Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Robotic Systems Lab

Msc. - Exercise Exam

Month Day, Year

Robot Dynamics - Exercise Exam

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Question	Points	Score
Multiple Choice	18	
Kinematics: Example Concrete Pump	8	
Robot Dynamics	3	
Legged Robots	4	
Optimal Airplane Flight Speeds	10	
Dynamic Modes of an Airplane	6	
Steering a helicopter	6	
Modeling and Control of a Hexacopter	12	
Total:	67	

Duration:	90min
Number of pages:	16
Allowed aids:	Calculator Two A4 sheets of personal notes, written on both sides Dictionary for foreign students

Write your name on every page in the box in the footer.

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Cooperation is strictly forbidden.

Please draw your answer in the respective figure if required to do so in the respective questions.

Name: ______________________________

Student number: ______________________________

Signature: ______________________________

A. Multiple Choice 18 pts

Decide whether the following statements are true or false. Cross the checkbox on the corresponding answer. You will be credited 1 point for a correct answer, while 1 pt will be subtracted from the total, if your answer is wrong.

(1) The number of generalized coordinates of a fixed-based robot is unique. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

(2) The choice of generalized coordinates for an articulated robot arm is unique. ◯ Correct ◯ Wrong [1 pt]

Solution: False

(3) For given generalized coordinates and velocities of a floating base system, the linear and angular velocity of the end-effector is always unique. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

(4) Inverse differential kinematics of a serial link robot can always be solved analytically. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

(5) The aerodynamic performance of an MAV glider is different from a manned glider, since the respective Reynolds numbers are totally different. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

(6) In a coordinated turn, the sideslip force causes the needed centripetal acceleration. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(7) Batteries carry much more exploitable energy per kilogram than hydrocarbon fuels. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(8) In stall, the flow accross an airplane wing or an airfoil is largely separated. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

(9) In order to assess dynamic airplane stability, it is sufficient to analyze aerodynamic coefficients. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(10) If not stalled, a wing will produce increased lift with increased angle of attack. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

(11) The rotation matrix that characterizes the orientation of an airplane with respect to an Earth-fixed frame has always singularities when parameterized with Euler angles (e.g. Tait-Bryan angles). ◯ Correct ◯ Wrong [1 pt]

Please fill in your name: ______________________________

Solution: Wrong

(12) The hub force on a rotor in forward flight results mostly due to an imbalance of the lift forces on the advancing and the retreating blade. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(13) BEMT can be used to model propeller characteristics, where momentum theory enables solving for induced velocities. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

(14) The lower rotor in the coaxial rotor configuration is generally more efficient than the upper rotor. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(15) A swashplate has generally three degrees of freedom. One to control the cyclic pitch and two to control the collective pitch. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(16) A rotor in forward motion has a reverse flow region on the advancing blade. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(17) In a front-rear rotor configuration, the yaw motion is steered by differential drag torques of the rotors. ◯ Correct ◯ Wrong [1 pt]

Solution: Wrong

(18) According to the momentum theory, the power consumption decreases to zero by increasing the disc area to infinity. ◯ Correct ◯ Wrong [1 pt]

Solution: Correct

B. Kinematics: Example Concrete Pump 8 pts

Mobile concrete pumps are used to deliver concrete on construction sites. The arm, which is typically connected to a heavy mobile base, has an extreme reach with many successive joints. The reason for having so many joints is the fact that the arm must be compactly folded for transport. In the following we assume an arm that has one rotational joint φ_0 around the vertical axis and 5 successive rotational joints in a single plane $[\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5]$ (see Figure 1). All joints are actuated.

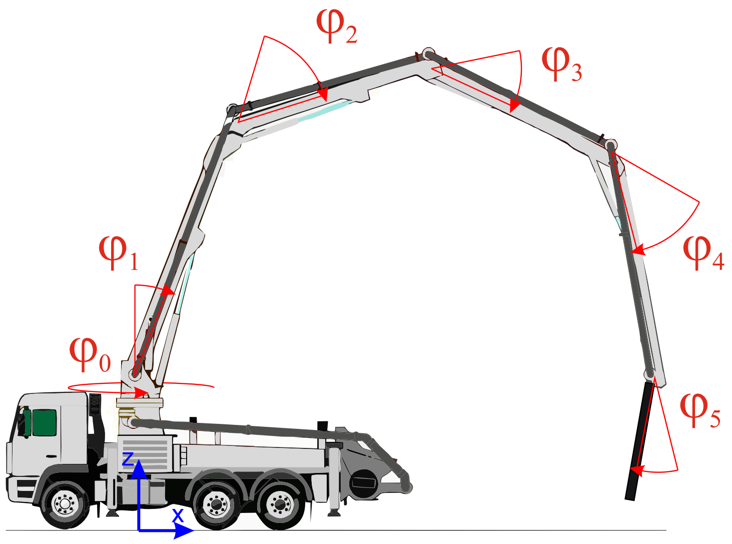

Figure 1: Mobile Concrete Pump

(1) What is the generalized coordinate vector **q**? [1 pt]

$\mathbf{q} = [\qquad\qquad\qquad\qquad]^\top$

Solution: $\mathbf{q} = [\varphi_0, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5]^\top$

(2) How many end-effector degrees of freedom can be controlled? Please describe them. [1 pt]

Solution: There are in total four controllable end-effector degrees of freedom, for example one could choose the position of the outlet (x, y, z) and orientation in the plane of the arm wrt to world (α)

Please fill in your name: ______________________________

(3) Choose the end-effector coordinates $\boldsymbol{\chi}$. [1 pt]

$\boldsymbol{\chi} = [\qquad\qquad\qquad\qquad\qquad]^\top$

Solution: $\boldsymbol{\chi} = [x, y, z, \alpha]^\top$

(4) Given a desired end-effector configuration $\boldsymbol{\chi}^*$, please write a pseudo-code of a numerical inverse kinematics algorithm to iteratively find the joint coordinates $\mathbf{q}$. To this end, assume that the functions to calculate the analytical Jacobian ($\boldsymbol{J}_A = \boldsymbol{J}_A(\mathbf{q})$) and end-effector configuration ($\boldsymbol{\chi} = \boldsymbol{\chi}(\mathbf{q})$) as function of joint configuration $\mathbf{q}$ are given. [3 pts]

Solution:
Define some variables required for this exemplary solution

- start configuration: $\mathbf{q} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$
- Max end-effector position error: $e_p = 1\,\mathrm{mm}$
- Max end-effector angle error: $e_r = 0.01\,\mathrm{rad}$
- max iterations: $MaxIterations = 100$
- initialize iterator: $iterator = 0$
- initialize stop variable: $stop = false$

While $!stop$
$iterator = iterator + 1$
$\boldsymbol{\chi} = \boldsymbol{\chi}(\mathbf{q})$
$\Delta\boldsymbol{\chi} = \boldsymbol{\chi}^* - \boldsymbol{\chi}$
$if\ \|\Delta\boldsymbol{\chi}(1:3)\| < e_p \quad \&\& \quad \Delta\boldsymbol{\chi}(4) < e_r$
$stop = true$
=> solution was found
$else$
$\mathbf{q} = \mathbf{q} + \boldsymbol{J}_A^+ \Delta\boldsymbol{\chi}$
end
if $iterator > MaxIterations$
$stop = true$
=> no solution was found
end
end

Note (not required in the exam): it is often useful to select $\mathbf{q} = \mathbf{q} + k\boldsymbol{J}_A^+ \Delta\boldsymbol{\chi}$ with $k \in (0, 1)$ to let the algorithm converge if the start configuration is chosen far away from the goal location.

(5) What will your algorithm do if the target end-effector configuration lies outside the reaching space? [1 pt]

Solution: It will become unstable and we need to use damped inverse or gradient decent

(6) Is it (generally) still possible to reach the desired end-effector configuration if we impose the constraints $\varphi_1 = 0$ and $\varphi_3 = \varphi_4$? Argue why [1 pt]

Solution: Yes, the arm has still four degrees of freedom if the constraints are active

C. Robot Dynamics 3 pts

(1) Given the link masses m_i, the location of the respective center of gravity $\mathbf{r}_{CoG\_i} = \mathbf{r}_{CoG\_i}(\mathbf{q})$ and the corresponding Jacobians $\mathbf{J}_{CoG\_i} = \mathbf{J}_{CoG\_i}(\mathbf{q})$, please derive **the analytical expression** for the joint torques that are necessary to compensate for gravity. [3 pts]

Solution:

$$\boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{b} + \mathbf{g} \overset{\text{no motion}}{=} \mathbf{g}$$

$$\mathbf{g} = \sum_{i=1}^{6} -\mathbf{J}_{CoG\_i}^{T} m_i \begin{pmatrix} 0 \\ 0 \\ -9.81 \end{pmatrix}$$

D. Legged Robots 4 pts

A quadrupedal robot as depicted in Figure 2 has three successive actuated joints per leg. Three legs are in ground contact and one leg is in motion.

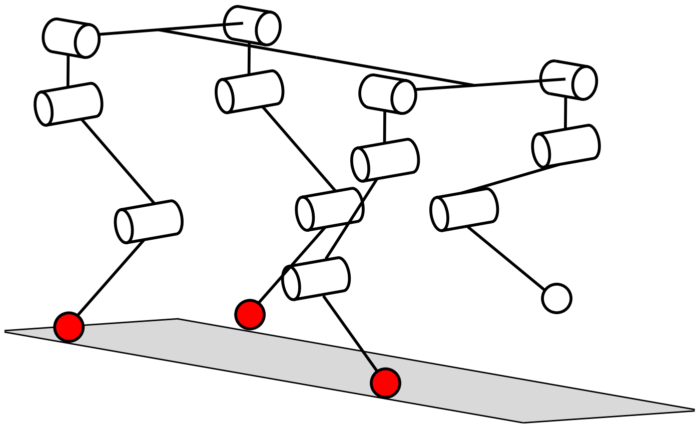

Figure 2: schematics of the quadrupedal robot

(1) How many degrees of freedom are actuated? [1 pt]

Solution: 12

(2) How many degrees of freedom are un-actuated? [1 pt]

Solution: 6

(3) If we assume no slippage at the contact points, how many contact constraints exist? [1 pt]

Solution: 3 times 3 = 9 constraints

(4) How many degrees of freedom remain controllable if the robot stands on three legs and is supposed to move the swing foot along a predefined trajectory? [1 pt]

Solution: 6

E. Optimal Airplane Flight Speeds 10 pts

A hobby pilot flies a Cessna 172 with the following properties:

Table 1: C172 parameters

Parameter	Value
Wing area:	16.1 m^2
Take-off mass:	1'010 kg

Table 2: C172 polar data (you do not need to interpolate for your calculations)

α [deg]	c_L $[-]$	c_D $[-]$	c_L/c_D $[-]$	c_L^3/c_D^2 $[-]$
-5.0	-0.100	0.0350	-2.86	-0.8
-2.5	0.130	0.0340	3.82	1.9
0.0	0.380	0.0360	10.56	42.3
2.5	0.620	0.0430	14.42	128.9
5.0	0.850	0.0510	16.67	236.1
7.5	1.090	0.0660	16.52	297.3
10.0	1.300	0.0810	16.05	334.9
12.0	1.450	0.0980	14.80	317.4
13.0	1.500	0.1060	14.15	300.4
14.0	1.540	0.1150	13.39	276.2
15.0	1.570	0.1230	12.76	255.8
16.0	1.590	0.1320	12.05	230.7
17.0	1.570	0.1400	11.21	197.4
18.0	1.550	0.1490	10.40	167.7
20.0	1.480	0.1670	8.86	116.2

(1) The pilot wants to fly from airfield A to airfield B using as little fuel as possible. He flies at 1000 m above mean sea level, where the air density amounts to 1.112 kg/m^3. What level flight speed should he choose (Assume a constant mass)? [4 pts]

Solution: max range: $\left.\frac{c_L}{c_D}\right|_{max}, c_L = 0.85$

force balance: $mg = \frac{1}{2}\rho v^2 A c_L$

$v = \sqrt{\frac{2mg}{\rho A c_L}} = 36.09\frac{m}{s}$

(2) Arriving at airfield B, the runway is blocked and the pilot is asked to circle above the airfield for some minutes. He chooses a bank (roll) angle of 15° (coordinated turn) and flies still constantly at 1000 m above mean sea level. Again trying to save fuel, what is the best speed now to circle for a fixed amount of time (Assume a constant mass)? Make a drawing of a front view showing the balance of forces that act on the airplane. [6 pts]

Solution: max endurance: $\left.\frac{c_L^3}{c_D^2}\right|_{max}, c_L = 1.3$

force balance: $\frac{mg}{\cos 15°} = \frac{1}{2}\rho v^2 A c_L$

$v = \sqrt{\frac{2mg}{\rho A c_L cos 15°}} = 29.69\frac{m}{s}$

F. Dynamic Modes of an Airplane 6 pts

The modes of an airplane describe the characteristic dynamic behaviors. In this context, please answer the following questions.

(1) List the longitudinal modes and describe their characteristics. [2 pts]

Solution: Short period: complex roots (fast), stable
Phugoid: complex roots (flow), stable

(2) List the lateral modes and describe their characteristics. [2 pts]

Solution: Roll: real root, (response depends on the aircraft), stable
Spiral: real root, often unstable or marginally stable.
Dutch Roll: complex root (response depends on aircraft), stable

(3) Assume you are designing an airplane where the parameters (aerodynamics, mass, inertia) are roughly known. Describe the steps you would have to go through, in order to find the poles in the imaginary plane that correspond to the modes. [2 pts]

Solution:

- Linearize the model around an operating point and separate longitudinal / lateral subsystems (1pt).
- Find the eigenvalues of the system matrix A of $\dot{x} = Ax + Bu$. (1pt)

G. Steering a helicopter 6 pts

Consider a rotorcraft in a standard helicopter configuration with one main and tail rotor. The direction of the blade azimuth angle ξ of the main rotor is depicted in Figure 3. The helicopter is thought of being in hover position at the start. For **initiating** different maneuvers, the helicopter needs to change the tip path plane of the main rotor.

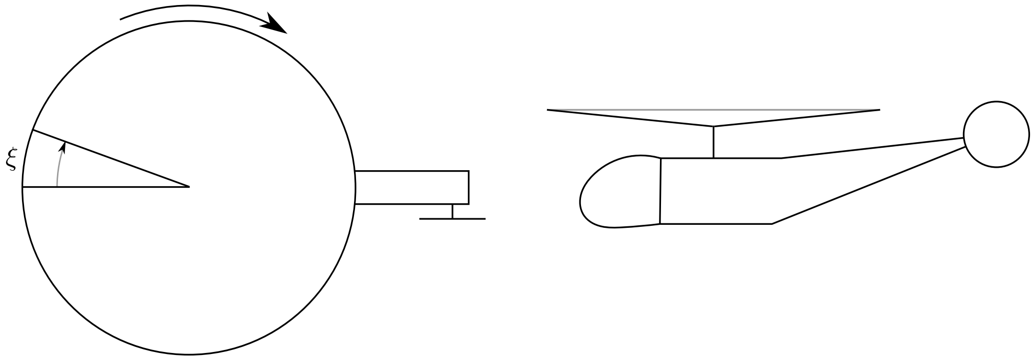

Figure 3: Helicopter configuration.

(1) Show how the **blade flapping angle** of the main rotor changes when initiating different maneuvers. Write for each maneuver if the statement in the corresponding column is **T**(true) or **F**(false). [3 pts]

Blade flapping

Maneuver	Coning angle: Increase	Coning angle: Decrease	Tip path plane: Tilt to front	Tip path plane: Tilt to right	Tip path plane: Tilt to back	Tip path plane: Tilt to left
Flying upwards						
Yawing						
Flying forward						

Solution:

Flying upwards	Increase of coning angle
Yawing	nothing, Steered with tail rotor
Flying forward	Increase coning, tip path plane tilt to front

(2) Show how the **blade pitch angle** has to change for the same maneuvers as in the question above. Assume a teetering rotorhead. Write for each maneuver if the statement in the corresponding column is **T**(true) or **F**(false). [3 pts]

Swashplate

Maneuver	Collective pitch: Increase	Collective pitch: Decrease	Cyclic pitch: Max. at front	Cyclic pitch: Max. at right	Cyclic pitch: Max. at back	Cyclic pitch: Max. at left
Flying upwards						
Yawing						
Flying forward						

Please fill in your name: ______

	Flying upwards	Increase of collective pitch
Solution:	Yawing	Nothing, steered with tail rotor
	Flying forward	Increase coollective pitch and tilt to left

H. Modeling and Control of a Hexacopter 12 pts

Consider a hexacopter with six propellers in star configuration depicted in Figure 4. You want to model this hexacopter in near hover condition to analyse the dynamics and design a controller. For modeling, please use the variables defined in Table 3.

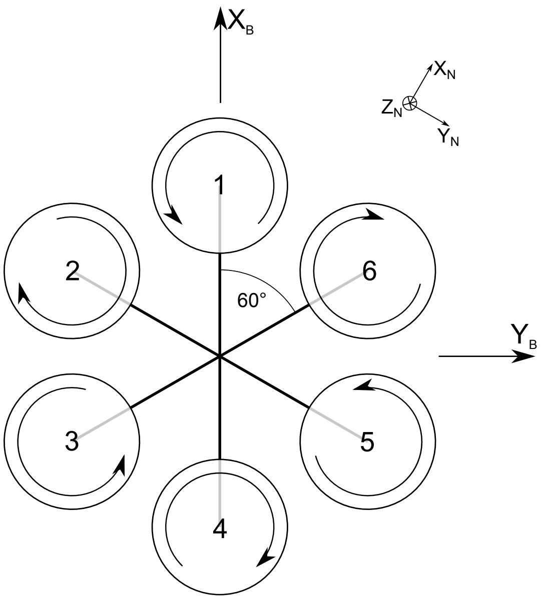

Figure 4: Hexacopter configuration.

Table 3: Properties of the hexacopter.

Property	Variable
Inertia	$\mathbf{J}$
Mass	m
Thrust constant	b
Drag constant	d
Arm length	l

(1) List all relevant forces and moments acting on the hexacopter. Write down its equations in the body frame. Use $\mathbf{R}_{nb}$ as the rotation matrix from the navigation to the body frame. Further you can use the position vector $\mathbf{p}_i = [l_{xi}, l_{yi}, 0]^\top$ for propeller i. [4 pts]

Solution:

Forces	
Gravity	$R^\top [0, 0, g]^\top$
Thrust	$\sum_{i=1}^{8} [0, 0, b\Omega_i^2]$

Moments	
Drag moment	$\sum_{i=1}^{8} [0, 0, d\Omega_i^2](-1)^i$
Thrust induced moment	$\sum_{i=1}^{8} [0, 0, b\Omega_i^2] \times \mathbf{p}_i$

(2) You want to build a control allocation for your controller. The control allocation calculates the desired propeller speeds out of a commanded thrust and commanded moments from the angular feedback controller. How many possible solutions exist for this allocation? Which solution would you implement for the controller? Briefly discuss your answer. (Hint: You can calculate the solution(s) for the propeller speeds which gives a constant thrust T and zero moments.) [4 pts]

Solution: Infinitely many solution for the control allocation. Mapping from 4 dimensional space to 6 dimensional space is under-defined. Can use an optimization, e.g. use the mapping which reduces power consumption.

(3) Draw a hierarchical control flow for controlling the **velocity** of the hexacopter in the body frame. Describe briefly the function of each block (no equations, description like "PID control of ...", "transformation from ... to ...") [4 pts]

Hexacopter Dynamics

Solution: No Latex solution available