problem 4(a) :-

Convolution as matrix multiplication using toeplitz matrices

Given $I = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $h = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

$\vec{I} = [1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9]^T$

Goal is to find a matrix H such that

$$I * h = H I^T$$

Below are the following steps to compute H. These steps are identified with the help of pdf available for study and making little changes.

1) Make kernel size equal to output size by zero padding

$$\text{Output size} = I_m + h_m - 1 \times I_n + h_n - 1 = 3+3-1 \times 3+3-1$$

Now, we will pad zeroes at the bottom and right of kernel

$$h = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) For each row compute toeplitz matrix, this matrix no. of columns = Input no. of columns

$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $T_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$T_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $T_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3) Now taking these toeplitz matrices we compute doubly toeplitz matrix as follows. The numbers of columns for this matrix would be same as number of rows of input image

doubly blocked toeplitz matrix =
$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & -1 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
-1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 & 0 \\
0 & -1 & 0 & & -1 & 0 & 0 & -1 & -1 \\
0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 0 & 0 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$
25×9

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$
9×1

4) Now we multiply the above two matrices to get a 25×1 output vector which is as follows

output vector = $[1 \;\; 2 \;\; 2 \;\; -2 \;\; -3 \;\; 5 \;\; 7 \;\; 4 \;\; -7 \;\; -9 \;\; 12 \;\; 15 \;\; 6 \;\; -15 \;\; -18 \;\; 11 \;\; 13 \;\; 4 \;\; -13 \;\; -15 \;\; 7 \;\; 8 \;\; 2 \;\; -8 \;\; -9]$

5) Now the output vector is reshaped into 5x5 to get the Convolution output

$$\text{output} = \begin{bmatrix} 1 & 2 & 2 & -2 & -3 \\ 5 & 7 & 4 & -7 & -9 \\ 12 & 15 & 6 & -15 & -18 \\ 11 & 13 & 4 & -13 & -15 \\ 7 & 8 & 2 & -8 & -9 \end{bmatrix}$$

0
0
1
0