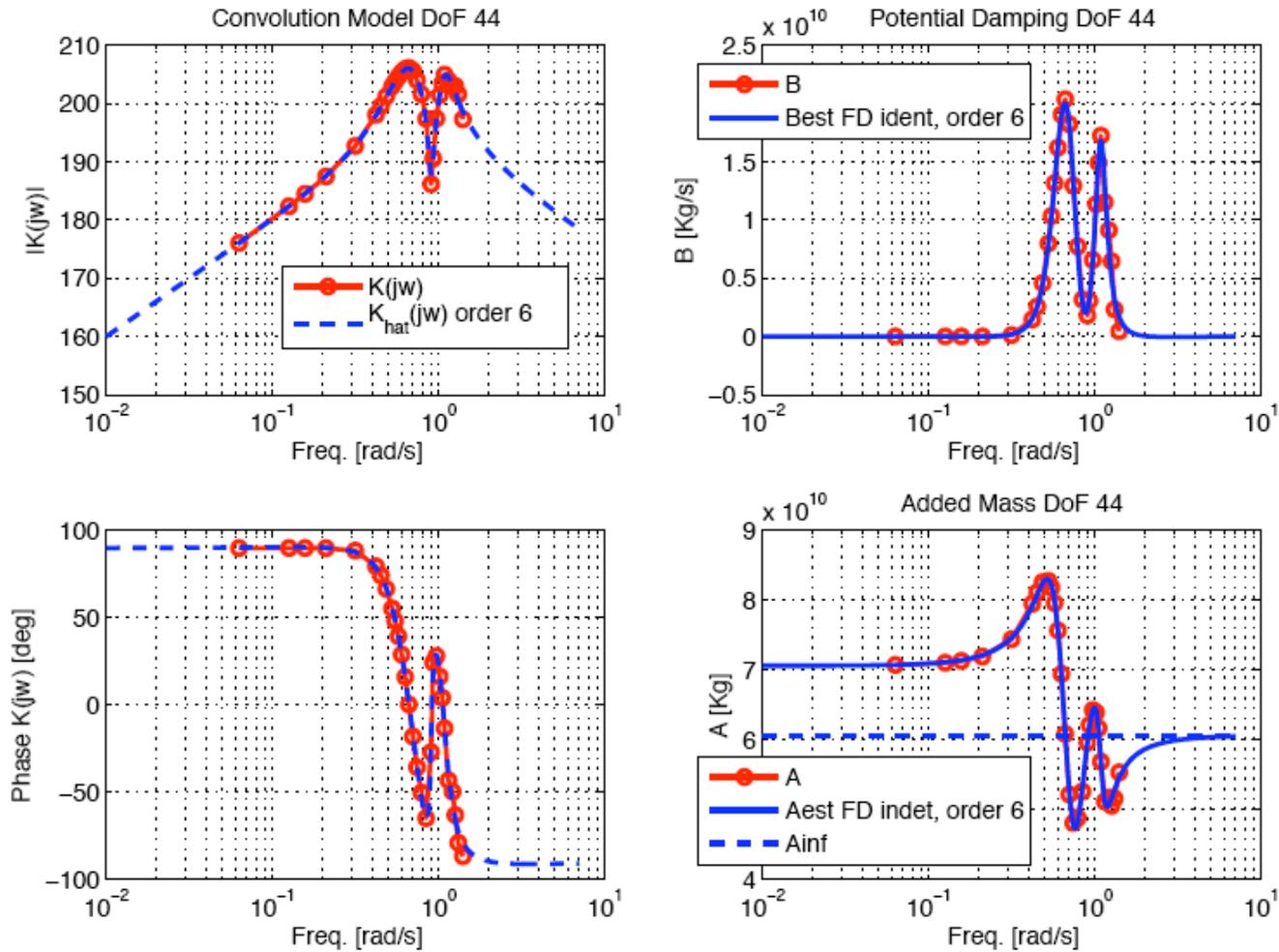
example semi-sub: sway (ord. 7)



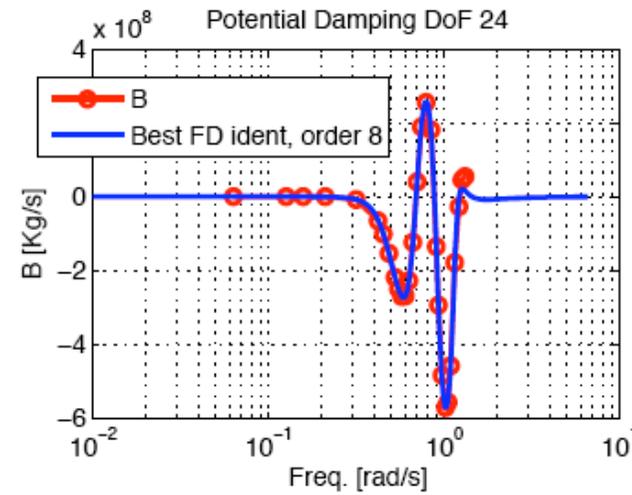
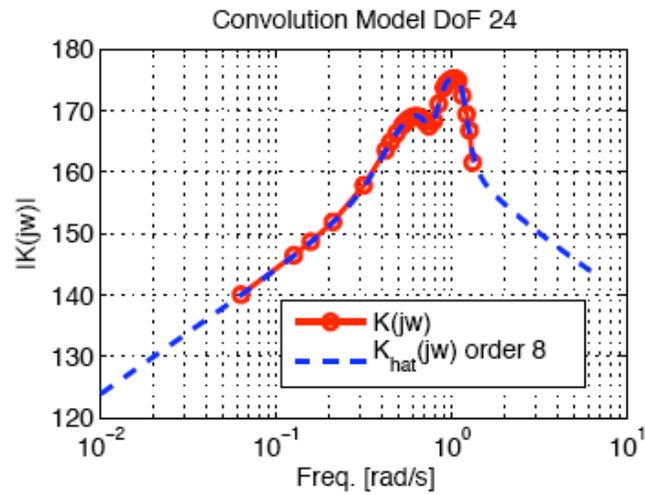
example semi-sub: heave (ord. 8)



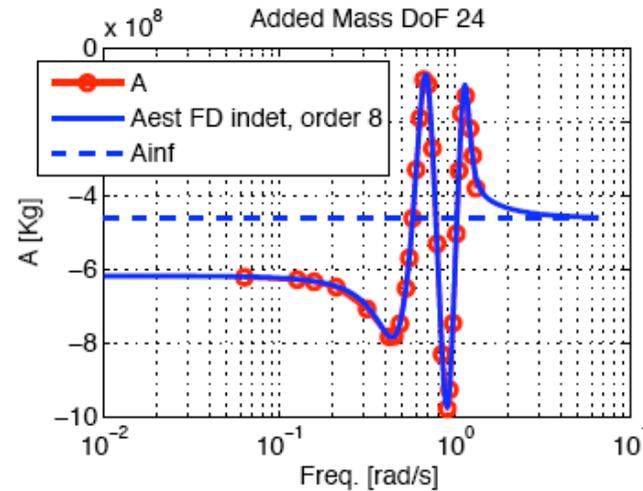
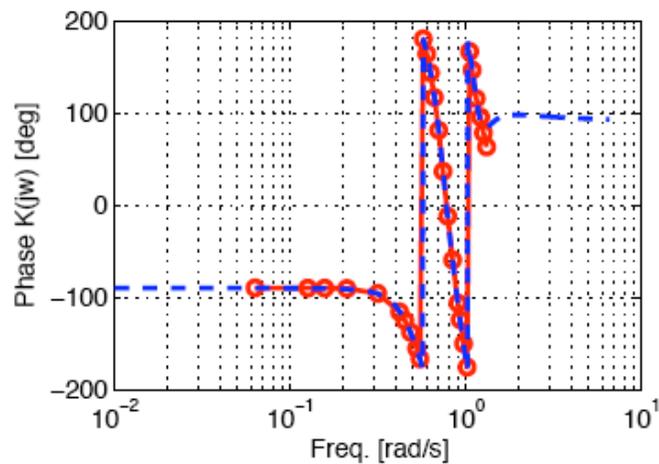
example semi-sub: roll (ord. 6)



example semi-sub: roll-sway (ord 8)



Couplings are not necessarily passive $B(\omega) < 0$



Frequency-domain methods

- Work directly in the FD, hence, there is no need to compute a distorted impulse response by going to time-domain.
- No need to compute data at very-low or very-high frequency. We can concentrate in the important range and compute many points.
- They can incorporate prior knowledge, and thus the models satisfy the physical characteristics of the problem. Good quality models.
- Order selection: one can start with the minimum order $n=2$ and increase it if necessary to improve the fit. (check passivity.)
- Parameter estimation method is simple: a sequence of Linear Least-Squares problems.

frequency-domain identification with 2D-data (strip-theory codes)

joint identification of infinite-frequency
added mass and memory models

indirect FD identification

If we do not have access to \mathbf{A}^∞ , as in the case of strip theory codes, we can identify it together with the $K(s)$.

The non-parametric radiation force models in the frequency domain can be expressed as

$$\tau_{rad,ik}(j\omega) = \left[\frac{B_{ik}(\omega)}{j\omega} + A_{ik}(\omega) \right] \ddot{\xi}(s)$$

From the parametric approximations, this can also be expressed as

$$\begin{aligned} \hat{\tau}_{rad,ik}(s) &= \left[A_{\infty,ik} s + \frac{P_{ik}(s)}{Q_{ik}(s)} \right] \dot{\xi}(s), \\ &= \left[A_{\infty,ik} + \frac{P'_{ik}(s)}{Q_{ik}(s)} \right] \ddot{\xi}(s), \end{aligned}$$

indirect FD identification

Then we can define the complex coefficient:

$$\tilde{A}(j\omega) \triangleq \frac{B_{ik}(\omega)}{j\omega} + A_{ik}(\omega)$$

And fit to it a rational approximation

$$\theta^* = \arg \min_{\theta} \sum_l w_l (\epsilon_l^* \epsilon_l), \quad \epsilon_l = \tilde{A}_{ik}(j\omega_l) - \frac{R_{ik}(j\omega_l, \theta)}{S_{ik}(j\omega_l, \theta)}$$

with the constraint

$$n = \deg S_{ik}(s) = \deg R_{ik}(s) \quad \Leftrightarrow \quad \hat{A}_{ik}(s) = \frac{R_{ik}(s)}{S_{ik}(s)} = \frac{A_{\infty, ik} Q_{ik}(s) + P'_{ik}(s)}{Q_{ik}(s)}$$

indirect FD identification

Once the $R(s)$ and $S(s)$ are obtained, we can obtain the added mass and fluid memory model from

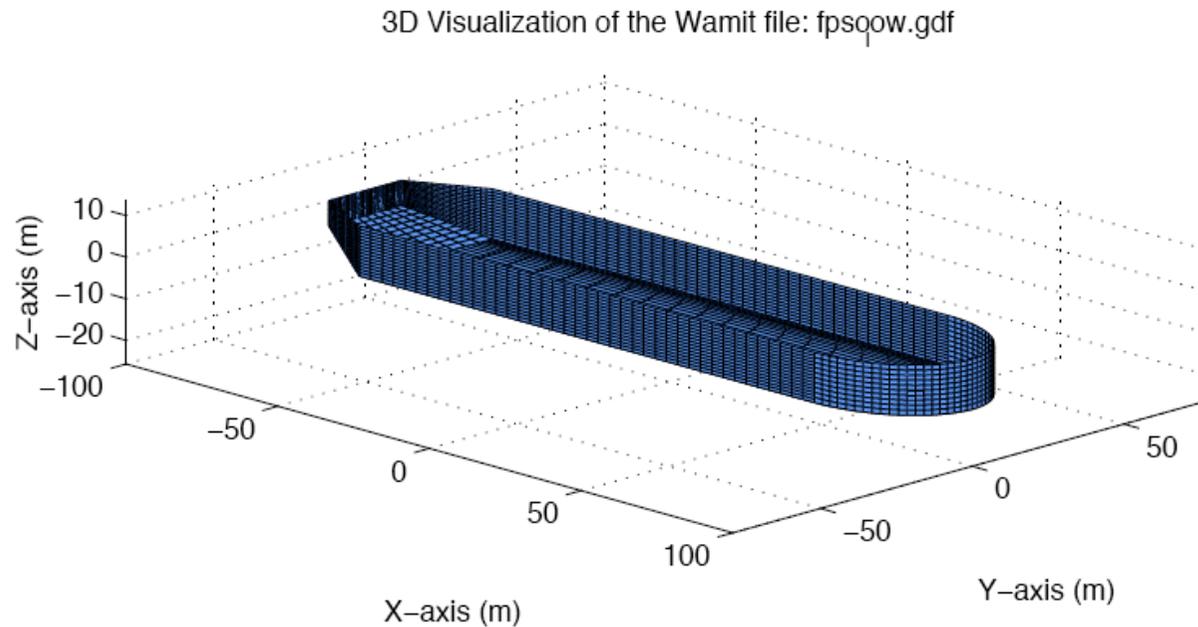
$$\hat{A}_{\infty,ik} = \lim_{\omega \rightarrow \infty} \frac{R_{ik}(s, \theta^*)}{S_{ik}(s, \theta^*)}$$

$$Q_{ik}(s, \theta^*) = S_{ik}(s, \theta^*),$$

$$P_{ik}(s, \theta^*) = R_{ik}(s, \theta^*) - \hat{A}_{\infty,ik} S_{ik}(s, \theta^*)$$

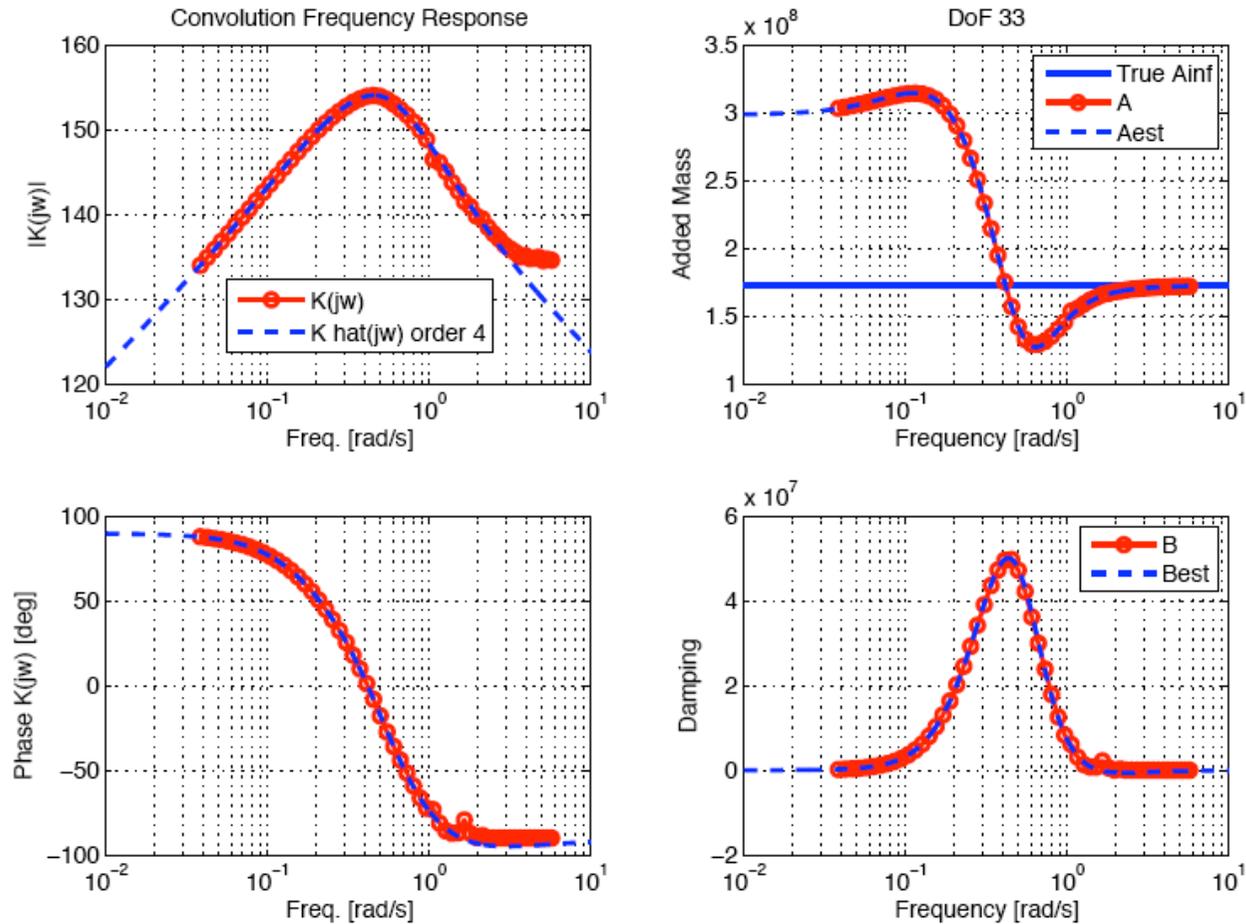
The coefficient $\hat{A}_{\infty,ik}$ is the coefficient of the higher-order term of $R_{ik}(s, \theta^*)$ if $S_{ik}(s, \theta^*)$ is monic.

example FPSO

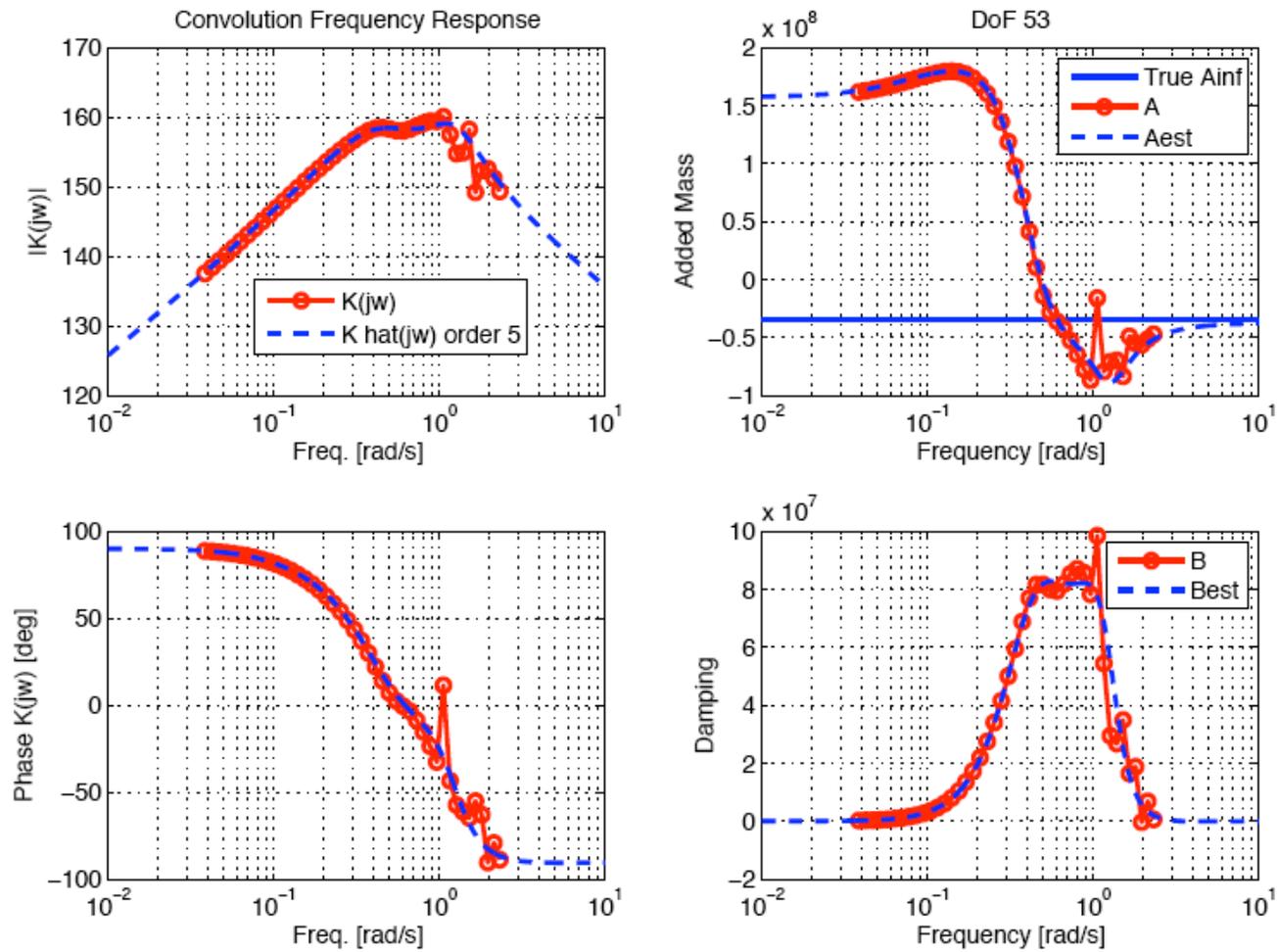


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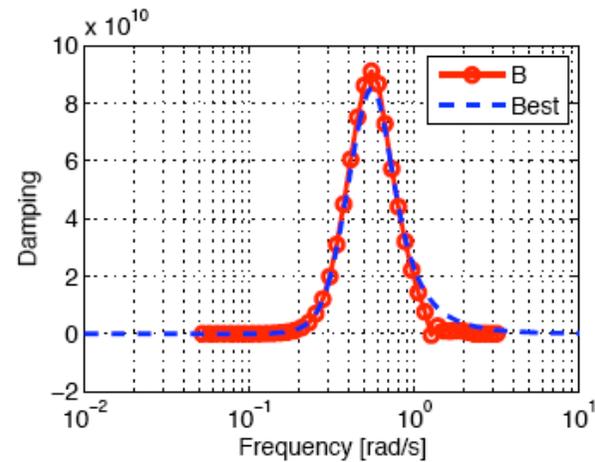
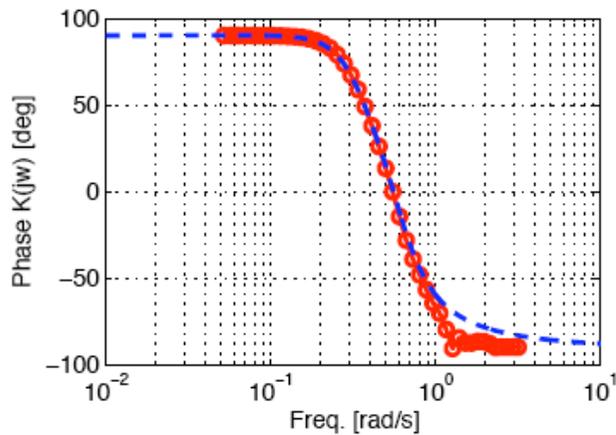
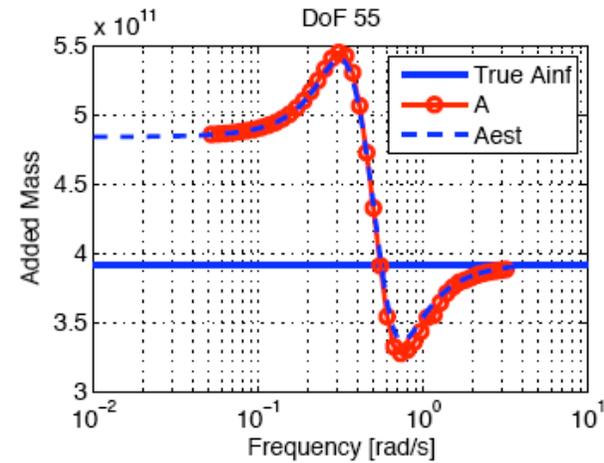
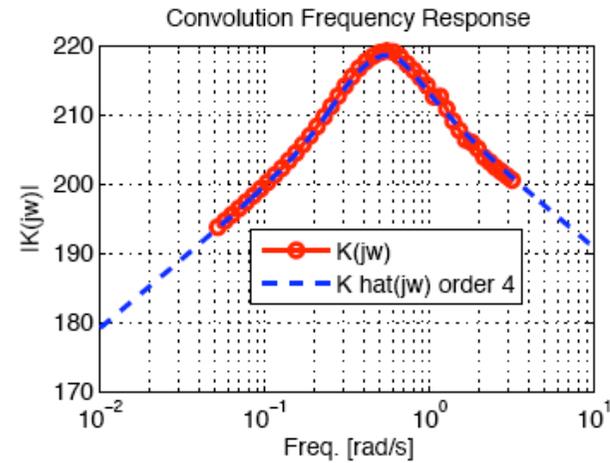
example FPSO (dof 33)



example FPSO (dof 35,53)



example FPSO (dof 55)



example FPSO

Infinite-frequency added mass coefficients.

True Value	Identified	Rel. Err.
$A_{33}=1.7283e8$	$\hat{A}_{33}=1.731e8$	1.5 %
$A_{35}=-3.463e7$	$\hat{A}_{35}=-3.7179e7$	0.18%
$A_{55} 3.9154e11$	$\hat{A}_{55}=3.9293e11$	0.35%

Similar accuracy is obtained for the other couplings.

Summary and Conclusions

- We have revisited different methods to the identification of time-domain models based on frequency domain computations.

- Time-domain methods
 - Require forming the impulse response function (distortion).
 - Make no use of prior knowledge (affects model quality).
 - Realization Theory is relatively easy to implement.
 - Applicable to 2D and 3D data.

- Frequency-domain methods
 - Simple to implement and use.
 - Incorporate prior knowledge as constraints (improved model quality).
 - Applicable 2D and 3D data.

This presentation

- This presentation summarizes the discussions in
 - Perez and Fossen (2008a) ***Time- vs frequency-domain Identification of parametric radiation force models for marine structures at zero speed.*** Modeling, Identification and Control Vol. 29 No.1, pp1-19.
 - Perez and Fossen (----) ***Joint Identification of Infinite-frequency Added Mass and Fluid-Memory Models of Marine Structures.*** Modeling, Identification and Control. Under review.
 - Perez and Fossen (----) ***Identification of Seakeeping Models from Frequency-response data Enforcing Model structure and parameter Constraints.*** To be submitted at Ocean Engineering