

problem - 1:-

Given $x(m,n) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 2 & 1 \end{bmatrix}$ $h(m,n) = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

Convolution output shape is given by $x_{m+h_m-1} \times x_{n+h_n-1}$

i.e. $3+2-1 \times 2+3-1 = 4 \times 4$

let output be $y = \begin{bmatrix} y_{00} & y_{01} & y_{02} & y_{03} \\ y_{10} & y_{11} & y_{12} & y_{13} \\ y_{20} & y_{21} & y_{22} & y_{23} \\ y_{30} & y_{31} & y_{32} & y_{33} \end{bmatrix}$

- 1) First, we need to flip the kernel in both horizontal and vertical directions

$$h = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \xrightarrow[\text{flip}]{\text{horizontal}} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} \xrightarrow{\text{vertical flip}} \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

- 2) slide the kernel over image $x(m,n)$

$$\begin{array}{ccc} -1 & 0 & 1 \\ 0 & 1 & \boxed{2} & 2 \\ & 3 & 4 \\ & 2 & 1 \end{array}$$

$$\Rightarrow y_{00} = -1 \times 0 + 0 \times 0 + 1 \times 0 + 0 \times 0 + 1 \times 0 + 2 \times 1 = 2$$

(Since, we assumed zero padding outside of x)

$$\begin{array}{ccc} -1 & 0 & 1 \\ \circ & \boxed{1} & \boxed{2} \\ & 3 & 4 \end{array}$$

$$\Rightarrow y_{01} = 1 \times 1 + 2 \times 2 = 5$$

$$\begin{array}{ccc} & 2 & 1 \\ & -1 & 0 & 1 \\ & \boxed{1} & \boxed{2} \\ & 3 & 4 \\ & 2 & 1 \end{array}$$

$$\Rightarrow y_{02} = 1 \times 0 + 1 \times 2 = 2$$

$$1 \quad \boxed{2 \quad 0} \quad 1 \quad 2$$

$$y_{03} = 2 \times 0 = 0$$

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

$$\begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 3 \\ \hline \end{array} \begin{array}{c} 2 \\ 4 \\ 2 \end{array}$$

$$y_{10} = 1 \times 1 + 2 \times 3 = 7$$

$$\begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \begin{array}{|c|c|} \hline 0 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \begin{array}{c} 1 \\ 2 \\ 2 \end{array}$$

$$y_{11} = 0 \times 1 + 1 \times 2 + 1 \times 3 + 2 \times 4 = 13$$

$$\begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \begin{array}{|c|c|} \hline 1 & 2 \\ \hline 0 & 3 \\ \hline \end{array} \begin{array}{c} 1 \\ 2 \\ 2 \end{array}$$

$$y_{12} = -1 \times 1 + 0 \times 2 + 0 \times 3 + 1 \times 4 = 3$$

$$\begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array} \begin{array}{|c|c|} \hline -1 & 0 \\ \hline 2 & 1 \\ \hline \end{array} \begin{array}{c} 0 \\ 1 \\ 1 \end{array}$$

$$y_{13} = -1 \times 2 + 0 \times 4 = -2$$

$$\begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & 2 \\ \hline \end{array} \begin{array}{c} 1 \\ 4 \\ 1 \end{array}$$

$$y_{20} = 1 \times 3 + 2 \times 2 = 7$$

$$\begin{array}{cc} 1 & 2 \\ 1 & 2 \end{array} \begin{array}{|c|c|} \hline 0 & 3 \\ \hline 1 & 2 \\ \hline \end{array} \begin{array}{c} 1 \\ 4 \\ 1 \end{array}$$

$$y_{21} = 0 \times 3 + 1 \times 4 + 1 \times 2 + 2 \times 1 = 8$$

	1	2	
-1	3	0	4
0	2	1	1

$$y_{22} = -1 \times 3 + 0 \times 4 + 0 \times 2 + 1 \times 1 = -2$$

	1	2	
3	-1	4	0
2	0	1	1

$$y_{23} = -1 \times 4 + 0 \times 1 = -4$$

	1	2	
	3	4	
-1	0	1	2
0	1	2	1

$$y_{30} = 1 \times 2 = 2$$

	1	2	
	3	4	
-1	0	2	1
0	1	2	1

$$y_{21} = 0 \times 2 + 1 \times 1 = 1$$

	1	2	
	3	4	
-1	2	0	1
0	1	2	1

$$y_{32} = -1 \times 2 + 0 \times 1 = -2$$

	1	2	
	3	4	
2	-1	1	0
0	1	2	1

$$y_{33} = -1 \times 1 = -1$$

output, Convolution = $x * h =$

$$\begin{bmatrix} 2 & 5 & 2 & 0 \\ 7 & 13 & 3 & -2 \\ 7 & 8 & -2 & -4 \\ 2 & 1 & -2 & -1 \end{bmatrix}$$