problem - 1 :-

Given $x(m,n) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$ $h(m,n) = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

Convolution output shape is given by $x_m + h_m - 1 \times x_n + h_n - 1$

i.e. $3+2-1 \times 2+3-1 = 4 \times 4$

let output be $y = \begin{bmatrix} y_{00} & y_{01} & y_{02} & y_{03} \\ y_{10} & y_{11} & y_{12} & y_{13} \\ y_{20} & y_{21} & y_{22} & y_{23} \\ y_{30} & y_{31} & y_{32} & y_{33} \end{bmatrix}$

1) First, we need to flip the kernel in both horizontal and vertical directions

$h = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \xrightarrow{\text{horizontal flip}} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \xrightarrow{\text{vertical flip}} \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

2) slide the kernel over image $x(m,n)$

-1 0 1
0 1 [2 1]
3 4
2 1

$\Rightarrow y_{00} = -1\times0 + 0\times0 + 1\times0 + 0\times0 + 1\times0 + 2\times1$

$= 2$

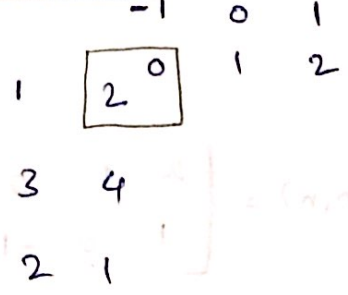
(Since, we assumed zero padding outside of x)

-1 0 1
0 [1 2
1 2]
3 4
2 1

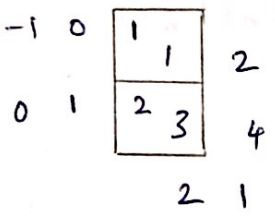
$\Rightarrow y_{01} = 1\times1 + 2\times2 = 5$

2 1
-1 0 1
[0 1
1 2] 2
3 4
2 1

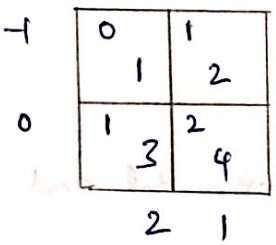
$\Rightarrow y_{02} = 1\times0 + 1\times2 = 2$



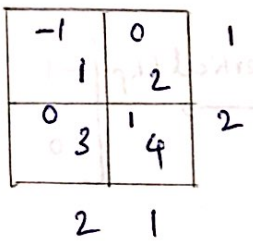
$y_{03} = 2 \times 0 = 0$



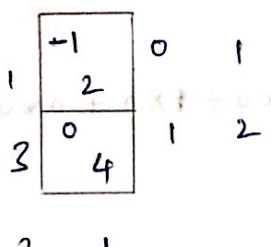
$y_{10} = 1 \times 1 + 2 \times 3 = 7$



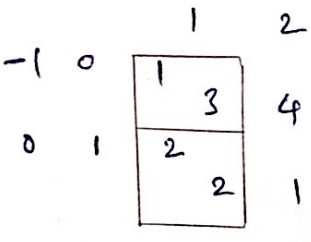
$y_{11} = 0 \times 1 + 1 \times 2 + 1 \times 3 + 2 \times 4 = 13$



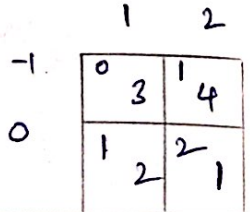
$y_{12} = -1 \times 1 + 0 \times 2 + 0 \times 3 + 1 \times 4 = 3$



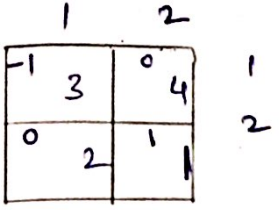
$y_{13} = -1 \times 2 + 0 \times 4 = -2$



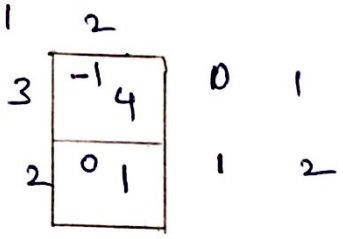
$y_{20} = 1 \times 3 + 2 \times 2 = 7$



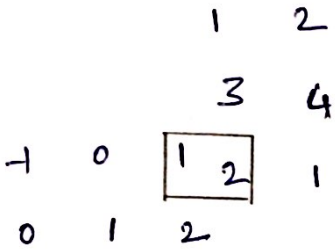
$y_{21} = 0 \times 3 + 1 \times 4 + 1 \times 2 + 2 \times 1 = 8$



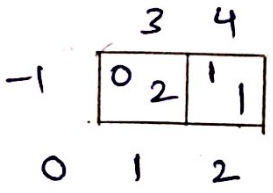
$y_{22} = -1 \times 3 + 0 \times 4 + 0 \times 2 + 1 \times 1 = -2$



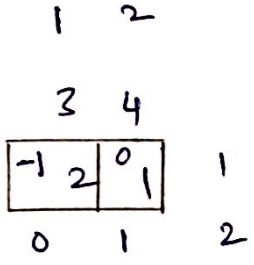
$y_{23} = -1 \times 4 + 0 \times 1 = -4$



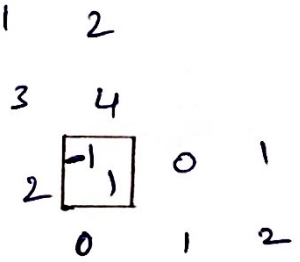
$y_{30} = 1 \times 2 = 2$



$y_{21} = 0 \times 2 + 1 \times 1 = 1$



$y_{32} = -1 \times 2 + 0 \times 1 = -2$



$y_{33} = -1 \times 1 = -1$

Output, Convolution $= x * h = \begin{bmatrix} 2 & 5 & 2 & 0 \\ 7 & 13 & 3 & -2 \\ 7 & 8 & -2 & -4 \\ 2 & 1 & -2 & -1 \end{bmatrix}$