ETH zürich

Lecture «Robot Dynamics»: Kinematics 1

151-0851-00 V

lecture:	CAB G11	Tuesday 10:15 – 12:00, every week
exercise:	HG E1.2	Wednesday 8:15 – 10:00, according to schedule (about every 2nd week)

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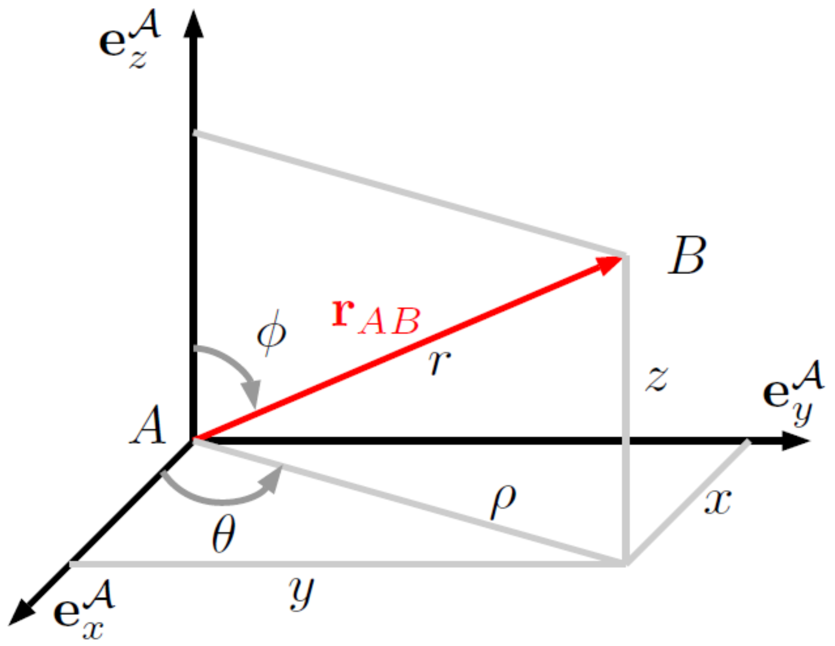
Recapitulation: Vectors, Position, and Vector Calculus

- Builds upon notation of other dynamics classes at ETH and IEEE standards

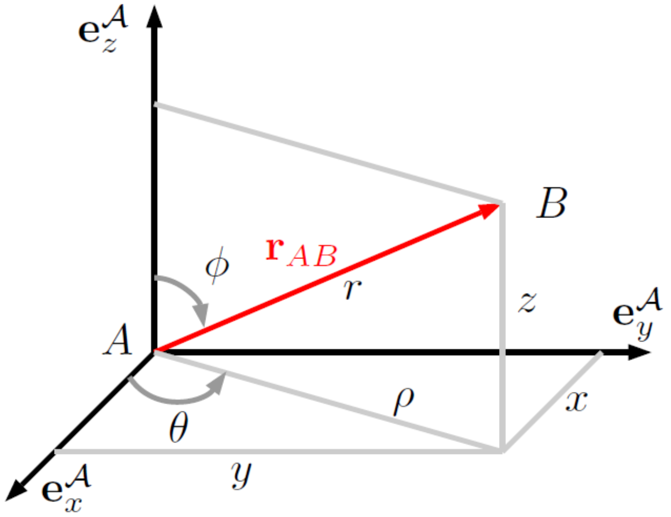
Parameterization of Vectors



- Cartesian coordinates
  - Position vector
- Cylindrical coordinates
  - Position vector
- Spherical coordinates
  - Position vector

Parameterization of Vectors

Example



$$_{\mathcal{A}}\mathbf{r}_{AP} = {}_{\mathcal{A}}\mathbf{r}_{AB} + {}_{\mathcal{A}}\mathbf{r}_{BP}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Differentiation of Representation ⇔ Linear Velocity

- The velocity of point P relative to point B, expressed in frame $\mathcal{A}$ is:

- Question: What is the relationship between the velocity $\dot{\mathbf{r}}$
  and the time derivative of the representation $\dot{\chi}$

Differentiation of Representation ⇔ Linear Velocity

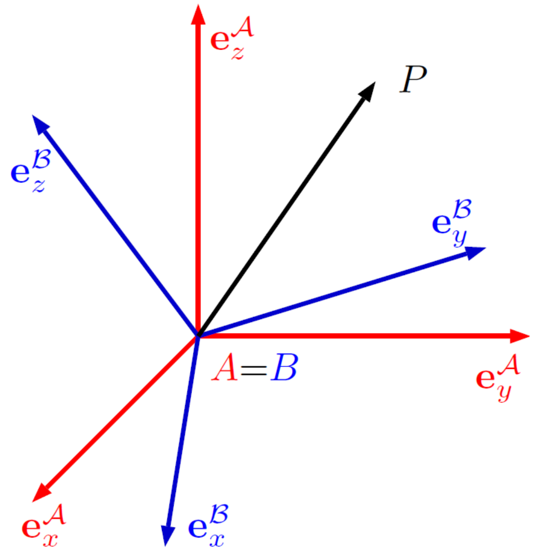
- Cartesian coordinates:

- Cylindrical coordinates:

Rotations

- Position of P with respect to A expressed in $\mathcal{A}$:

- Position of P with respect to A expressed in $\mathcal{B}$:



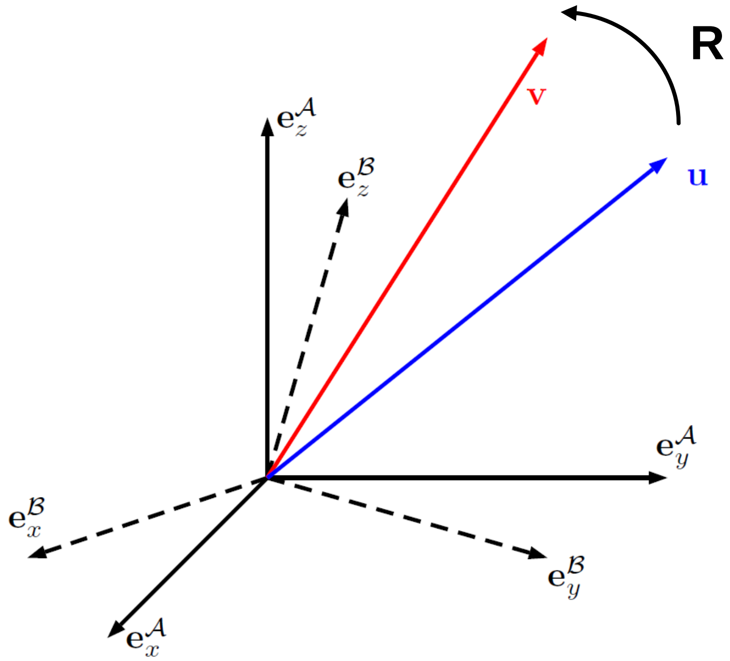
Rotation Matrix

- The rotation matrix transforms vectors expressed in $\mathcal{B}$ to $\mathcal{A}$:

Passive and Active Rotation

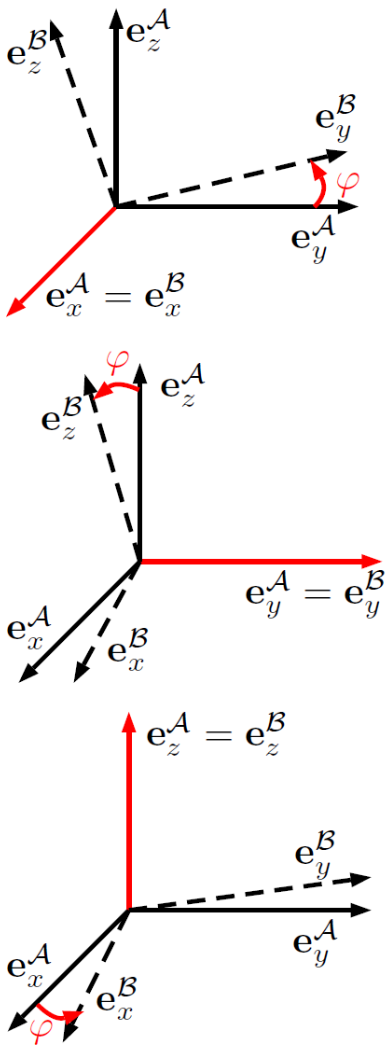
- Passive rotation = mapping of the same vector from frame $\mathcal{B}$ to $\mathcal{A}$

- Active rotation = rotating a vector in the same frame

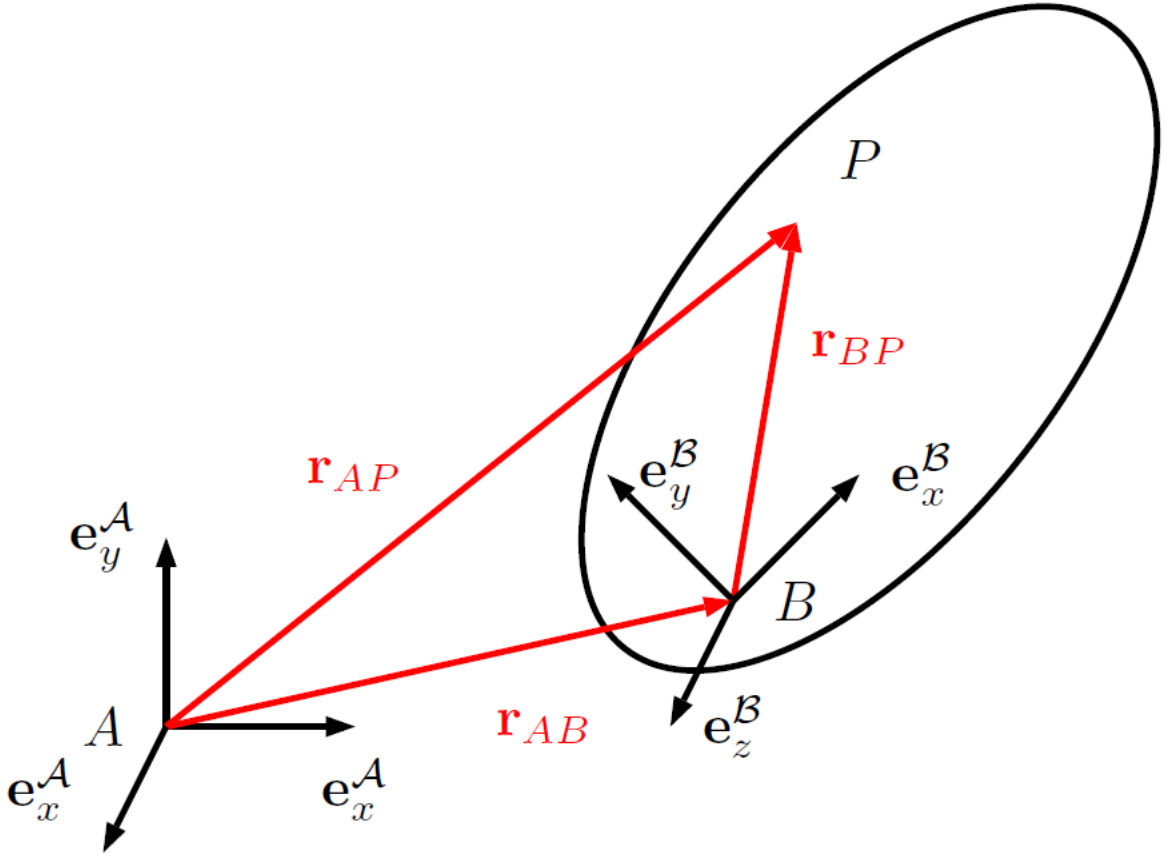


Elementary Rotation

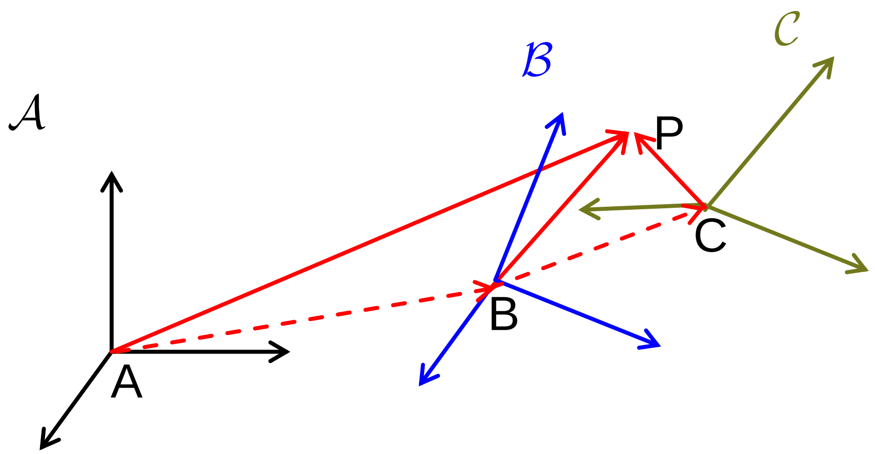
- Find the elementary rotation matrix s.t $_{\mathcal{A}}\mathbf{u} = \mathbf{C}_{\mathcal{AB}} \cdot {}_{\mathcal{B}}\mathbf{u}$



Homogeneous Transformation
Combined Translation and Rotation



Homogeneous Transformations
Consecutive Transformation

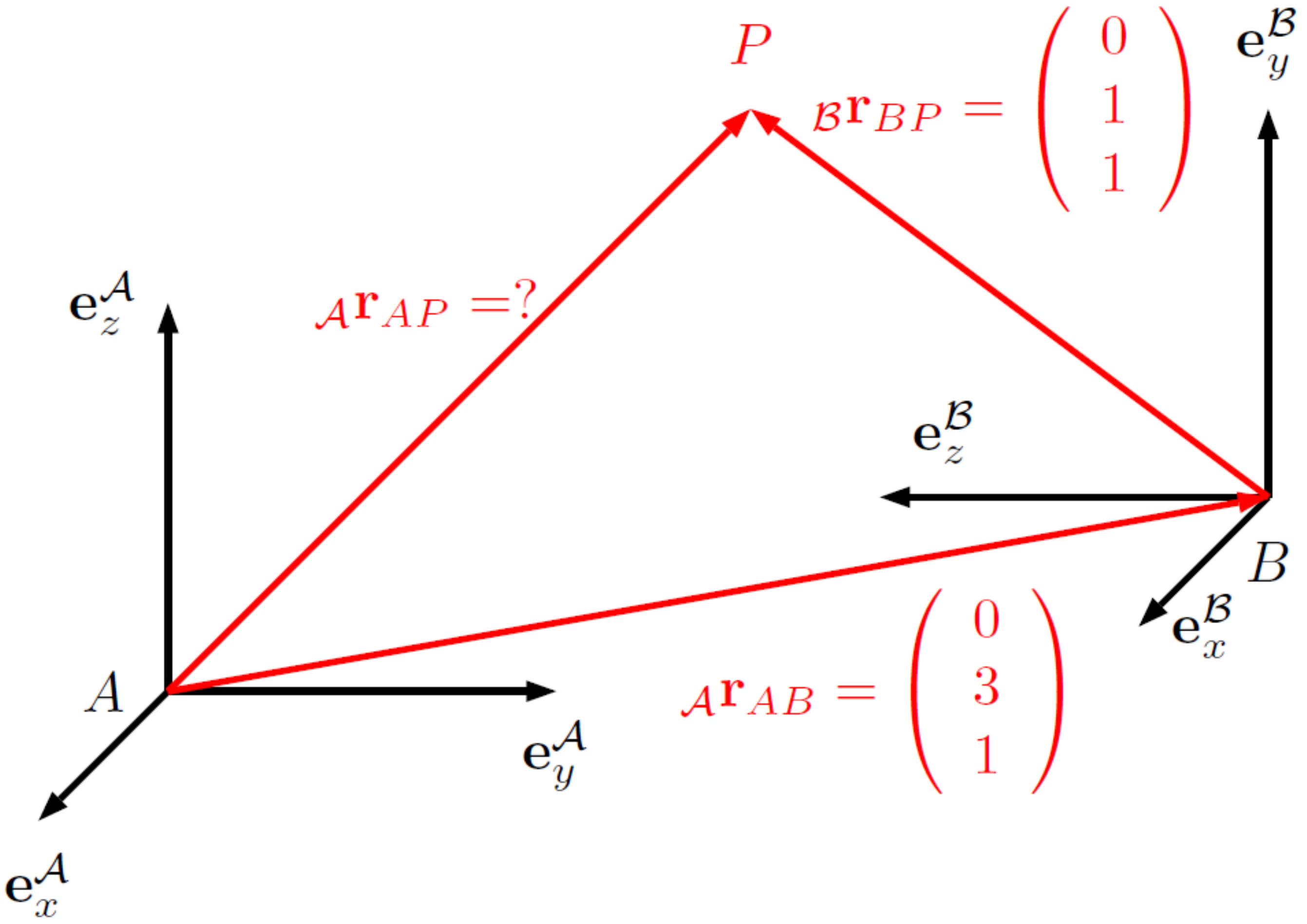


- This allows to transform an arbitrary vector between different reference frames (classical example: mapping of features in camera frame to world frame)

Homogeneous Transformation
Simple Example

- Find the position vector ${}_{\mathcal{A}}\mathbf{r}_{AP}$
  - Find the transformation matrix

  - Find the vector



Angular Velocity

- Angular velocity ${}_{\mathcal{A}}\boldsymbol{\omega}_{\mathcal{AB}}$ describes the relative rotational velocity of $\mathcal{B}$ wrt. $\mathcal{A}$ expressed in frame $\mathcal{A}$
- The relative velocity of $\mathcal{A}$ wrt. $\mathcal{B}$ is:
- Given the rotation matrix $\mathbf{C}_{\mathcal{AB}}(t)$ between two frames, the angular velocity is
- Transformation of angular velocity:
- Addition of relative velocities:

Angular Velocity
Simple Example

- Given the rotation matrix $$\mathbf{C}_{\mathcal{AB}}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha(t)) & \sin(\alpha(t)) \\ 0 & -\sin(\alpha(t)) & \cos(\alpha(t)) \end{bmatrix}$$

  determine ${}_{\mathcal{A}}\boldsymbol{\omega}_{\mathcal{AB}}$

Outlook (next week)
Rotation Parameterization

- Rotation matrix:
  -
  -
- Euler Angles
  -
  -
- Angle Axis
  -
  -
- Quaternions
  -
  -