Manoeuvring Models

(Module 4)

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Vectorial Representation for Ships

From robotics to ship modeling (Fossen 1991)

It is here assumed that the hydrodynamic coefficients are <u>frequency independent.</u> This will be relaxed later!

Consider the classical robot manipulator model:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{q} = \boldsymbol{\tau}$$

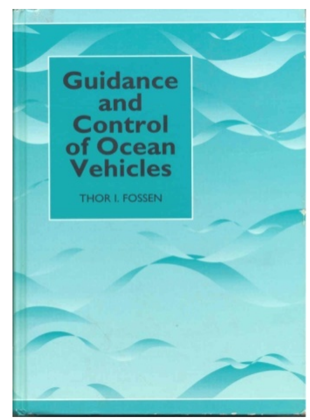
- **q** is a vector of joint angles
- $\boldsymbol{\tau}$ is a vector of torque
- **M** and **C** are the system inertia and Coriolis matrices

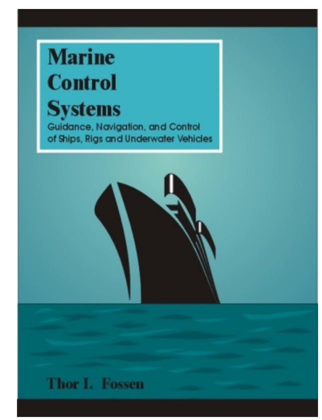
This model structure can be used as foundation to write the 6 DOF marine vessel equations of motion in a compact *vectorial* setting (**Fossen 1994, 2002**):

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}$$

- body velocities: $\boldsymbol{\nu} = [u,v,w,p,q,r]^T$
- position and Euler angles: $\boldsymbol{\eta} = [x,y,z,\phi,\theta,\psi]^T$
- **M, C** and **D** denote the system inertia, Coriolis and damping matrices
- **g** is a vector of gravitational and buoyancy forces and moments





Rigid-Body Equations of Motion

Newtonian Formulation (Body Frame)

$$\mathbf{M}_{RB}\dot{\boldsymbol{\nu}} + \mathbf{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}_{RB}$$

where

$\boldsymbol{M}_{RB}$ rigid-body system inertia matrix
$\boldsymbol{C}_{RB}$ rigid-body Coriolis/centripetal matrix

$$\mathbf{M}_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & -I_{xy} & -I_{xz} \\ mz_g & 0 & -mx_g & -I_{yx} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix}$$

Rigid-body system inertia matrix

See **Fossen (1994, 2002)** for parameterizations of $\boldsymbol{C}_{RB}$

The generalized forces on a floating vessel are <u>superpositioned</u>:

$$\boldsymbol{\tau}_{RB} = \boldsymbol{\tau}_H + \boldsymbol{\tau}_{wave} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{current} + \boldsymbol{\tau}_{control}$$

Hydrodynamic radiation-induced forces + viscous damping

Radiation-Induced Hydrodyn. Forces

- Forces on the body when the body is forced to oscillate with the wave excitation frequency and there are no incident waves (Faltinsen 1990):

  (1) *Added mass* due to the inertia of the surrounding fluid

  (2) Radiation-induced (linear) *potential damping* due to the energy carried away by generated surface waves

  (3) Restoring forces due to *Archimedes* (weight and buoyancy)

$$\boldsymbol{\tau}_R = \underbrace{-\mathbf{M}_A\dot{\boldsymbol{\nu}} - \mathbf{C}_A(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{added mass}} \underbrace{- \mathbf{D}_P(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{potential damping}} \underbrace{- \mathbf{g}(\boldsymbol{\eta})}_{\text{restoring forces}}$$

"hydrodynamic mass-damper-spring"

Faltinsen (1990). *Sea Loads on Ships and Offshore Structures*, Cambridge.

Added Mass and Inertia

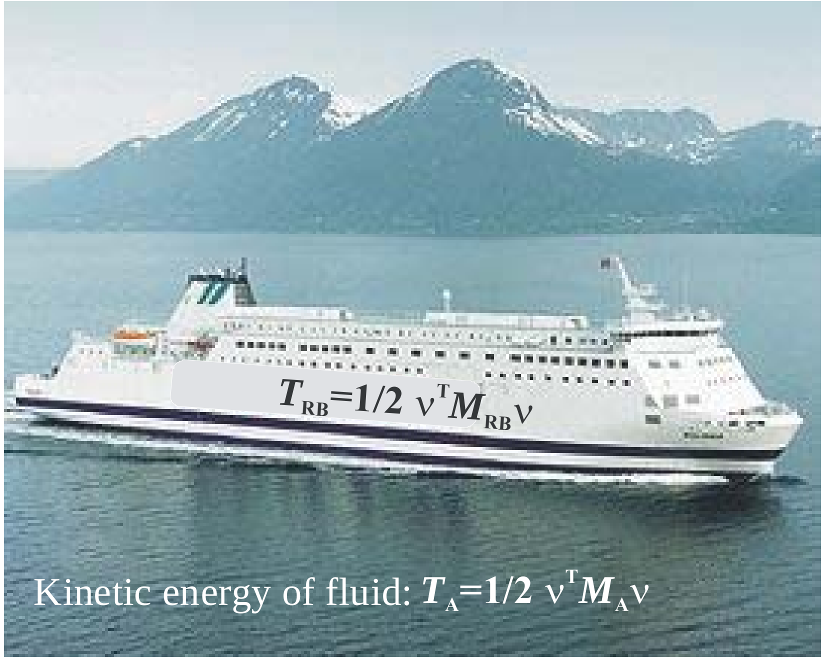
- Fluid Kinetic Energy
- The concept of fluid kinetic energy:

$$T_A = \frac{1}{2}\boldsymbol{\nu}^\top \mathbf{M}_A \boldsymbol{\nu}$$

- can be used to derive the added mass terms.

$$\mathbf{M}_A = -\begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix}$$

- Any motion of the vessel will induce a motion in the otherwise stationary fluid. In order to allow the vessel to pass through the fluid, it must move aside and then close behind the vessel.
- Consequently, the fluid motion possesses kinetic energy that it would lack otherwise (**Lamb 1932**).



6 DOF Body-Fixed Representation for Added Mass (Includes Coriolis/Centripetal Terms due to Added Mass)

$$
\begin{aligned}
X_A &= X_{\dot u}\dot u + X_{\dot w}(\dot w + uq) + X_{\dot q}\dot q + Z_{\dot w}wq + Z_{\dot q}q^2 \\
&\quad + X_{\dot v}\dot v + X_{\dot p}\dot p + X_{\dot r}\dot r - Y_{\dot v}vr - Y_{\dot p}rp - Y_{\dot r}r^2 \\
&\quad - X_{\dot v}ur - Y_{\dot w}wr \\
&\quad + Y_{\dot w}vq + Z_{\dot p}pq - (Y_{\dot q} - Z_{\dot r})qr \\
Y_A &= X_{\dot v}\dot u + Y_{\dot w}\dot w + Y_{\dot q}\dot q \\
&\quad + Y_{\dot v}\dot v + Y_{\dot p}\dot p + Y_{\dot r}\dot r + X_{\dot v}vr - Y_{\dot w}vp + X_{\dot r}r^2 + (X_{\dot p} - Z_{\dot r})rp - Z_{\dot p}p^2 \\
&\quad - X_{\dot w}(up - wr) + X_{\dot u}ur - Z_{\dot w}wp \\
&\quad - Z_{\dot q}pq + X_{\dot q}qr \\
Z_A &= X_{\dot w}(\dot u - wq) + Z_{\dot w}\dot w + Z_{\dot q}\dot q - X_{\dot u}uq - X_{\dot q}q^2 \\
&\quad + Y_{\dot w}\dot v + Z_{\dot p}\dot p + Z_{\dot r}\dot r + Y_{\dot v}vp + Y_{\dot r}rp + Y_{\dot p}p^2 \\
&\quad + X_{\dot v}up + Y_{\dot w}wp \\
&\quad - X_{\dot v}vq - (X_{\dot p} - Y_{\dot q})pq - X_{\dot r}qr \\
K_A &= X_{\dot p}\dot u + Z_{\dot p}\dot w + K_{\dot q}\dot q - X_{\dot v}wu + X_{\dot r}uq - Y_{\dot w}w^2 - (Y_{\dot q} - Z_{\dot r})wq + M_{\dot r}q^2 \\
&\quad + Y_{\dot p}\dot v + K_{\dot p}\dot p + K_{\dot r}\dot r + Y_{\dot w}v^2 - (Y_{\dot q} - Z_{\dot r})vr + Z_{\dot p}vp - M_{\dot r}r^2 - K_{\dot q}rp \\
&\quad + X_{\dot w}uv - (Y_{\dot v} - Z_{\dot w})vw - (Y_{\dot r} + Z_{\dot q})wr - Y_{\dot p}wp - X_{\dot q}ur \\
&\quad + (Y_{\dot r} + Z_{\dot q})vq + K_{\dot r}pq - (M_{\dot q} - N_{\dot r})qr \\
M_A &= X_{\dot q}(\dot u + wq) + Z_{\dot q}(\dot w - uq) + M_{\dot q}\dot q - X_{\dot w}(u^2 - w^2) - (Z_{\dot w} - X_{\dot u})wu \\
&\quad + Y_{\dot q}\dot v + K_{\dot q}\dot p + M_{\dot r}\dot r + Y_{\dot p}vr - Y_{\dot r}vp - K_{\dot r}(p^2 - r^2) + (K_{\dot p} - N_{\dot r})rp \\
&\quad - Y_{\dot w}uv + X_{\dot v}vw - (X_{\dot r} + Z_{\dot p})(up - wr) + (X_{\dot p} - Z_{\dot r})(wp + ur) \\
&\quad - M_{\dot r}pq + K_{\dot q}qr \\
N_A &= X_{\dot r}\dot u + Z_{\dot r}\dot w + M_{\dot r}\dot q + X_{\dot v}u^2 + Y_{\dot w}wu - (X_{\dot p} - Y_{\dot q})uq - Z_{\dot p}wq - K_{\dot q}q^2 \\
&\quad + Y_{\dot r}\dot v + K_{\dot r}\dot p + N_{\dot r}\dot r - X_{\dot v}v^2 - X_{\dot r}vr - (X_{\dot p} - Y_{\dot q})vp + M_{\dot r}rp + K_{\dot q}p^2 \\
&\quad - (X_{\dot u} - Y_{\dot v})uv - X_{\dot w}vw + (X_{\dot q} + Y_{\dot p})up + Y_{\dot r}ur + Z_{\dot q}wp \\
&\quad - (X_{\dot q} + Y_{\dot p})vq - (K_{\dot p} - M_{\dot q})pq - K_{\dot r}qr
\end{aligned}
$$

Kirchhoff's Equations (1869)

$$T = \tfrac{1}{2}\mathbf{v}^\top \mathbf{M}_A \mathbf{v}$$

kinetic energy due to the fluid

$$
\begin{aligned}
\frac{d}{dt}\left(\frac{\partial T}{\partial \mathbf{v}_1}\right) + \mathbf{S}(\mathbf{v}_2)\frac{\partial T}{\partial \mathbf{v}_1} &= \boldsymbol{\tau}_1 \\
\frac{d}{dt}\left(\frac{\partial T}{\partial \mathbf{v}_2}\right) + \mathbf{S}(\mathbf{v}_2)\frac{\partial T}{\partial \mathbf{v}_2} + \mathbf{S}(\mathbf{v}_1)\frac{\partial T}{\partial \mathbf{v}_1} &= \boldsymbol{\tau}_2
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial T_A}{\partial u} &= r\frac{\partial T_A}{\partial v} - q\frac{\partial T_A}{\partial w} - X_A \\
\frac{d}{dt}\frac{\partial T_A}{\partial v} &= p\frac{\partial T_A}{\partial w} - r\frac{\partial T_A}{\partial u} - Y_A \\
\frac{d}{dt}\frac{\partial T_A}{\partial w} &= q\frac{\partial T_A}{\partial u} - p\frac{\partial T_A}{\partial v} - Z_A \\
\frac{d}{dt}\frac{\partial T_A}{\partial p} &= w\frac{\partial T_A}{\partial v} - v\frac{\partial T_A}{\partial w} + r\frac{\partial T_A}{\partial q} - q\frac{\partial T_A}{\partial r} e - K_A \\
\frac{d}{dt}\frac{\partial T_A}{\partial q} &= u\frac{\partial T_A}{\partial w} - w\frac{\partial T_A}{\partial u} + p\frac{\partial T_A}{\partial r} - r\frac{\partial T_A}{\partial p} - M_A \\
\frac{d}{dt}\frac{\partial T_A}{\partial r} &= v\frac{\partial T_A}{\partial u} - u\frac{\partial T_A}{\partial v} + q\frac{\partial T_A}{\partial p} - p\frac{\partial T_A}{\partial q} - N_A
\end{aligned}
$$

M_A $\quad$ $C_A(\nu)$

Viscous Hydrodynamic Damping

- In addition to potential damping we have to include other dissipative viscous terms like *skin friction, wave drift damping etc*:

$$\boldsymbol{\tau}_D = -\underbrace{\mathbf{D}_S(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{skin friction}} - \underbrace{\mathbf{D}_W(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{wave drift damping}} - \underbrace{\mathbf{D}_M(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{damping due to vortex shedding}}$$

- Total *hydrodynamic damping matrix*:

$$\mathbf{D}(\boldsymbol{\nu}) := \mathbf{D}_P(\boldsymbol{\nu}) + \mathbf{D}_S(\boldsymbol{\nu}) + \mathbf{D}_W(\boldsymbol{\nu}) + \mathbf{D}_M(\boldsymbol{\nu})$$

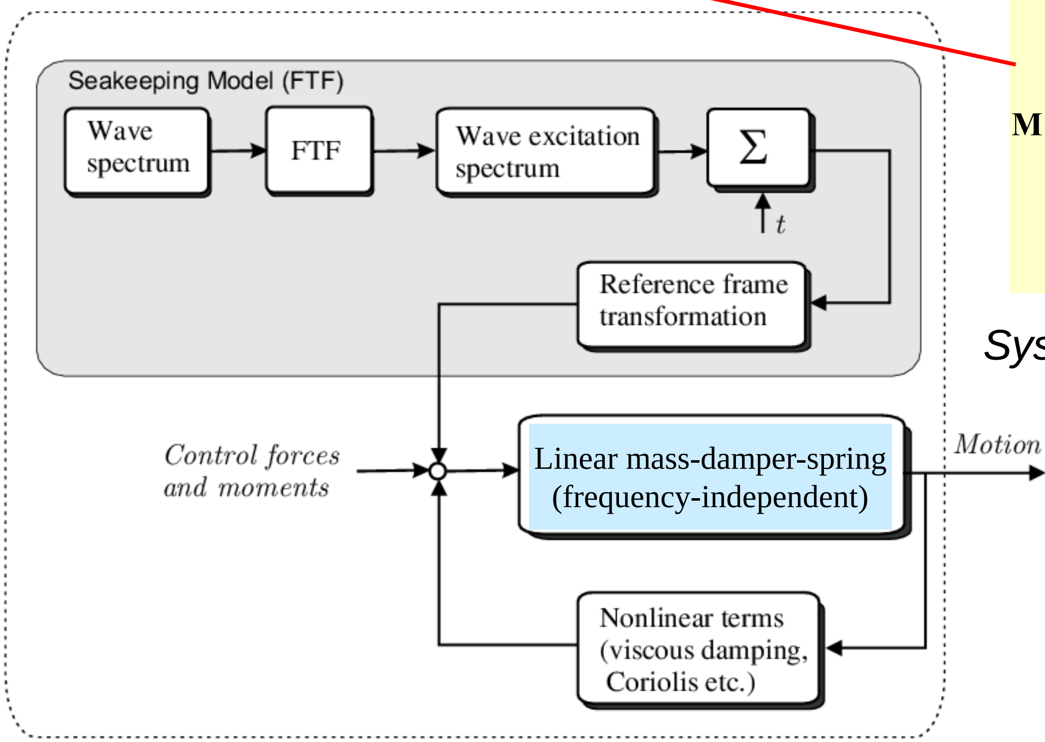
- The hydrodynamic forces and moments $\boldsymbol{\tau}_H$ can be now be written as the sum of $\boldsymbol{\tau}_R$ and $\boldsymbol{\tau}_D$:

$$\boldsymbol{\tau}_H = -\mathbf{M}_A\dot{\boldsymbol{\nu}} - \mathbf{C}_A(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{g}(\boldsymbol{\eta})$$

Equations of Motion

The resulting model is (frequency-independent coefficients):

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{wave} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{current} + \boldsymbol{\tau}_{control}$$



$$\mathbf{M} = \begin{bmatrix} m - X_{\dot{u}} & -X_{\dot{v}} & -X_{\dot{w}} & -X_{\dot{p}} & mz_g - X_{\dot{q}} & -my_g - X_{\dot{r}} \\ -X_{\dot{v}} & m - Y_{\dot{v}} & -Y_{\dot{w}} & -mz_g - Y_{\dot{p}} & -Y_{\dot{q}} & mx_g - Y_{\dot{r}} \\ -X_{\dot{w}} & -Y_{\dot{w}} & m - Z_{\dot{w}} & my_g - Z_{\dot{p}} & -mx_g - Z_{\dot{q}} & -Z_{\dot{r}} \\ -X_{\dot{p}} & -mz_g - Y_{\dot{p}} & my_g - Z_{\dot{p}} & I_x - K_{\dot{p}} & -I_{xy} - K_{\dot{q}} & -I_{zx} - K_{\dot{r}} \\ mz_g - X_{\dot{q}} & -Y_{\dot{q}} & -mx_g - Z_{\dot{q}} & -I_{xy} - K_{\dot{q}} & I_y - M_{\dot{q}} & -I_{yz} - M_{\dot{r}} \\ -my_g - X_{\dot{r}} & mx_g - Y_{\dot{r}} & -Z_{\dot{r}} & -I_{zx} - K_{\dot{r}} & -I_{yz} - M_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix}$$

System inertia matrix including added mass

$$\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A$$

$$\mathbf{C}(\boldsymbol{\nu}) = \mathbf{C}_{RB}(\boldsymbol{\nu}) + \mathbf{C}_A(\boldsymbol{\nu})$$

Manoeuvring Hydrodynamics

In classical manoeuvring theory, the forces are modelled at a general non-linear function:

$$\mathbf{M}\dot{\mathbf{v}} = \mathbf{f}(\dot{\mathbf{v}}, \mathbf{v}, \boldsymbol{\eta}) + \boldsymbol{\tau}$$

A particular affine parameterization is then used, and the coefficients are estimated linear regression from the data.

The disadvantage of this model representation to a energy-based (Lagrangian) approach is that model reduction, symmetry/skew-symmetry properties, positive matrices, etc. are difficult to exploit in simulation and control design.

This model can, however, be related to the Lagrangian model: as shown by Ross et al. 2007:

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}$$

Parameterisations

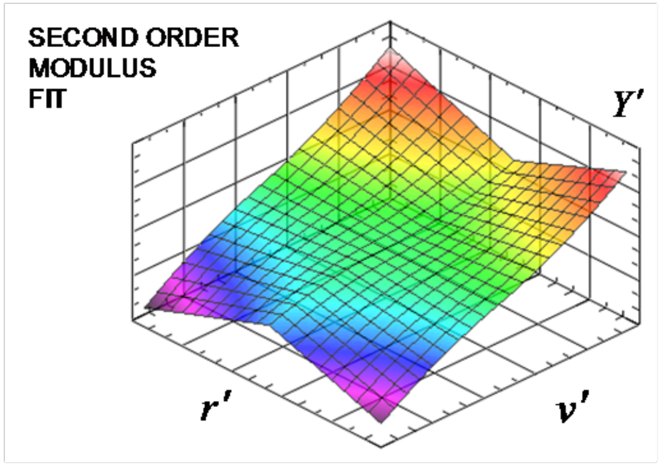
Two types of parameterisations for the hydrodynamic forces are generally used in classical manoeuvring theory:

- Truncated Taylor-series expansions:
  - Davison and Shiff (1946): 1st-order (linear) terms.
  - Abkowitz (1964): odd terms up to 3rd order.
- 2nd -order modulus
  - Fedyaevsky and Sobolev (1963)
  - Norrbin (1970)

Parameterisations

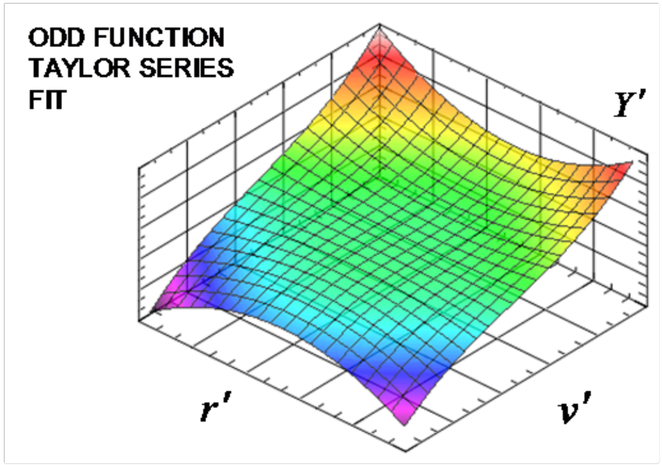
- 2nd -order modulus

$$Y' = Y'_v v' + Y'_r r' + Y'_{v|v|} v'|v'| + Y'_{v|r|} v'|r'|$$
$$+ Y'_{|v|r} |v'|r' + Y'_{r|r|} r'|r'|$$
$$N' = N'_v v' + N'_r r' + N'_{v|v|} v'|v'| + N'_{v|r|} v'|r'|$$
$$+ N'_{|v|r} |v'|r' + N'_{r|r|} r'|r'|$$



- Taylor-series

$$Y' = Y'_v v' + Y'_r r' + Y'_{vvv} v'^3 + Y'_{vvr} v'^2 r'$$
$$+ Y'_{vrr} v' r'^2 + Y'_{rrr} r'^3$$
$$N' = N'_v v' + N'_r r' + N'_{vvv} v'^3 + N'_{vvr} v'^2 r'$$
$$+ N'_{vrr} v' r'^2 + N'_{rrr} r'^3$$



Parameterisations

As commented by *Clarke (2003),*

- Taylor expansions give rise to a smooth representation of the forces, but have no physical meaning.

- 2^{nd-}order modulus expansions represent well the hydrodynamic forces at angles of incidence: <u>cross-flow drag</u>.

Taylor-Series Expansions

$$\boldsymbol{\tau}_{hyd} = \mathbf{f}_{hyd}(\mathbf{x}) + \frac{\partial \mathbf{f}_{hyd}}{\partial \mathbf{x}}(\mathbf{x} - \bar{\mathbf{x}}) + \frac{\partial^2 \mathbf{f}_{hyd}}{\partial \mathbf{x}^2}(\mathbf{x} - \bar{\mathbf{x}})^2 + \ldots$$

$$\mathbf{x} = \begin{bmatrix} \dot{\mathbf{v}} & \mathbf{v} & \boldsymbol{\eta} \end{bmatrix}^T$$

Where the partial derivatives are taken at an equilibrium:

$$\bar{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & \bar{\mathbf{v}} & \mathbf{0} \end{bmatrix}^T \qquad \bar{\mathbf{v}} = \begin{bmatrix} U & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Model of Abkowitz (1964)

$$\begin{aligned}\mathcal{X} &= \mathcal{X}_0 + \mathcal{X}_u \dot{u} + \mathcal{X}_u \Delta u + \mathcal{X}_{uu} \Delta u^2 + \mathcal{X}_{uuu} \Delta u^3 + \mathcal{X}_{vv} v^2 + \mathcal{X}_{rr} r^2 \\ &+ \mathcal{X}_{\delta\delta}\delta^2 + \mathcal{X}_{rv} rv + \mathcal{X}_{r\delta} r\delta + \mathcal{X}_{v\delta} v\delta + \mathcal{X}_{vvu} v^2 \Delta u + \mathcal{X}_{rru} r^2 \Delta u \\ &+ \mathcal{X}_{\delta\delta u}\delta^2 \Delta u + \mathcal{X}_{rvu} rv\Delta u + \mathcal{X}_{r\delta u} r + \delta\Delta u + \mathcal{X}_{v\delta u} v\Delta u \\ &+ (1-t)T + \mathcal{X}_{ext}\end{aligned}$$

$$\begin{aligned}\mathcal{Y} &= \mathcal{Y}_0 + \mathcal{Y}_u \Delta u + \mathcal{Y}_{uu} \Delta u^2 + \mathcal{Y}_r r + \mathcal{Y}_v v + \mathcal{Y}_{\dot{r}} \dot{r} + \mathcal{Y}_{\dot{v}} \dot{v} + \mathcal{Y}_\delta \delta \\ &+ \mathcal{Y}_{rrr} r^3 + \mathcal{Y}_{vvv} v^3 + \mathcal{Y}_{\delta\delta\delta}\delta^3 + \mathcal{Y}_{rr\delta} r^2 \delta + \mathcal{Y}_{\delta\delta r}\delta^2 r + \mathcal{Y}_{rrv} r^2 v \\ &+ \mathcal{Y}_{vvr} v^2 r + \mathcal{Y}_{\delta\delta v}\delta^2 v + \mathcal{Y}_{vv\delta} v^2\delta + \mathcal{Y}_{\delta vr}\delta vr + \mathcal{Y}_{vu} v\Delta u + \mathcal{Y}_{ru} r\Delta u \\ &+ \mathcal{Y}_{vuu} v\Delta u^2 + \mathcal{Y}_{ruu} r\Delta u^2 + \mathcal{Y}_{\delta u}\delta\Delta u + \mathcal{Y}_{\delta uu}\delta\Delta u^2 + \mathcal{Y}_{ext}\end{aligned}$$

$$\begin{aligned}\mathcal{N} &= \mathcal{N}_0 + \mathcal{N}_u \Delta u + \mathcal{N}_{uu} \Delta u^2 + \mathcal{N}_r r + \mathcal{N}_v v + \mathcal{N}_{\dot{r}} \dot{r} + \mathcal{N}_{\dot{v}} \dot{v} + N_\delta \delta \\ &+ \mathcal{N}_{rrr} r^3 + \mathcal{N}_{vvv} v^3 + \mathcal{N}_{\delta\delta\delta}\delta^3 + \mathcal{N}_{rr\delta} r^2 \delta + \mathcal{N}_{\delta\delta r}\delta^2 r + \mathcal{N}_{rrv} r^2 v \\ &+ \mathcal{N}_{vvr} v^2 r + \mathcal{N}_{\delta\delta v}\delta^2 v + \mathcal{N}_{vv\delta} v^2\delta + \mathcal{N}_{\delta vr}\delta vr + \mathcal{N}_{vu} v\Delta u + \mathcal{N}_{ru} r\Delta u \\ &+ \mathcal{N}_{vuu} v\Delta u^2 + \mathcal{N}_{ruu} r\Delta u^2 + \mathcal{N}_{\delta u}\delta\Delta u + \mathcal{N}_{\delta uu}\delta\Delta u^2 + \mathcal{N}_{ext}\end{aligned}$$

The coefficients are called hydrodynamic derivatives.

Many terms are set to zero by exploiting physically properties. If not, there will thousands of coefficients.

Model of Norrbin (1970)

Speed equation:

$$(1 - X''_{\dot u})\dot u = \frac{1}{2}L^{-1}X''_{uu}u^2 + \frac{1}{24}L^{-2}g^{-1}X''_{uuuu}u^4 + g(1-t)T'' + (1 + X''_{vr})vr$$
$$+ L(x''_g + \frac{1}{2}X''_{rr})r^2 + \frac{1}{6}L^{-2}g^{-1}X''_{uvvv}u|v|v^2 + \frac{1}{4}L^{-1}X_{c|c|\delta\delta}|c|c\delta_\epsilon^2$$

Steering equations:

$$(1 - Y''_{\dot v})\dot v = L(Y''_{\dot r} - x''_g)\dot r + (Y''_{ur} - 1)ur + \frac{1}{2}(Lg)^{-1/2}Y''_{uur}u^2 r$$
$$+ L^{-1}Y''_{uv}uv + \frac{1}{2}L^{-3/2}g^{-1/2}Y''_{uuv}u^2 v + \frac{1}{2}L^{-1}Y''_{|v|v}|v|v + \frac{1}{2}LY''_{|r|r}|r|r$$
$$+ Y''_{|v|r}|v|r + Y''_{v|r|}v|r| + \frac{1}{2}L^{-1}Y''_{|c|c\delta}|c|c\delta_\epsilon + k_\gamma g T''$$

$$((k''_z)^2 - N''_{\dot r})\dot r = L^{-1}(N''_{\dot v} - x''_g)\dot v + L^{-1}(N''_{ur} - x''_g)ur$$
$$+ \frac{1}{2}L^{-3/2}g^{-1/2}N''_{uur}u^2 r + L^{-2}N''_{uv}uv + \frac{1}{2}L^{-5/2}g^{-1/2}N''_{uuv}u^2 v$$
$$+ \frac{1}{2}L^{-2}N''_{|v|v}|v|v + \frac{1}{2}N''_{|r|r}|r|r + L^{-1}N''_{|v|r}|v|r$$
$$+ L^{-1}N''_{v|r|}v|r| + \frac{1}{2}L^{-2}N''_{|c|c\delta}|c|c\delta_\epsilon + L^{-1}gk_N T''$$

2nd-Order Modulus

From Blanke and Christiansen (1986):

Sway terms

$$\begin{aligned}\tau^b_{2\text{hyd}} &= Y_{\dot v}\dot v + Y_{\dot r}\dot r + Y_{\dot p}\dot p \\ &+ Y_{|u|v}\,|U|\,v + Y_{ur}Ur + Y_{v|v|}v\,|v| + Y_{v|r|}v\,|r| + Y_{r|v|}r\,|v| \\ &+ Y_{\phi|uv|}\phi\,|Uv| + Y_{\phi|ur|}\phi\,|Ur| + Y_{\phi uu}\phi\,U^2.\end{aligned} \tag{4.46}$$

Roll terms

$$\begin{aligned}\tau^b_{4\text{hyd}} &= K_{\dot v}\dot v + K_{\dot p}\dot p \\ &+ K_{|u|v}\,|U|\,v + K_{ur}Ur + K_{v|v|}v\,|v| + K_{v|r|}v\,|r| + K_{r|v|}r\,|v| \\ &+ K_{\phi|uv|}\phi\,|Uv| + K_{\phi|ur|}\phi\,|Ur| + K_{\phi uu}\phi\,U^2 + K_{|u|p}\,|U|\,p \\ &+ K_{p|p|}p\,|p| + K_p p + K_{\phi\phi\phi}\phi^3 - \rho g \nabla GZ(\phi).\end{aligned} \tag{4.47}$$

Yaw terms

$$\begin{aligned}\tau^b_{6\text{hyd}} &= N_{\dot v}\dot v + N_{\dot r}\dot r \\ &+ N_{|u|v}\,|U|\,v + N_{|u|r}\,|U|\,r + N_{r|r|}r\,|r| + N_{r|v|}r\,|v| \\ &+ N_{\phi|uv|}\phi\,|Uv| + N_{\phi u|r|}\phi\,U\,|r| + N_p p + N_{|p|p}|p|p + N_{|u|p}|U|p \\ &+ N_{\phi u|u|}\phi\,U\,|U|.\end{aligned} \tag{4.48}$$

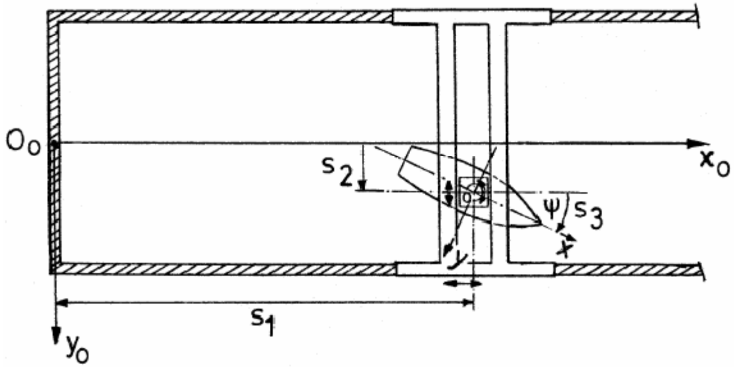
Measurement of Hydrodynamic Derivatives

- Experiments with model tests.
- Full scale sea trials and system identification.
- Theoretical prediction methods.
- Regression analysis results from similar designs.

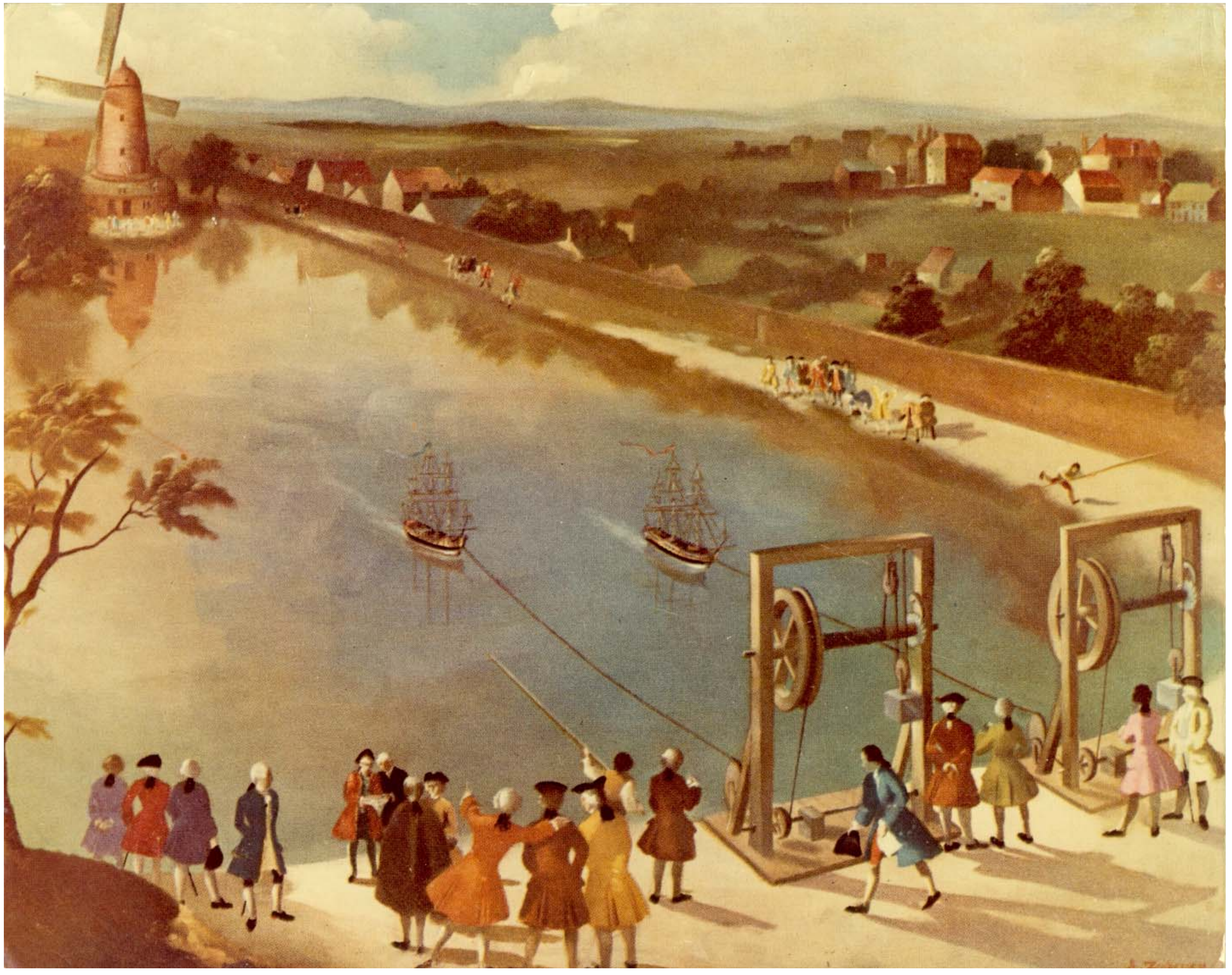
PMM

Model tests that can be performed

- Straight line in a towing tank,
- Rotating arm,
- Planar motion mechanism PMM,
- Oscillator tests,
- Free running (radio controlled).



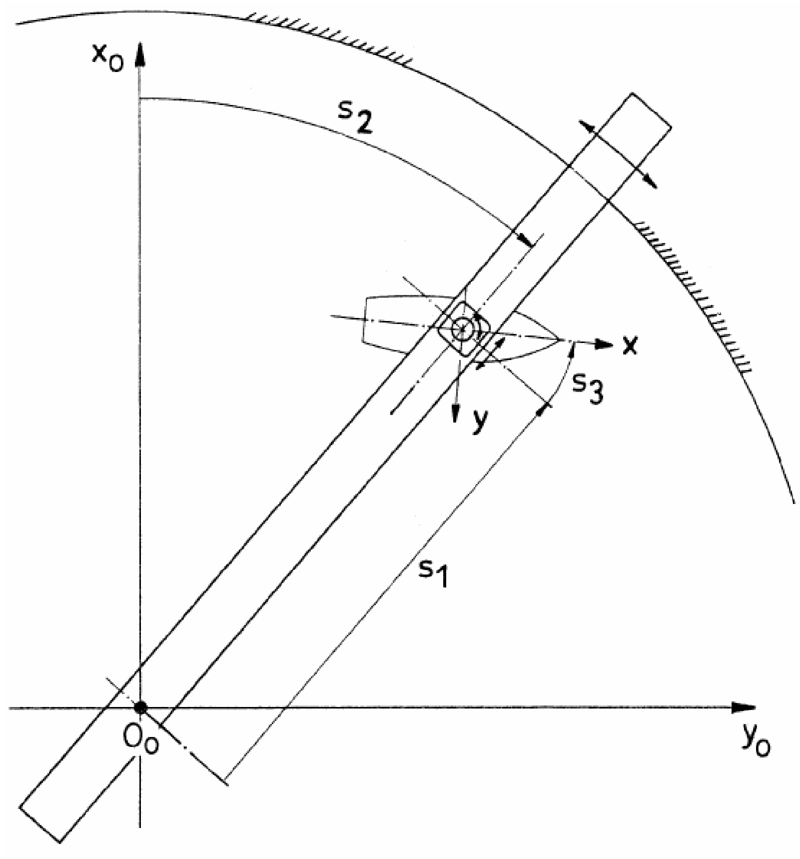
Experimental Methods



Model testing in Peerlesspool in London

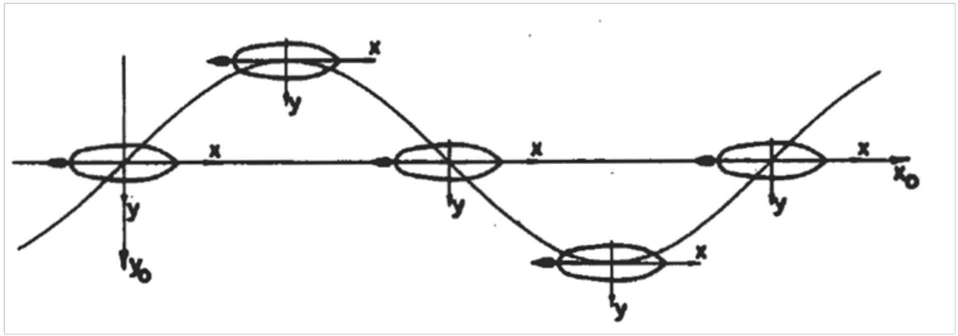
Measurement of Hydrodynamic Derivatives

Rotating arm

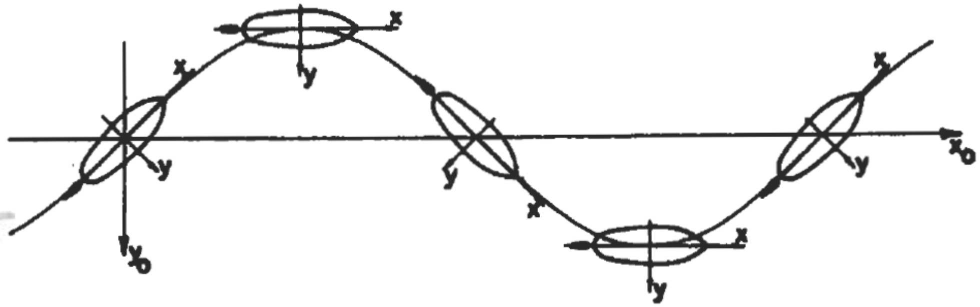


Typical Tests

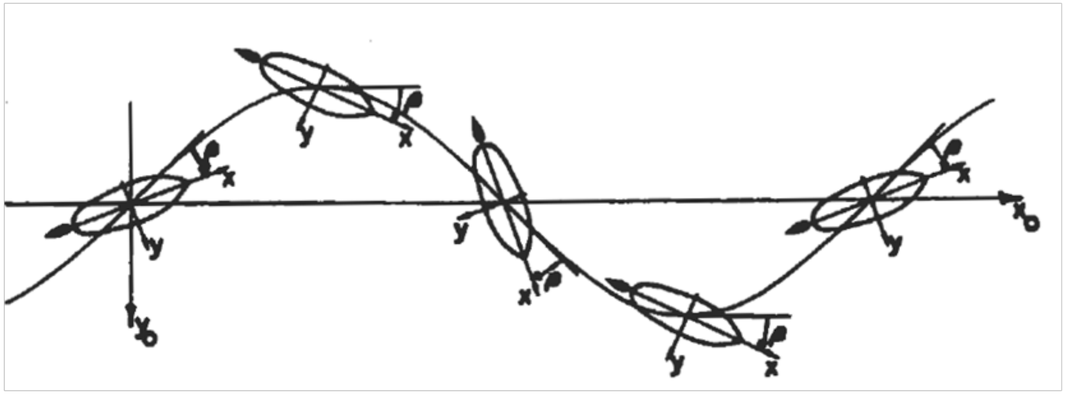
Pure Sway:



Pure yaw:



Drift and yaw:

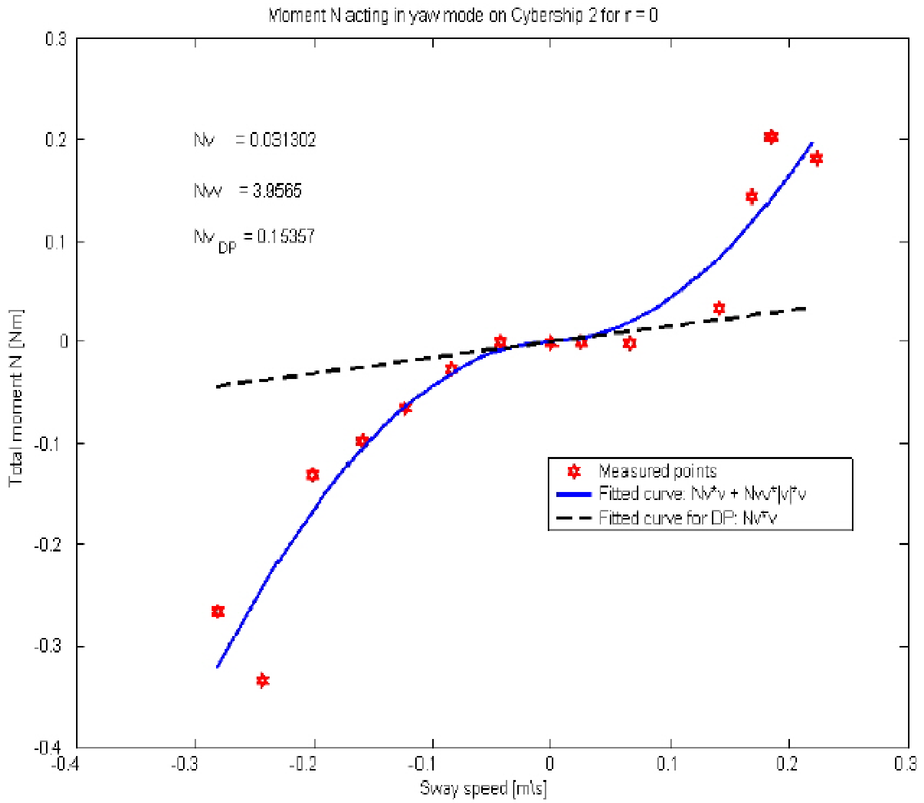


Different tests are used to fit different parts of the model.

Measurement of Hydrodynamic Derivatives

During the model tests, the model is forces to move and forces velocities and accelerations are recorded.

Then the hydrodynamic derivatives are estimated from regression analysis.



A Novel 4 DOF Manoeuvring Model

Ross et. al. (2007) has reassessed the manoeuvring models in the literature, and formulated a novel 4 DOF (surge, sway, roll, yaw) Lagrangian model using first principles and superposition of:

- Potential (added mass)
- Circulation effects: lift and drag
- Effect of roll on circulation effects
- Cross-flow drag.

$$
\begin{aligned}
\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{N}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) &= \boldsymbol{\tau}, \\
\dot{\boldsymbol{\eta}} &= \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}
\end{aligned}
$$

The advantage of the Lagrangian model is its vector representation which is tailor made for energy-based control design (Lyapunov).

Added Mass and Coriollis

The 4 DOF solution of Kirchhoff's equations can be expressed as (Fossen, 2002)

$$\left[X_A \; Y_A \; K_A \; N_A\right]^\top = -\mathbf{M}_A\dot{\boldsymbol{\nu}} - \mathbf{C}_A\left(\boldsymbol{\nu}\right)\boldsymbol{\nu}$$

Added mass

Added mass Coriollis and Centripetal terms

$$\mathbf{C}_A\left(\boldsymbol{\nu}\right) = \begin{bmatrix} 0 & 0 & 0 & Y_{\dot{v}}v + Y_{\dot{p}}p + Y_{\dot{r}}r \\ 0 & 0 & 0 & -X_{\dot{u}}u \\ 0 & 0 & 0 & 0 \\ -Y_{\dot{v}}v - Y_{\dot{p}}p - Y_{\dot{r}}r & X_{\dot{u}}u & 0 & 0 \end{bmatrix}$$

Model of Ross et al. (2007)

Circulation effects (lift and drag), effect of roll on circulation effects and cross-flow drag (modulus representation) are derived in Ross et al. (2007):

$$\begin{aligned}\mathbf{N}(\boldsymbol{\nu})\boldsymbol{\nu} &\triangleq (\mathbf{C}(\boldsymbol{\nu})+\mathbf{D}(\boldsymbol{\nu}))\boldsymbol{\nu} \\ &= \begin{bmatrix} X_N & Y_N & K_N & N_N \end{bmatrix}^\top\end{aligned}$$

where the components are:

$$\begin{aligned}
X_N &= -X_{uu}^L u^2 - X_{uuu}^L u^3 - X_{vv}^L v^2 - X_{rr}^L r^2 - X_{rv}^L rv - X_{uvv}^L uv^2 \\
&\quad -X_{rvu}^L rvu - X_{urr}^L ur^2 - X_{uv\phi\phi}^L uv\phi^2 + Y_{\dot{v}} vr + Y_{\dot{p}} pr + Y_{\dot{r}} r^2 \\
Y_N &= -Y_{uv}^L uv - Y_{ur}^L ur - Y_{uur}^L u^2 r - Y_{uuv}^L u^2 v - Y_{vvv}^L v^3 - Y_{rrr}^L r^3 - Y_{rrv}^L r^2 v \\
&\quad -Y_{vvr}^L v^2 r - Y_{uv\phi\phi}^L uv\phi^2 - Y_{|v|v}|v|v - Y_{|r|v}|r|v - Y_{|v|r}|v|r - Y_{|r|r}|r|r - X_{\dot{u}} ur \\
K_N &= -K_p p - K_{ppp} p^3 - K_{uv}^L uv - K_{ur}^L ur - K_{uur}^L u^2 r - K_{uuv}^L u^2 v - K_{vvv}^L v^3 - K_{rrr}^L r^3 - K_{rrv}^L r^2 v \\
&\quad -K_{vvr}^L v^2 r - K_{uv\phi\phi}^L uv\phi^2 - K_{|v|v}|v|v - K_{|r|v}|r|v - K_{|v|r}|v|r - K_{|r|r}|r|r \\
N_N &= -N_{uv}^L uv - N_{ur}^L ur - N_{uur}^L u^2 r - N_{uuv}^L u^2 v - N_{vvv}^L v^3 - N_{rrr}^L r^3 - N_{rrv}^L r^2 v - N_{vvr}^L v^2 r - N_{uu\phi\phi}^L uv\phi^2 \\
&\quad -N_{|v|v}|v|v - N_{|r|v}|r|v - N_{|v|r}|v|r - N_{|r|r}|r|r - Y_{\dot{p}} pu - Y_{\dot{r}} ru + (X_{\dot{u}} - Y_{\dot{v}})\, uv.
\end{aligned}$$

Manoeuvring Model

Combining all the terms in a matrix for, we obtain the manoeuvring equations in Lagrangian form (Fossen 1994, 2002).

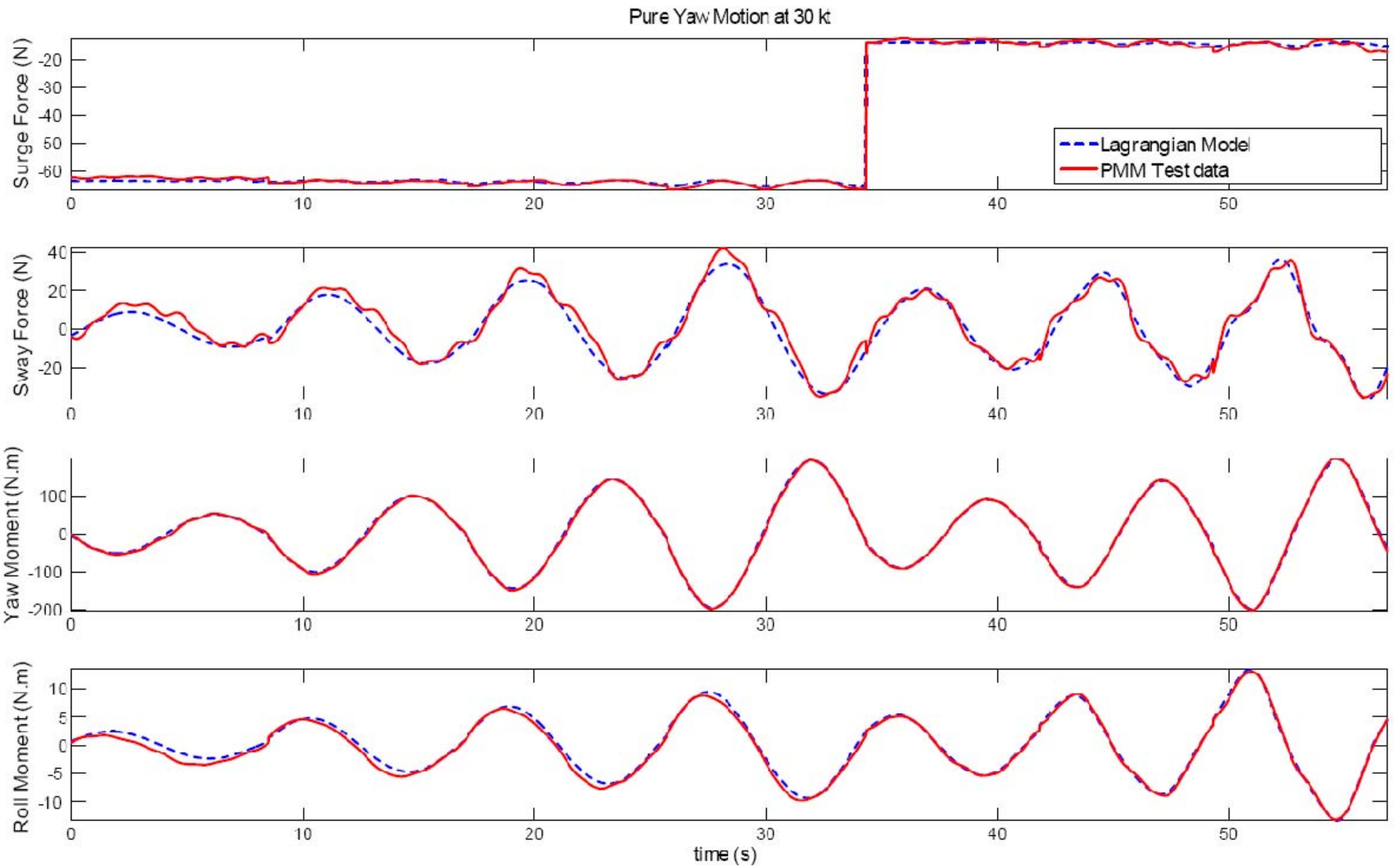
$$
\begin{aligned}
\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{N}\left(\boldsymbol{\nu}\right)\boldsymbol{\nu} + \mathbf{g}\left(\boldsymbol{\eta}\right) &= \boldsymbol{\tau}, \\
\dot{\boldsymbol{\eta}} &= \mathbf{J}\left(\boldsymbol{\eta}\right)\boldsymbol{\nu}
\end{aligned}
$$

Model Validation with PMM Data

To validate the model, Ross et al. (2007) used data of several PMM tests, and perform a regression based on the model structure derived.

Then compared the fit with that of a model fitted by a tank testing facility to the same dataset.

Fitting Using PMM Data @ 30kt



Validation in Full Scale (Perez et al.,2007)

Perez et al. (2007) fitted a simplified model to data recorded on full scale manoeuvres of Austal's Trimaran Hull 260.

The parameters were fitted with data of a 20-20 zig-zag test, and then the model validated with data of a 10-10 zig-zag test.

Simplified Model

The model was simplified according to the that of Blanke (1981). This was done because the excitation signal was not rich enough to estimate all the parameters—the zig-zag test is not designed for system identification!

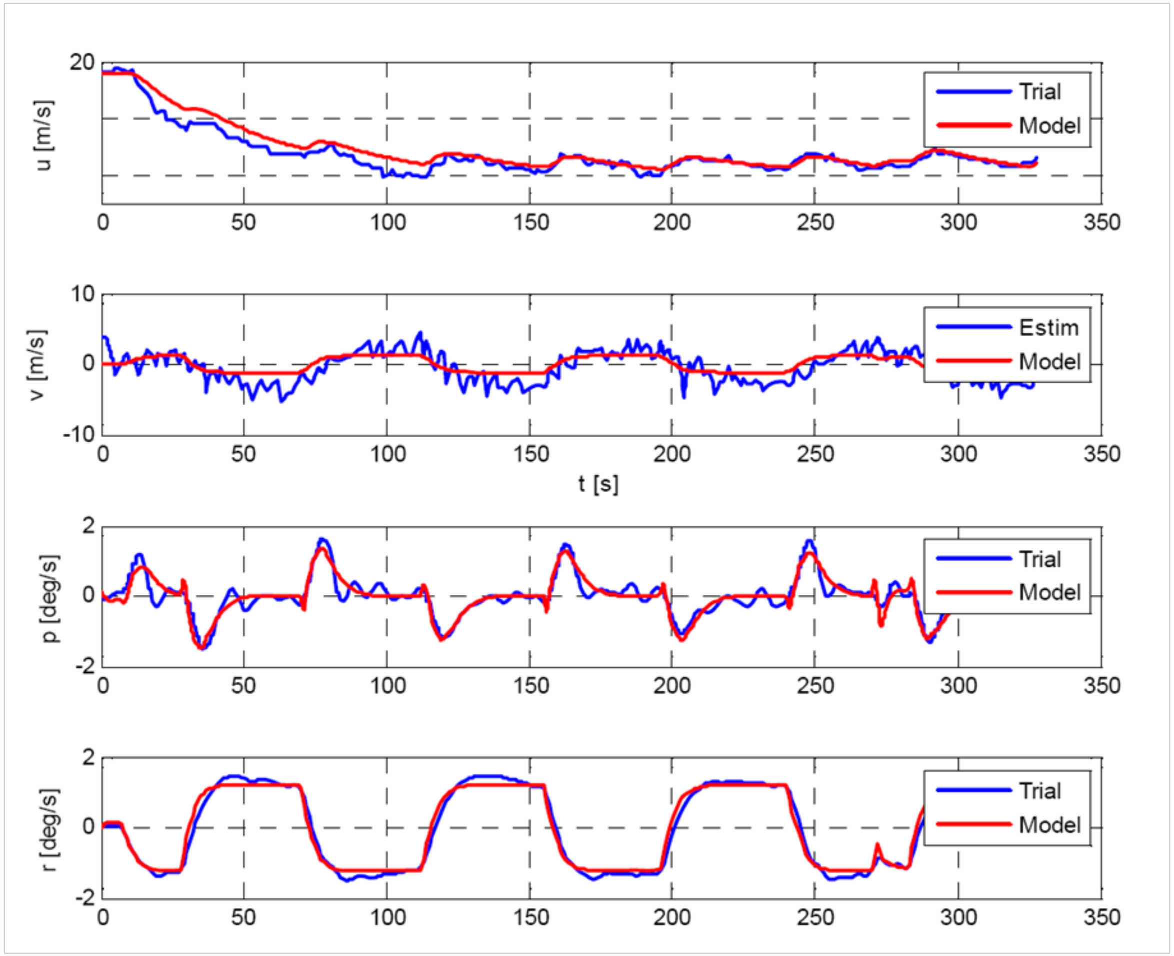
$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\ \mathbf{v}$$

$$\mathbf{M}\ \dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\ \mathbf{v} + \mathbf{D}(\mathbf{v})\ \mathbf{v} + \mathbf{G}(\boldsymbol{\eta}) = \boldsymbol{\tau}$$

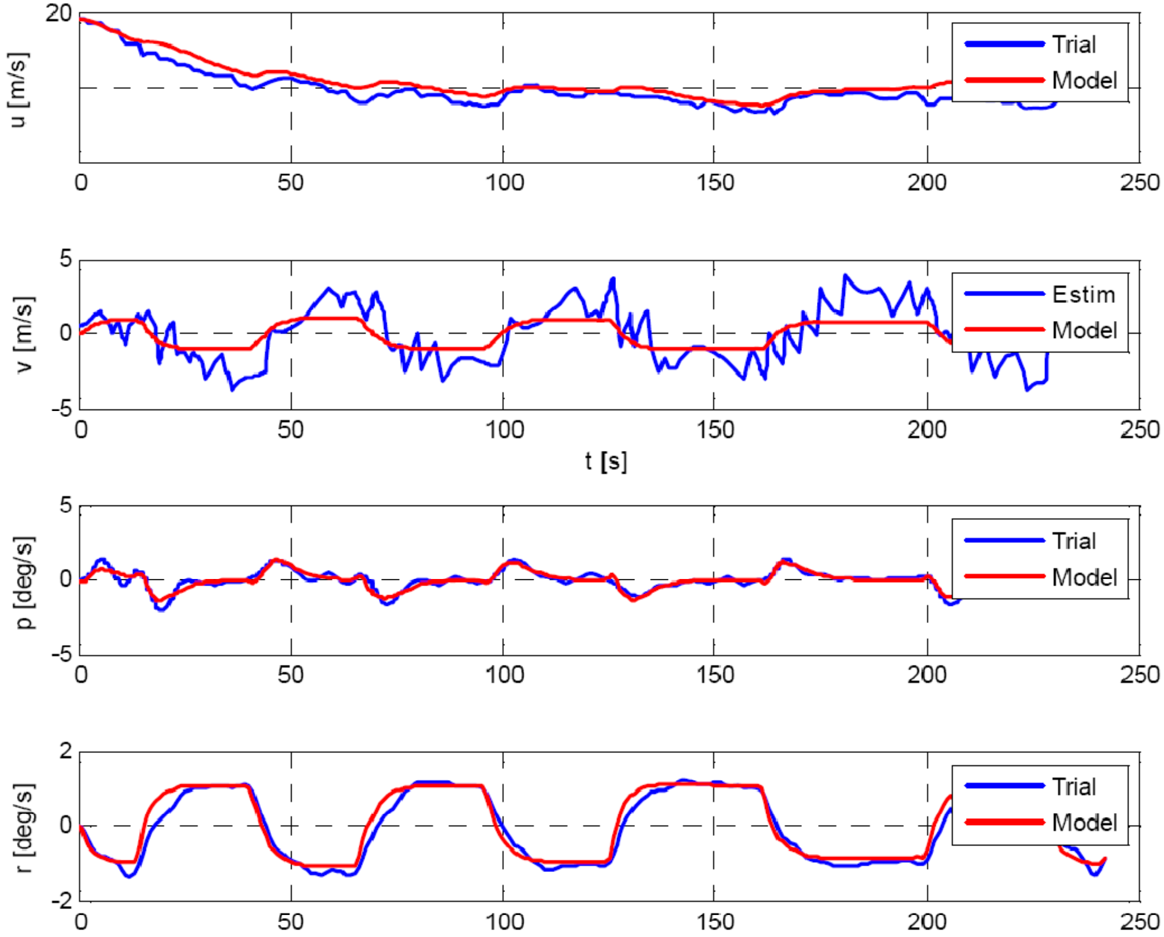
$$\mathbf{D}(\mathbf{v}) = \mathbf{D}_{LD}(\mathbf{v}) + \mathbf{D}_{NL}(\mathbf{v})$$

$$\mathbf{D}_{LD}(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & Xrv\ v \\ 0 & Yuv\ u & 0 & Yur\ u \\ 0 & Kuv\ u & 0 & Kur\ u \\ 0 & Nuv\ u & 0 & Kur\ u \end{bmatrix} \quad \mathbf{D}_{NL}(\mathbf{v}) = \begin{bmatrix} X_{|u|u} & 0 & 0 & 0 \\ 0 & Y_{|v|v}\,|v| + Y_{|r|v}\,|r| & 0 & Y_{|v|r}\,|v| + Y_{|r|r}\,|r| \\ 0 & 0 & K_{|p|p}\,|p| + Y_p & 0 \\ 0 & N_{|v|v}\,|v| + N_{|r|v}\,|r| & 0 & N_{|v|r}\,|v| + N_{|r|r}\,|r| \end{bmatrix}$$

Model Fitting (20-20 ZZ)



Model Validation (10-10 ZZ)



Effects of Currents

In some applications, where positioning is important, the effects of current must be considered:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu}$$

$$(\mathbf{M}_{RB} + \mathbf{M}_{A})\dot{\boldsymbol{\nu}} + \mathbf{C}_{RB}(\boldsymbol{\nu}) + \mathbf{C}_{A}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{D}_{A}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \mathbf{G}(\boldsymbol{\eta}) = \boldsymbol{\tau}$$

$$\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c$$

The current has to effects, which are represented with the velocity of the vessel relative to the current velocity:

- Potential: The Munk moment is incorporated in the added mass Coriollis-Centripetal terms.
- Viscous: eddy making and skin friction. These are incorporated in the cross-flow drag.

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