BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

Course No.: EEE212 Group No.: **16**

Project Description

Project Title:

Solution of higher order polynomial equation for real roots and determination of their multiplicities by the Newton-Raphson method

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Project planning:

The real roots of the polynomial equation P(x) = 0 may be found from the graphical plot of the function y=P(x). Plotting y=P(x) results in a continuous curve which intersects the x axis at points where the x coordinates are equal to the real roots of the polynomial equation.

We may apply Newton-Raphson method to get the real roots of the polynomial equation. However, as the slope of the plotted curve changes its sign (+/-) in different segments of the curve, we must apply this method in different curve segments separated by the stationary points.

The stationary points may be found by solving the equation P'(x)=0 which itself is polynomial equation of order (n-1).

To find the real roots of P(x) = 0 we will separate the curve y=P(x) by the stationary points and consider the following:-

- (i) A stationary point itself may give a root of the equation if the curve touches the x axis at that point.
- (ii) If two consecutive stationary points are located on the opposite sides of the x axis we will find a root by applying Newton-Raphson method between two points.
- (iii) We may find a root left to the leftmost stationary point by applying Newton-Raphson method.
- (iv) We may find a root right to the rightmost stationary point by applying Newton-Raphson method.

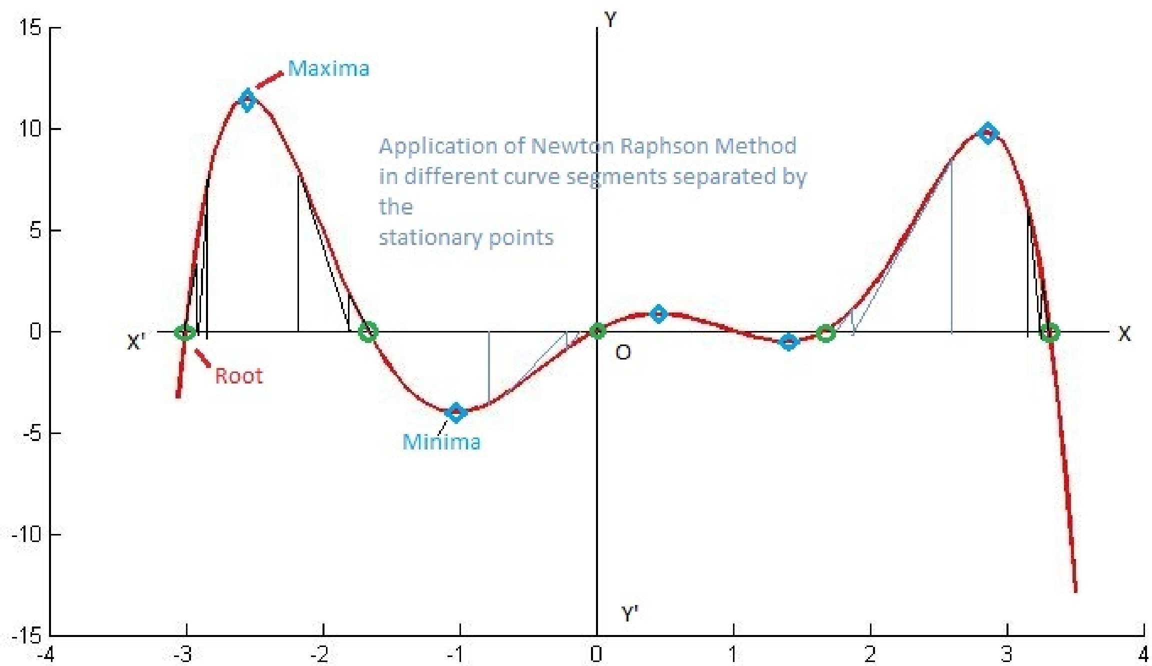
By considering all the above, we can find all the real roots of the equation. The multiplicities of the roots can then be found by successive differentiation.

Algorithm:

- (i) If the equation P(x) = 0 is of degree 1 i.e. linear then solve by normal algebra and return.
- (ii) Differentiate P(x) to find P'(x).
- (iii) Solve P'(x) = 0 by recursion to find the critical points.
- (iv) If there are no critical points then assume an arbitrary starting point (0) as a pseudo critical point.
- (v) If the curve converges with the x axis left to the left most critical point apply NR method to find a root.
- (vi) Take the critical points in consecutive pairs and:
  - (a) Check if the first critical point itself is a root, if not :-
  - (b) Check if the second critical point is located on the side of the x axis opposite to the first critical point, if so find a root by applying NR method by taking a starting point amid the 2 critical points.
- (vii)
  - (a) Check if the right most critical point itself is a root, if not :-

(b) If the curve converges with the x axis right to the right most critical point apply NR method to find a root.

(viii) Determine the multiplicities of the found roots



Numerical methods used:

Differentiation of a Polynomial:

The differentiation of a polynomial is straightforward. Consider a polynomial of degree n

P(x) = $\sum_{i=0}^{n} a_i x^i$

Then

P'(x) = $\sum_{i=1}^{n} i a_i x^{i-1}$

Which itself is another polynomial of degree (n – 1)

Recursion:

To solve a polynomial equation P(x) = 0 of order n we need to solve another polynomial equation P'(x) = 0 of order (n-1) in order to find the stationary points. As the order decreases in each calculation we will use recursion to solve this problem. The base case is a linear equation which may be solved easily by ordinary algebra.

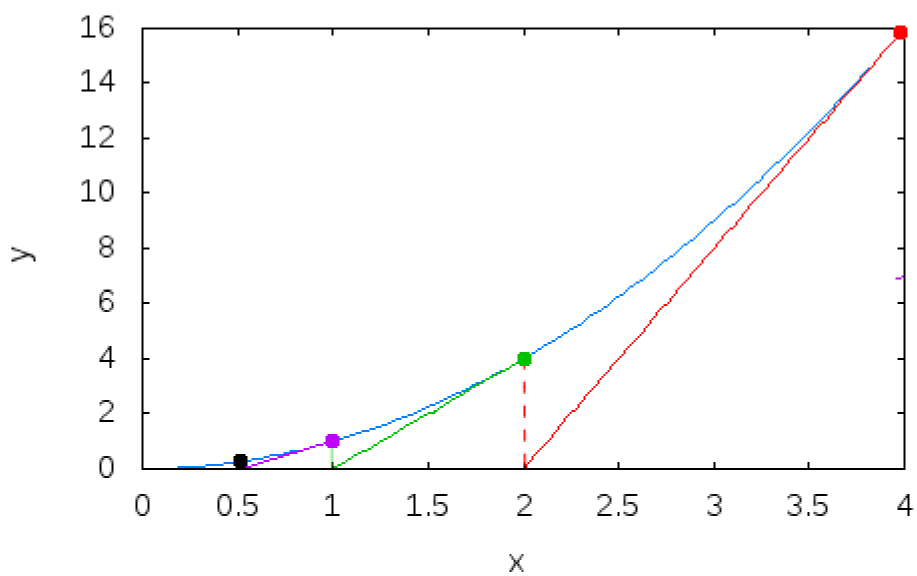
Newton-Raphson Method:

Newton-Raphson method is a recursive algorithm for approximating the root of a differentiable function. We know simple formulas for finding the roots of linear and quadratic equations, and there are also more complicated formulae for cubic and quartic equations. At one time it was hoped that there would be formulas found for equations of quantic and higher-degree, though it was later shown by Neils Henrik Abel that no such equations exist. The Newton-Raphson method is a method for approximating the roots of polynomial equations of any order. In fact the method works for any equation, polynomial or not, as long as the function is differentiable in a desired interval.

Let f(x) be a differentiable function. We select a point x_0 based on a first approximation to the root, arbitrarily close to the function's root. To approximate the root we then recursively calculate using:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

As we recursively calculate, the x_{n+a}'s often become increasingly better approximations of the function's root.



A few iterations of Newton's method applied to $y=x^2$ starting with $x_0=4$

Determination of multiplicity of a root:

The multiplicity of a root can be found by successive differentiation of the polynomial. If r is a root of P(x) = 0 and the m^{th} order differential of a polynomial P(x)

$P^m(x)\ |_{x=r} \neq 0$, such that m is the least possible integer then then the multiplicity of r is m.