Modelling and Simulation of Environmental Disturbances

(Module 5)

Dr Tristan Perez

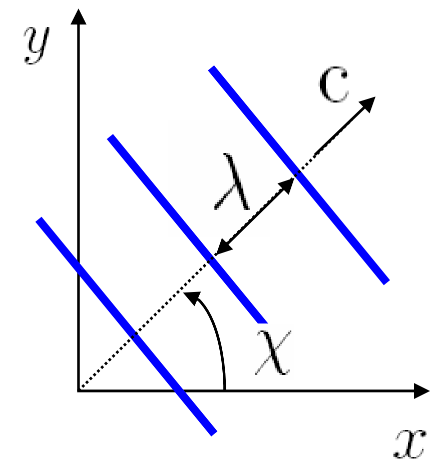
Centre for Complex Dynamic Systems and Control (CDSC)

Prof. Thor I Fossen

Department of Engineering Cybernetics

Regular waves in deep water

The sea surface elevation is described by:



$$\boxed{\zeta(x,y,t) = \bar{\zeta}\sin[\omega t + \varepsilon - kx\cos(\chi) - ky\sin(\chi)]}$$

where

$$\omega = \frac{2\pi}{T}$$

Wave frequency (rad/s)

$$k = \frac{\omega^2}{g}$$

Wave number (rad/m)

$$\lambda = \frac{g}{2\pi}T^2$$

Wave length (m)

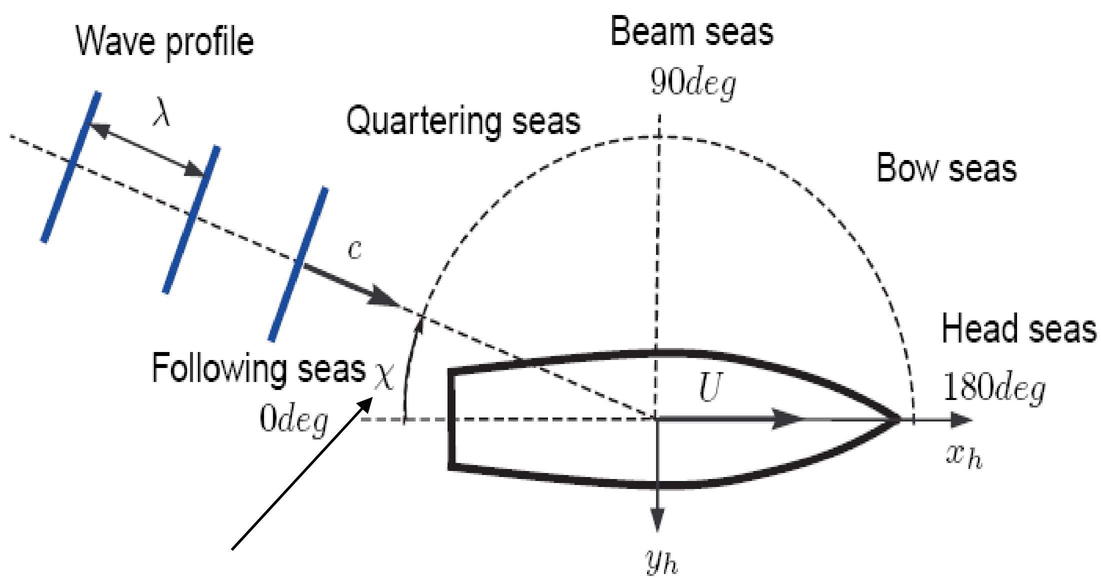
$$c = \sqrt{\frac{g\lambda}{2\pi}} = \frac{\lambda}{T}$$

Phase velocity (m/sec)

These expressions are only valid in deep water $h \geq \lambda/2$, where *h* is the water depth.

Sailing condition

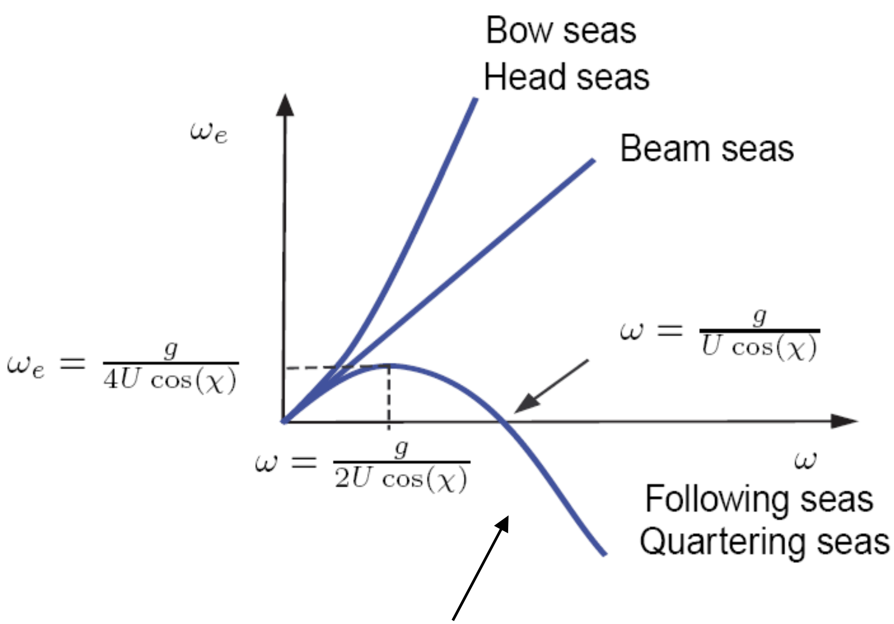
The sailing condition of a vessel is given by its forward speed U and its encounter angle, i.e., the heading angle relative to the waves.



Encounter angle

Encounter frequency

If the waves are observed from a reference frame that moves at a constant speed, the frequency observed is called Encounter Frequency.



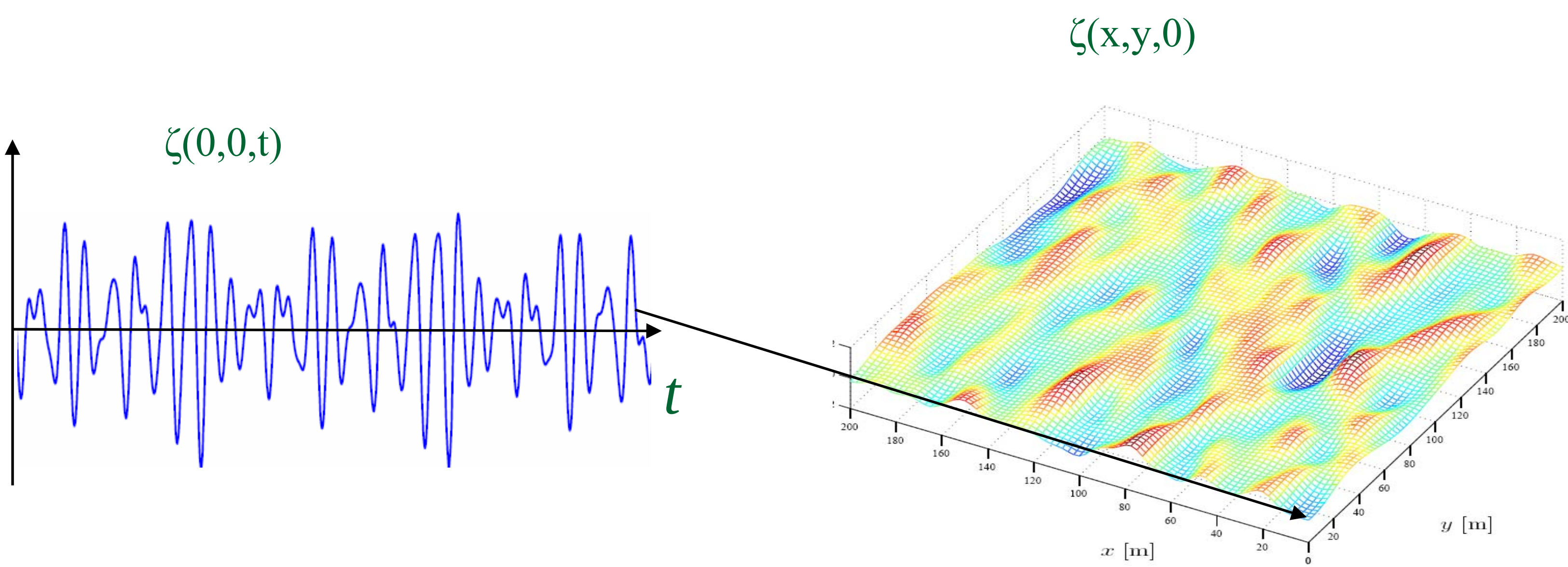
$$\boxed{\omega_e = \omega - \frac{\omega^2 U}{g}\cos(\chi)}$$

This is a Doppler effect

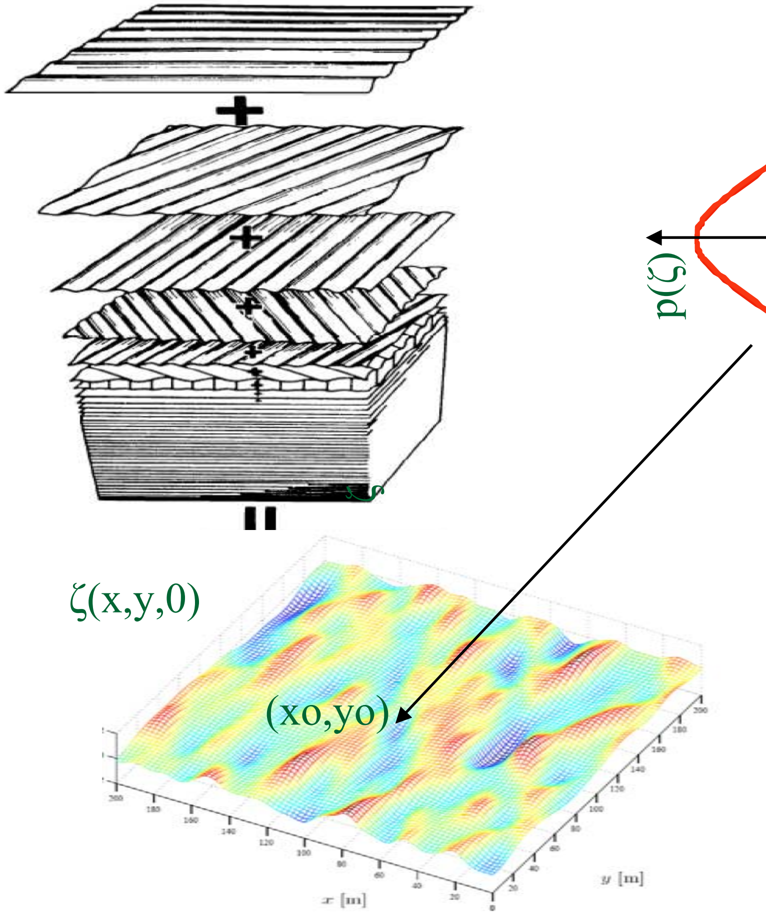
Negative encounter frequency => vessel overtakes the waves

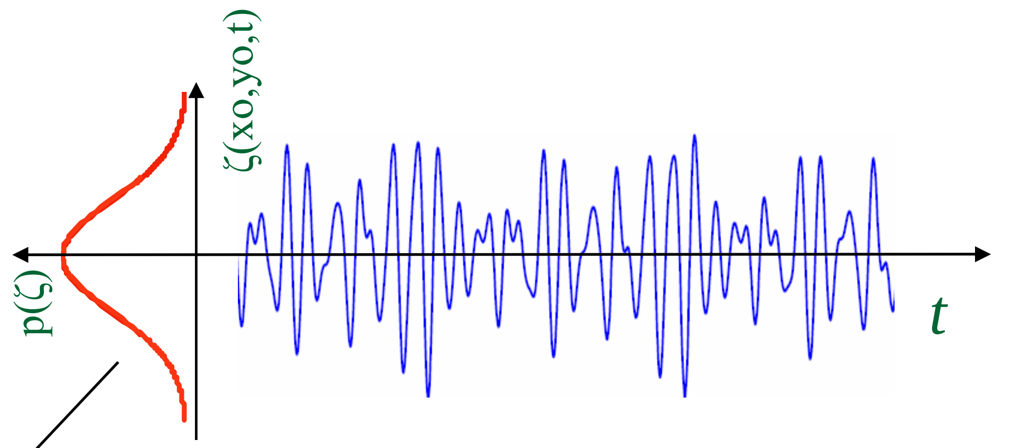
Ocean waves

Ocean waves present, in general, irregularity in time and space and cannot be predicted exactly: Stochastic Process.



Gaussian waves





The elevation of the sea surface can be thought as being generated as the sum of many sinusoidal waves with different amplitudes, frequencies, and phases.

How good are these hypotheses?

From data collected at sea (Haverre & Moan, 1985), it can be stated that

- For low and moderate seas (< 4m), the sea can be considered stationary for periods over 20 min. For more severe sea states, stationarity can be questioned even for periods of 20 min.

- For medium seas (4m to 8m), Gaussian models are still accurate, but deviations from Gaussianity slightly increase with the increasing severity of the sea state.

- If the water is sufficiently deep, wave elevation can be consider Gaussian regardless of the sea state.

Random sea characterization

Under the Gaussianity assumption, the process is completely described by

- Mean
- Variance

The sea surface elevation is described relative to the mean free surface; therefore, the mean of the SP is zero.

The variance is described in terms of a sea power spectral density or sea spectrum.

Power spectral density definition

If we use the frequency in Hz to define the FT, then

$$S_{xx}(f) = \int_{-\infty}^{+\infty} R_{xx}(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$R_{xx}(\tau) = \int_{-\infty}^{+\infty} S_{xx}(f)\, e^{j2\pi f\tau}\, df$$

and

$$\mathrm{var}[x] = R_{xx}(0) = \int_{-\infty}^{+\infty} S_{xx}(f)\, df$$

from which the name power spectral density follows.

This definition is common in the literature of electrical communications and signal processing.

PSD alternative definition

When we use the circular frequency (rad/s) to define the FT,

$$S_{xx}(\omega) = a \int_{-\infty}^{+\infty} R_{xx}(\tau)\, e^{-j\omega\tau} d\tau$$

$$R_{xx}(\tau) = b \int_{-\infty}^{+\infty} S_{xx}(\omega)\, e^{j\omega\tau} d\omega$$

$$a\, b = \frac{1}{2\pi}$$

We need to be careful on how we compute power!!!

Random sea characterization

$$\mathbf{E}[\zeta(t)] = 0, \qquad \mathbf{E}[\zeta(t)^2] = \int_0^\infty \mathbf{S}_{\zeta\zeta}(\omega)\, d\omega$$

The spectral moments of order *n* are defined as:

$$m_\zeta^n = \int_0^\infty \omega^n \mathbf{S}_{\zeta\zeta}(\omega) d\omega$$

Then the variance and standard deviation (RMS value) are

$$\mathbf{var}[\zeta] = \mathbf{E}[\zeta^2] = \mathbf{R}_{\zeta\zeta}[0] = m_\zeta^0 \qquad \sigma = \sqrt{m_\zeta^0}.$$

Statistics of wave period

- *Average wave period* (1/average frequency of the spectrum)

$$T \quad \texttt{or} \quad T_1 = 2\pi m_\zeta^0 / m_\zeta^1,$$

- *Zero-crossing wave period* (average period of zero up-crossings)

$$T_z = 2\pi \sqrt{m_\zeta^0 / m_\zeta^2},$$

- *Average period between response maxima (crests)*

$$T_c = 2\pi \sqrt{m_\zeta^2 / m_\zeta^4}.$$

Note that the 2π factor in the expressions above appears only if the moments are calculated in the circular frequency domain.

Statistics of wave height

Assuming narrow bandness,

- *mean value of wave amplitude*

$$\overline{\zeta} = 1.5\sqrt{m^0_\zeta}.$$

- *Significant wave amplitude*

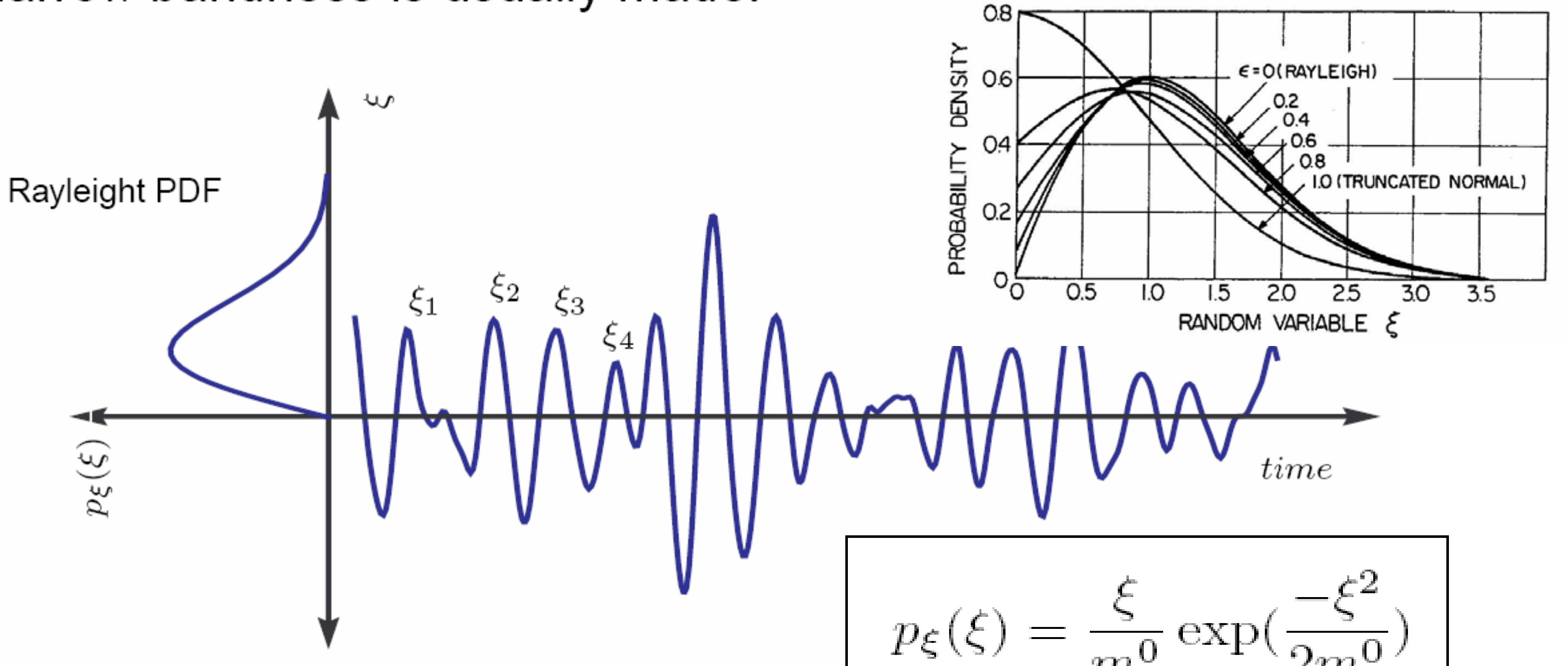
$$\zeta_{1/3} = 2\sqrt{m^0_\zeta}.$$

- *Significant wave height*

$$H_{1/3} = 4\sqrt{m^0_\zeta}.$$

Statistics of Maxima

In marine applications $\epsilon \leq 0.6$, and the assumption of narrow bandness is usually made.



$$p_\xi(\xi) = \frac{\xi}{m_\zeta^0} \exp\left(\frac{-\xi^2}{2m_\zeta^0}\right)$$

(This holds not only for wave elevation, but also for wave induced motion of ships.)

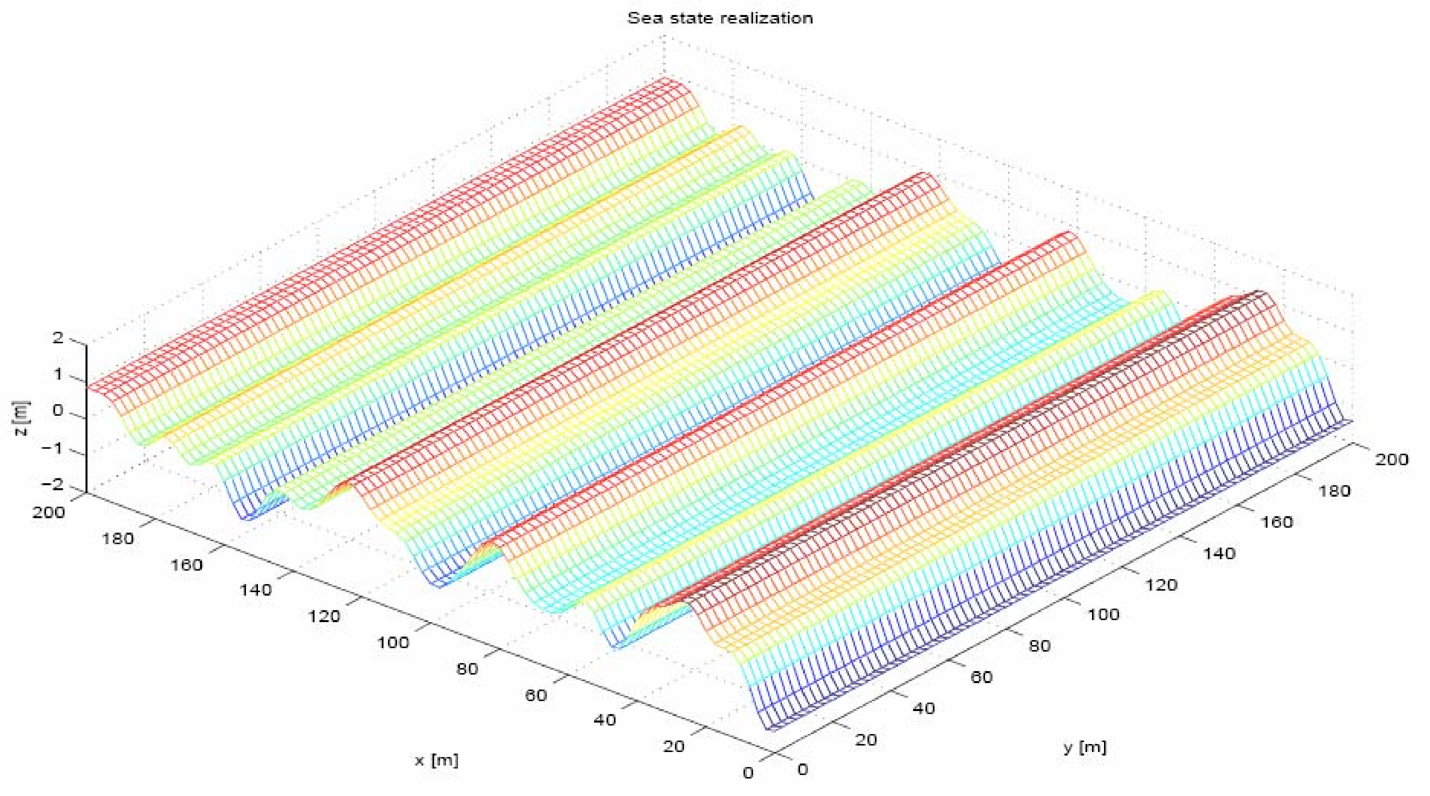
Long- and short-crested Seas

- After the wind has blown constantly for a certain period of time, the sea elevation can be assumed statistically stable. In this case, the sea is referred to as *fully-developed*.

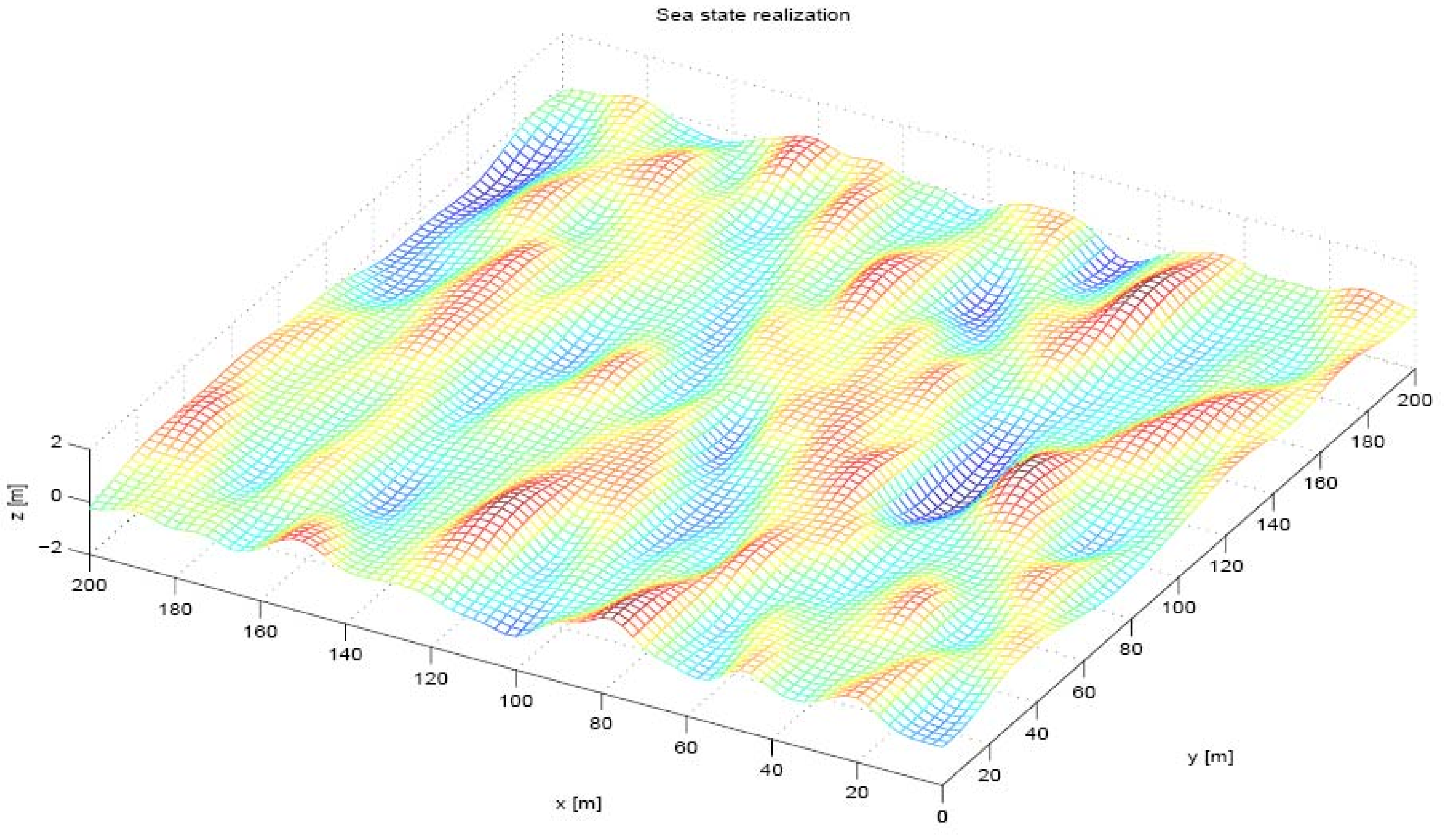
- If the irregularity of the observed waves are only in the dominant wind direction, so that there are mainly uni-directional wave crests with varying separation but remaining parallel to each other, the sea is referred to as a *long-crested* irregular sea.

- When irregularities are apparent along the wave crests at right angles to the direction of the wind, the sea is referred to as *short crested or confused sea.*

Long-crested Seas



Short-crested Seas



Idealised spectra Long-crested seas

In cases where no wave records are available, standard, idealized, formulae can be used. One family of idealized spectra is the *Bretschneither* family which was developed in early 1950s:

Modal frequency

$$\mathbf{S}_{\zeta\zeta}(\omega) = \frac{A}{\omega^5}\exp\left(\frac{-B}{\omega^4}\right)$$

$$\left.\frac{dS}{d\omega}\right|_{\omega=\omega_0} = 0; \qquad \omega_0 = \left(\frac{4B}{5}\right)^{\frac{1}{4}},$$

Spectral moments

$$m_\zeta^0 = \frac{A}{4B} \qquad m_\zeta^1 = 0.3\frac{A}{B^{3/4}} \qquad m_\zeta^2 = \sqrt{\frac{\pi A^2}{16B}}.$$

(This family can be used to represent rising and falling seas, as well as fully developed seas with no swell and unlimited fetch.)

Idealised Spectra

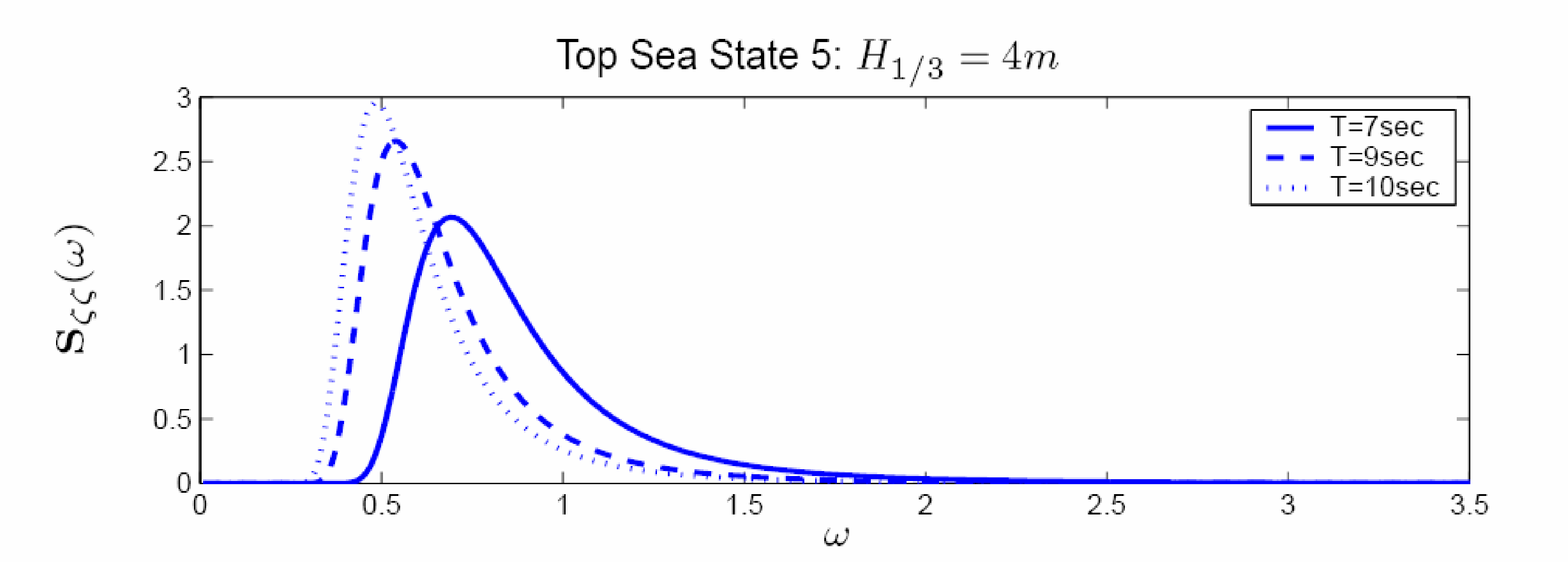
In the 1960's, the *Pierson-Moskowitz* family was developed to forecast storm waves at a single point in fully developed seas with no swell using wind data. This family relates the parameters *A* and *B* to the average wind speed at 19.5m above the sea surface:

$$A = 8.1 \times 10^{-3} g^2 \qquad B = \frac{0.74g}{\bar{V}_{19.5}}$$

The ITTC recommends the use of the *Modified Pierson-Moskowitz* family; significant wave height and zero-crossing period or the average wave period is used:

$$A = \frac{4\pi^3 H_{1/3}^2}{T_z^4}; \quad B = \frac{16\pi^3}{T_z^4} \qquad \text{or} \qquad A = \frac{172.75 H_{1/3}^2}{T_1^4}; \quad B = \frac{691}{T_1^4}$$

Example ITTC spectra



Other Spectra

Other families:

- The JONSWAP spectral family accounts for the case of limited fetch wind waves
- The TMA spectral family is an extension of the JONSWAP for finite water depth,
- The double-peak Torsethaugen spectral family accounts for both swell and wind waves,
- The Ochi six-parameter spectral family can be fitted to almost any wave record (it could account for swell and wind waves.)

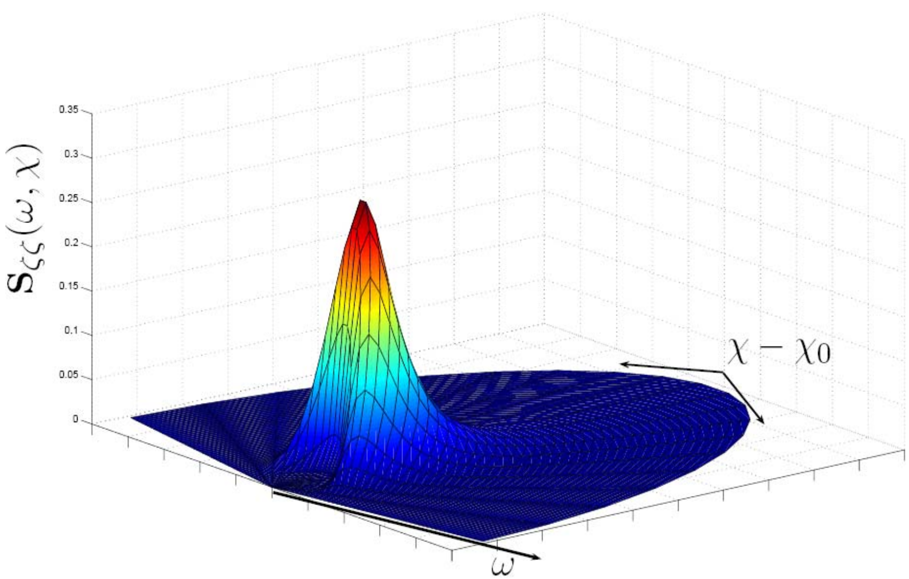
For further details see Ochi, K. (1998) Ocean Waves: The Stochastic Approach, Cambridge University Press. Ocean Tech. Series.

Spectra for Short-crested Seas

Directional spectra are more realistic and are very important to calculate loads on marine structures since the motion response depends highly on the encounter angle. For simulation, it is common to separate the directional spectrum as a product of two functions:

$$\boxed{\mathbf{S}_{\zeta\zeta}(\omega,\chi) = \mathbf{S}(\omega)M(\chi)}$$

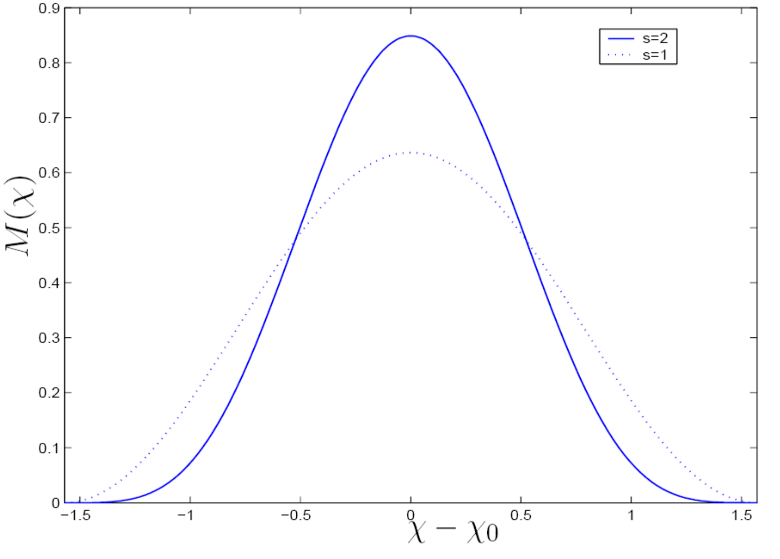
$M(\chi)$ is the spreading function



Spreading Function

$$M(\chi) = \begin{cases} \frac{2^{(2s-1)} s!(s-1)!}{\pi(2s-1)!} \cos^{2s}(\chi - \chi_0) & \text{for } -\frac{\pi}{2} < \chi - \chi_0 < \frac{\pi}{2} \\ 0 & \text{Otherwise} \end{cases}$$

where χ_0 is the dominant wave propagation direction, and the values of s = 1, 2 are commonly used. See Lloyd (1989) for a more general form where $|\chi - \chi_0| < \alpha$; with α not necessarily equal to $\pi/2$



Time-domain Simulations

If $\zeta(t)$ is stationary and Gaussian on time interval $[0, T]$, its realizations can be approximated to any degree of accuracy by

$$\boxed{\zeta(t) = \sum_{n=1}^{N} \underbrace{\sqrt{2}\sigma_n}_{\bar{\zeta}_n} \cos(\omega_n t + \varepsilon_n),}$$

with N sufficiently large; where σ_n are constants and the phases ε_n are independent identically distributed random variables with uniform distribution in $[0, 2\pi]$.

The autocorrelation is given by

$$\mathbf{R}_{\zeta\zeta}(\tau) = \mathbf{E}[\zeta(t)\zeta(t+\tau)] = \sum_{n=1}^{N} \sigma_n^2 \cos(\omega_n \tau).$$

Why is it done this way?

Since the autocorrelation for τ=0 gives the energy of $\eta(t)$, it follows that

$$\int_0^\infty \mathbf{S}_{\zeta\zeta}(\omega)\, d\omega \approx \sum_{n=1}^{N} \sigma_n^2,$$

and we can write

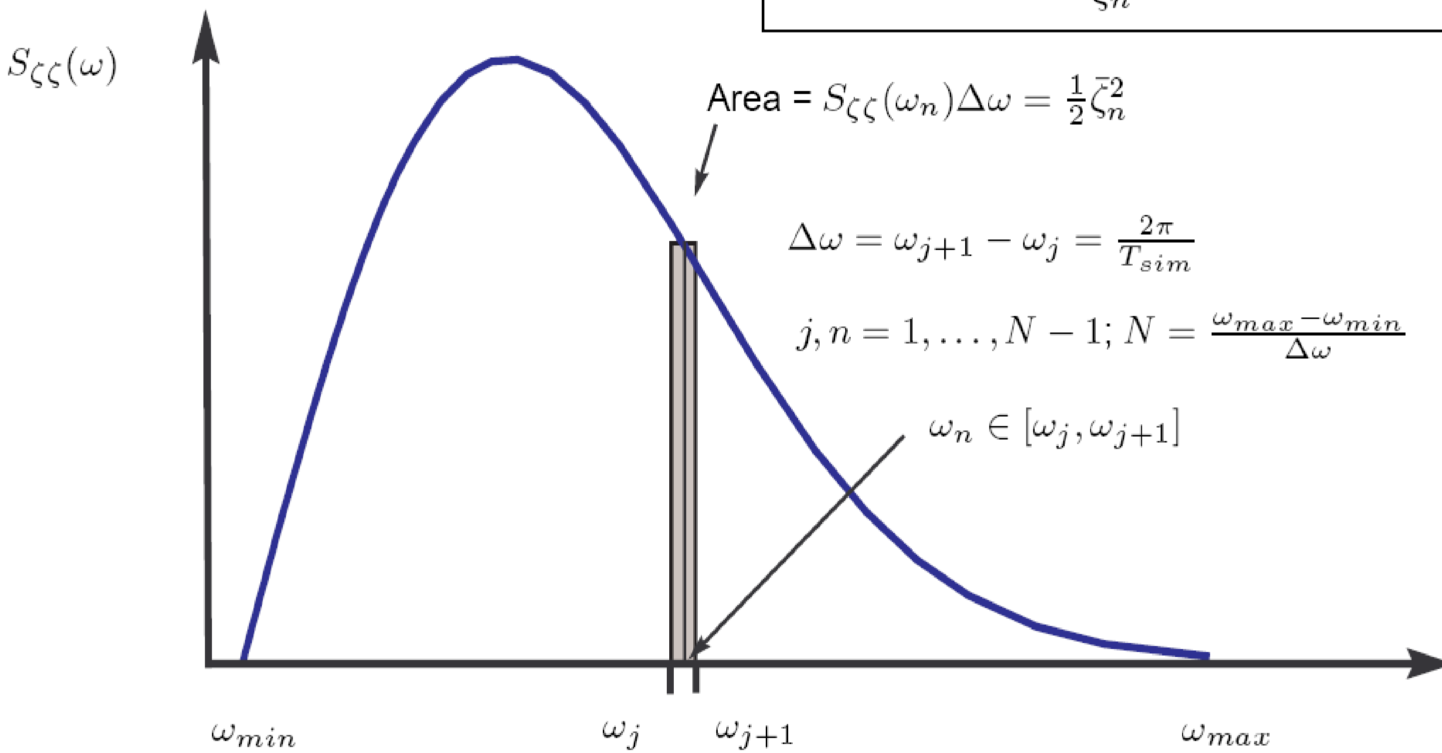
$$\sum_{n=1}^{N} \int_{\omega_n-\Delta\omega/2}^{\omega_n+\Delta\omega/2} \mathbf{S}_{\zeta\zeta}(\omega)\, d\omega = \sum_{n=1}^{N} \sigma_n^2,$$

and take

$$\sigma_n^2 = \int_{\omega_n-\Delta\omega/2}^{\omega_n+\Delta\omega/2} \mathbf{S}_{\zeta\zeta}(\omega)\, d\omega = \mathbf{S}_{\zeta\zeta}(\omega_n)\Delta\omega$$

Time-domain Simulations

$$\zeta(t) = \sum_{n=1}^{N} \underbrace{\sqrt{2}\sigma_n}_{\bar{\zeta}_n} \cos(\omega_n t + \varepsilon_n),$$



Formulae for Time-damin Simulations

- Long Crested Sea:

$$\zeta(x,y,t) = \sum_{n=1}^{N} \sqrt{2\mathbf{S}_{\zeta\zeta}(\omega_n)\Delta\omega}\cos(\omega_n t + \varepsilon_n - k_n(x\cos\chi - y\sin\chi))$$

- Short Crested Sea:

$$\zeta(x,y,t) = \sum_{n=1}^{N}\sum_{m=1}^{M} \sqrt{2\mathbf{S}_{\zeta\zeta}(\omega_n,\chi_m)\Delta\omega\Delta\chi}$$
$$\cos(\omega_n t + \varepsilon_{n,m} - k_n(x\cos\chi_m - y\sin\chi_m)).$$

Spectral Factorization Approach

The realizations of a sea surface elevation are modelled by filtered white noise:

$$\mathbf{S}_{\zeta\zeta}(\omega) \approx |H(j\omega)|^2 \, \mathbf{S}_{ww}$$

A typical model is a 2nd-order system :

$$H(s) = \frac{s}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

The parameters can be adjusted as follows (Perez, 2005):

- $\mathbf{S}_{ww} = \max \mathbf{S}_{\zeta\zeta}$
- ω_n chosen to be the modal frequency.
- ξ is chose to the variance is the same; and thus the RMS value.

Wind

Wind is commonly divided in two components; a mean value and a fluctuating component, or gust.

It is a 3D phenomenon, but in marine applications we restrict it to 2D, and velocities are considered only in the horizontal plane.

Wind is parameterized by the velocity U and the direction ψ. The direction is taken with respect to the North-East.

Note that, in general, the direction of the wind is the direction from where the wind is coming from, e.g., a SE wind blows from the SE towards NW.

Wind Mean-velocity Component

Slowly-varying fluctuations in the mean wind velocity can be modeled by a 1st order Gauss-Markov Process:

$$\boxed{\dot{\bar{U}} + \mu \bar{U} = w}$$

where *w* is Gaussian white noise and μ ≥ 0 is a constant.

The magnitude of the velocity should be restricted by saturation elements

$$0 \leq \bar{U}_{\min} \leq \bar{U} \leq \bar{U}_{\max}$$

Wind Mean-direction component

Slowly-varying fluctuations in the mean wind direction can also be implemented by a 1st order Gauss-Markov Process (Sorensen, 2005):

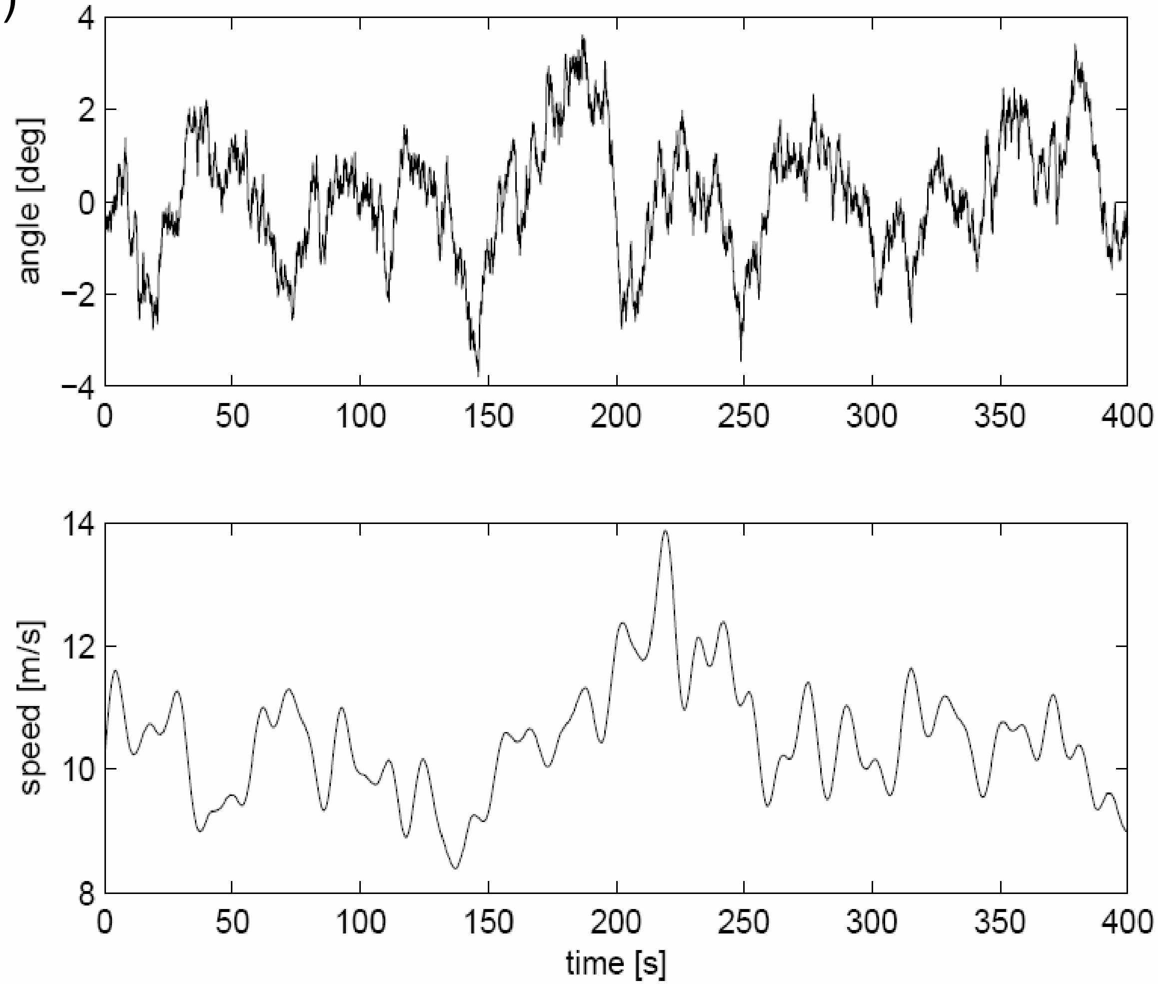
$$\begin{aligned}\dot{\psi} + \mu_2 \psi &= w_2 \\ \psi_{\min} &\leq \psi \leq \psi_{\max}\end{aligned}$$

The direction is taken with respect to the (n-frame).

NOTE: This direction is **often** the direction from where the wind is blowing: North-East (NE) wind blows from the NE

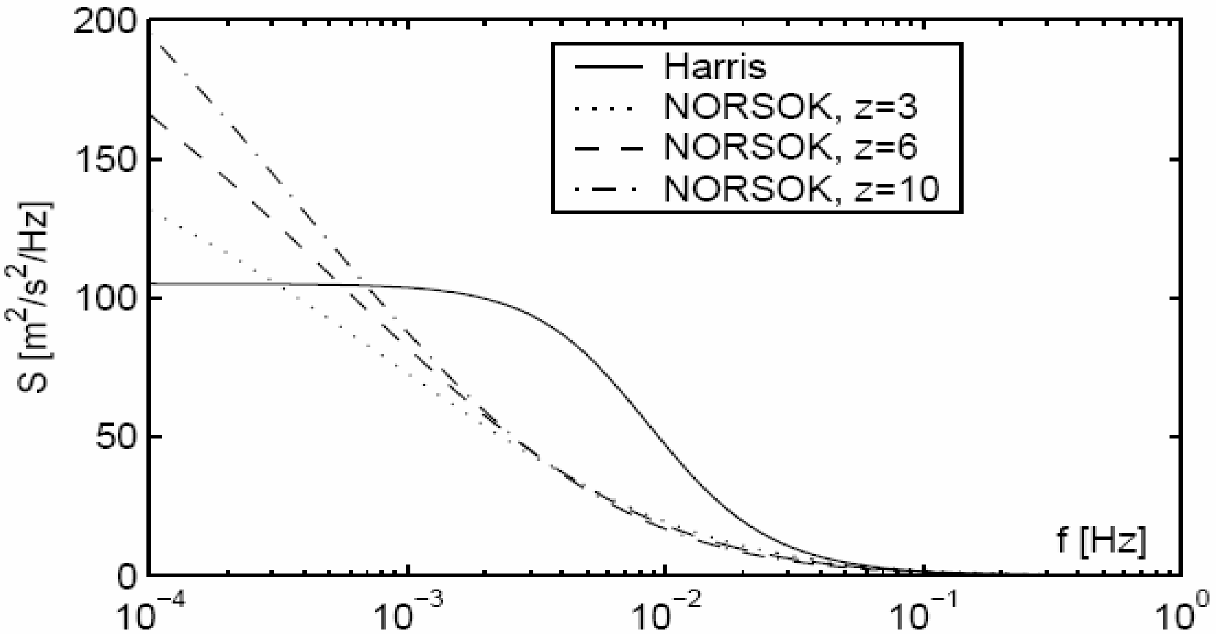
Wind velocity and direction

Sorensen, (2005)



Wind Gust

The wind gust is model as a realization of a stochastic process with a particular spectrum (Sorensen, 2005).



Current

We may divide current modelling in two levels of detail:

- Surface current, for use in modelling of surface vessel response
- Full current profile, for use in modelling of risers, anchor lines etc.

Surface Current

Current velocity magnitude:

$$\dot{V}_c + \mu V_c = w.$$

$$V_{c,\min} \leq V_c \leq V_{c,\max}$$

Current direction:

$$\begin{aligned} \dot{\psi}_c + \mu_2 \psi_c &= w_2 \\ \psi_{c,\min} &\leq \psi_c \leq \psi_{c,\max} \end{aligned}$$

The direction is taken with respect to the (n-frame). NOTE: This direction is the direction to where the current flows : North-East (NE) current flow towards the NE. (This is different from the convention for wind)

References

- Perez, T. (2005) **Ship Motion Control**. Springer.
- Ochi, M. (2005) **Ocean Waves: the stochastic approach**. Cambridge University Press.
- Sorensen, A.J. (2005) “***Marine Cybernetics***.” Lecture notes, Dept. of Marine Technology NTNU, NORWAY.