Method 2

$AX = X\Lambda$ —————— Same eigenvalue!

$$\underbrace{S^{-1}AS}_{B}\ \underbrace{S^{-1}X}_{Y} = \underbrace{S^{-1}X}_{Y}\Lambda$$

$$B\ Y = Y\Lambda \Rightarrow BY = Y\Lambda$$

RANDOM NUMBERS. → cannot be generated on a digital computer. (4/3/19)

→ unpredictability → incompressibility

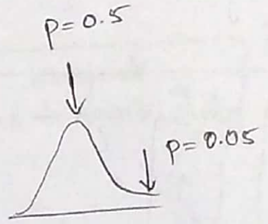
Pseudorandom numbers.

numbers are chosen from a particular distribution.

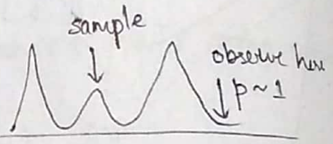
→ all outcomes are equally probable [randomize]

1,00,00,000 — 1 million

1,00,000 should be zeroes



$\cancel{^{8}C_{8}}$ $\quad ^{10\text{ million}}C_{1}$



p - value : measure of lack of surprise.

what does it quantify? → Statistical significance.

∴ high significance ⇒ lower p value.

2 branches of statistics : (Non-) Parametric

→ given a new sample, what is the probability that it is from the original distribution?

z score : $\dfrac{x-\mu}{\sigma}$

Read some Biostatistics.
* Coding the Matrix

* CASI
computer age statistical inference

History:

① John Von Neumann.
middle ~~man~~ number square.

② Knuth's Algorithm K. — extremely weird.
moral: random nos. should be generated in non-random way.

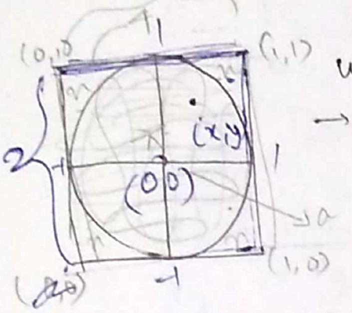
RANDOM NUMBER GENERATION.

l x_i A(

→ Given x people, how many pairs shared birthday? ↓ code pls. [Ans. 23]

Buffon's
~~Buffense~~-Needle expt:

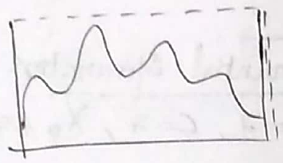
→ To Estimate π with dart board → code pliss



unit circle: $\boxed{x^2+y^2 < 1}$ i++

→ probability of dart falling inside circle $= \frac{\pi}{4}$.

To generate area:



throw dart again!
no. of times it falls below the curve → area.

For that, we need eqn of curve.

Boot strapping

Linear Congruential Generator.

$$x_{n+1} = (a x_n + c) \bmod (m).$$

m: modulus, $m > 0$.

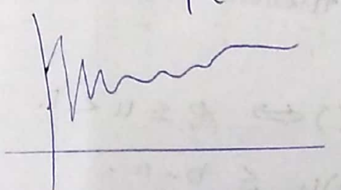
a: multiplier: $0 < a < m$

c: increment: $0 < c < m$.

x_0: starting value.

$\frac{j}{i} = \pi/4$

$p(i) = (4*j)/i$



Birthday problem:

given 2 people → same day b'day $= {}^{365}C_1 = \frac{365!}{364!1!} = 365$ $\left(1-\frac{1}{365}\right)\left(\frac{1}{365}\right)$

$= \left(\frac{365}{365}\right)\left(\frac{1}{365}\right) = \left(\frac{1}{365}\right)$ $\left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{1}{365}\right)$

3 people → at how many pairs
↳ choose 2 dates.

A B B
A A B
A B A

$\left(\frac{1}{365}\right)(1)\left(\frac{364}{365}\right)$

4 people → 3 dates so that A B C D, 1 1 2 3, shared bday $(1)(1)\left(\frac{1}{364}\right)($

Given n people: $P_n = \left(\frac{1}{365}\right)\left(\frac{2}{365}\right)\left(\frac{n}{}\right)\left(\frac{n-1}{365}\right)$

$P_n = \frac{(n-1)!}{(365)^n}$ $\frac{n-1}{365}$

Scanned by CamScanner

n people → ① ${}^nC_2$ pairs can be formed

2 → ② chance of 1 pair = $\left(\frac{364}{365}\right)$

P(atleast 1 pair) = 1 − P(no pair)

$$= 1 - \left[\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdots\frac{365-(n-1)}{365}\right]$$

$$= 1 - \frac{1}{365^n}\left[365\cdot 364\cdot 363 \cdots 365-(n-1)\right]$$

$$P\binom{\text{atleast 1}}{\text{pair}} = 1 - \frac{1}{365^n}\left[\frac{365!}{(365-n)!}\right] = 1 - \frac{{}^{365}P_n}{365^n}$$

6/3/19. **Linear Congruential Generator**

Try with → $m = 10,\ a = 7,\ c = 7,\ X_0 = 7$

→ $m = 4096,\ a = 109,\ c = 853,\ X_0 = 0.$

Key Principles:

→ perform event many times (n), and count occurrence of A (n_a)

→ relative f. of occurrence of A = $\frac{n_a}{n}$.

→ frequency theorem : as $n \to \infty$, we get true value.

$u = \text{rand}() \Leftrightarrow 0 \le u < 1.$

- $0 \le (b-a)u \le b-a.$
- $0 \le \lceil (b-a)u \rceil \le b-a.$
- $a \le a + \lceil (b-a)u \rceil \le b.$

- rand
- randi – rand integer

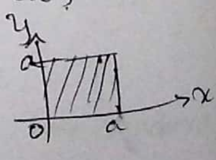
randi(6) = any integer b/w 1 ↔ 6

- randperm

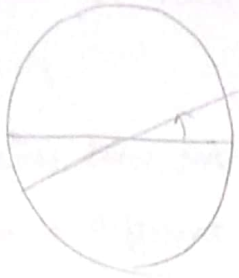
given vector, it'll get random permutation of vector.

- Generate coin toss = randi(2)
- die roll = randi(6)
- roll of two dice = randi(6) + randi(6)
- random DNA seq. of length l = randi(4, 1, l)
- random pt. on ◯le → pick θ alone. ± rand(360)
- random pt. within a □ → x = rand(0, a)

y = rand(0, a)

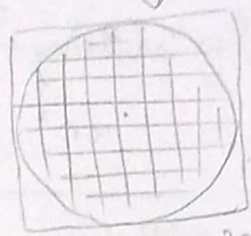


• random pt within ⊙le :- $\boxed{\text{Accepance - Rejection method}}$



$r = \mathrm{sqrt}(\mathrm{rand}(0, r^2))$

$2r$



$2r$

$\{x^2 + y^2 \le r^2\}$

zero = $\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ & \\ & \end{bmatrix}$ 1000, 2

all plot the x_i, y_i

every time x_i, y_i is picked b/w $x(0, 2r)$ $y(0, 2r)$

& $x_i^2 + y_i^2 \le r^2$ is kept and rest are equated to zero

---

→ Queing theory

→ Data structure/analytics

· Coursera course

→

```
for i = 1: 1000n      a=0
    x = randi(6);
    y = randi(6);
    z = randi(6);
    if x+y+z = 10
        a = a+1;
    end
end  p = a/n;
```

$\frac{1}{i} \times \frac{1}{i} = \frac{1}{i^2} = -1$

i^4

$i^3 = (-i)^2 = i^3 \; i^6 \; i^2 \ldots$

$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

$a^2 + bc = 1$

$ab + cd = 0$

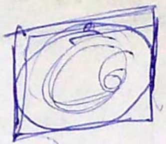
$-1 \quad 0$

i^3

---

given n = 55, find no of shared b'days

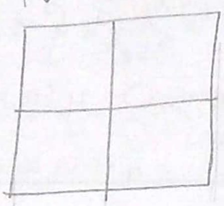
$1 \cdot \left(\frac{1}{365}\right)$

$\int_0^{4\pi} \sin^3 x \, dx =$  πr^2

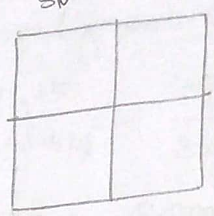
$9\pi = \frac{36}{4} \cdot 9$

$\pi r^2 =$

FV

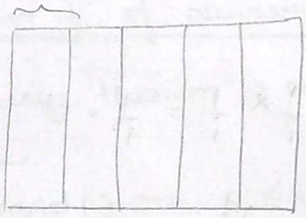


SV



TV?

same width = a



drop a needle onto the floor

what is the probability that the needle will lie across a line b/w 2 strips?

8/3/19.

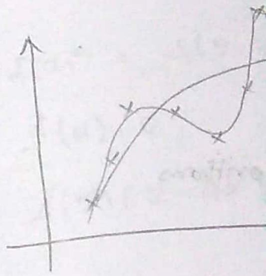
- why do we nee
- AC = y

captures how

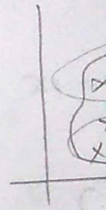
∴ given a differ

But when we hav

$f(\theta) =$



Example:



∴ Take rand

what is optimis

→ minimise f

Optimisation

Special case :

→ $f(\alpha x +$

→ convexity

→ convexity

8/3/19.

- why do we need a model? $\longrightarrow$ "To predict".
- $\underbrace{Ac = y}$

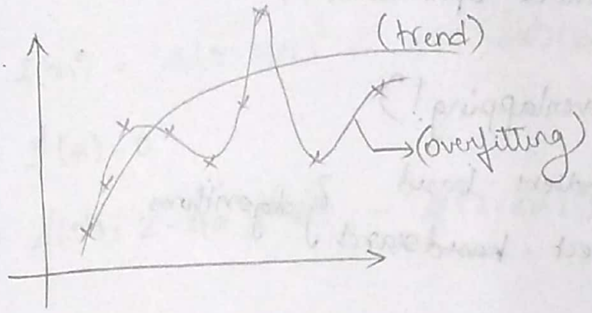
captures how we map $c \rightarrow y$. $\Rightarrow$ we are modelling 'y'!.

$\therefore$ given a different 'x', what is 'y' ?!

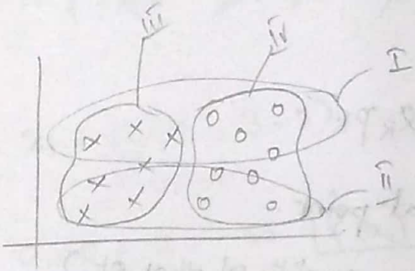
But when we have non-linear systems :

$$f(\theta) = \sum_i \left[\frac{x_{m,i} - x_{p,i}(\theta)}{x_{m,i}} \right]^2$$

$\leftarrow$ optimization : find $\underline{\underline{\theta}}$ such that $f(\theta)$ is maximum.



Example:



i) how do you define a cluster?

Basically,

a = $\downarrow$ intra distance.

b = $\uparrow$ inter cluster distance.

$\therefore$ Take random clusters and $\boxed{\text{maximise } (b-a)}$. $\rightarrow$ Optimisation.

what is optimisation?

$\rightarrow$ minimise $\underbrace{f_0(x)}_{\text{objective}}$ subject $\underbrace{f_i(x) < b_i}_{\text{constraint}}$, $i = 1, 2, \ldots m$

Optimisation $\rightarrow$ constrained / unconstrained.

Convex set:

Set of points such that a line b/w them contains all points which belong to the set.

Special case : Convex optimisation $\rightarrow$ where both objective & constraints are convex.

$\rightarrow$ $f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y)$ $\quad \alpha + \beta = 1, \alpha \geq 0, \beta \geq 0$.

$\rightarrow$ convexity is more general than linearity.

$\rightarrow$ convexity has only 1 minima.

Common optimisation:

① Linear least squares ② Linear programming

③ Quadratic prog. ④ Integer prog.

⑤ Dynamic prog ⑥ Non linear optimisation. *

$\downarrow$

> Practical methods of optimisation - R. Fletcher
> Clever Algorithms
> How 2 solve it: Modern Heuristics.
> Convex Optimisation.

Classification of methods: (overlapping!)

local , convex , gradient - based
global , stochastic , direct - ~~based~~ search } algorithms

$\underbrace{\text{local, convex; global, stochastic}}_{\text{opt. methods}}$

GRADIENT - BASED METHOD:

$$x_{k+1} = x_k + \alpha_k p_k.$$

① Start with opt. guess x_0 $\rightarrow$ initial point.

② Test for convergence. (test the quality of that pt.) i.e., is $f(x)$ low enough?

③ Find a search-direction, p_k. $\longrightarrow$ (just based on gradient of f)

④ decide step length, α_k $\longrightarrow$ (differenciate, equate to zero)

⑤ update, i.e, compute x_{k+1}.

Example: $f(x,y) = 4x^2 - 4xy + 2y^2$.

Let. $x_0 = (2,3)$ $\quad f(x_0) = 4(4) - 4(6) + 2(9) = 4(-2)+18$ $\boxed{= 10} = f(x_0)$

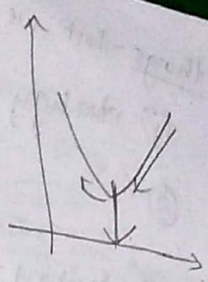
(i) to find p_k, find gradient $= \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y}$

$$\frac{\partial f}{\partial x} = 8x - 4y \qquad \frac{\partial f}{\partial y} = -4x + 4y.$$

we need to move in opposite direction. $\therefore$

move through $-\left(8x-4y,\ 4y-4x\right) = -p_k$.

$p_k = -4(2x-y,\ y-x)$



$\therefore x_1 = x_0 + \alpha(-4)(2x-y,\ y-x)$

$= (2,3) + (-4\alpha)(4-3,\ 3-1) = (2,3) - 4\alpha(1,1)$

$x_1 = (2-4\alpha,\ 3-4\alpha)$

To check if x_1 is a good fit: compute $f(x_1)$.

$\Rightarrow f(x_1) = 4(2-4\alpha)^2 - 4(2-4\alpha)(3-4\alpha) + 2(3-4\alpha)^2$.

$\Rightarrow f'(\alpha) = 0$.

$\Rightarrow \cancel{4}(2)(2-4\alpha)(-\cancel{4}) - \cancel{4}(2-4\alpha)(-\cancel{4}) - \cancel{4}(3-4\alpha)(-\cancel{4}) +$

$\cancel{2}(3-4\alpha)(\cancel{2})(-\cancel{4}) = 0$.

$\Rightarrow 4-8\alpha - 2+4\alpha - \cancel{3} + \cancel{4\alpha} + \cancel{3} - \cancel{4\alpha} = 0 \Rightarrow 2-4\alpha = 0$.

$\Rightarrow \boxed{\alpha = 0.5}$

$\therefore x_1 = (2-2,\ 3-2) = (0,1)$

$f(x_1) = 0 - 0 + 2 \Rightarrow \boxed{f(x_1) = 2} \rightarrow$ better than $f(x_0)$!

Now, start with x_1 & find x_2 . . . .

[tedious step].

$x_2 = x_1 + \alpha_1 p_1$.

Gradient - Descent method. (11/3/18)

Steepest descent

Conjugate gradient

Newton's method (modified) - Hessian.

Quasi - Newton. $\rightarrow$ DFP, $\rightarrow$ SR1

DIRECT ~~SERC~~ SEARCH METHOD.

why $\rightarrow$ non-diff. objective f^n. $\rightarrow$ many local minima.

$\rightarrow$ non-convex search spaces

$\rightarrow$ discrete search method. [IPL schedule / clusters]

$\rightarrow$ mixed variables.

$\rightarrow$ very high dimensionality

2 things that we need to know regarding - Direction - Search

① strategy to vary parameter vector. (guess)

② " " accept / reject a new parameter vector. (greedy)

Eg. Hill climbing.

→ always walk in the direction that improves your optimisation (greedy)

→ walk in random upward direction (guess).

→ Greedy criterion → short sightedness

→ ~~for~~ might get stuck @ a local minima.

→ converges faster

★ Overcome by:

① multiple runs.

② occasionally choose wrong direction hoping to find a better result.

→ ~~ability~~ easy to use: few controllable variable to ~~that~~ steer the minimisation.

→ parallelisability to cope with computation intensive cost f^n.

CLASSIC METHODS:

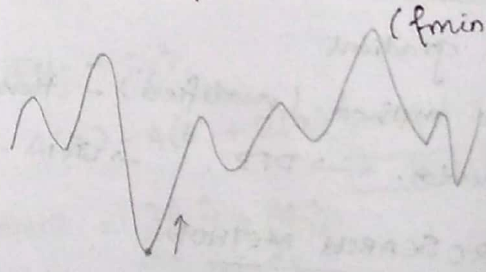
① Hooke - Jeeves Pattern Search.

② Nelder - Simplex / Downhill Simplex method (fminsearch) (fmincon)

③ Grid search

④ Random "

⑤ Hill climbing



STOCHASTIC SEARCH ALGO

① Simulated annealing

② Swarm algo

③ Ant colony optimisation algorithm

④ Genetic algo.

⑤ Tabu search

inspired from nature

⑥ Interactive EAs.

(meandering around)

SIMULATED ANNEALING - Metropolis algo

→ always reduce cost (accept those move) (i)

more so at the beginning → occasionly accept moves that ↑ cost. (ii)

annealing of metals: high temp (heat) (first) & slowly cool down (second).

↓ atoms excited. ↓ do (ii) first

↓ atoms stable ↓ do (i).

Key parameters:

① initial T.
② annealing schedule
③ length of run.
④ stopping condn
⑤ often decided by trial & error.

while ($x_{(new)} - x_0$ <= tolerance)
~~run~~ → stop the algo.

M(:,1)

EVOLUTIONARY ALGO.

① Sphingobium chlorophenylium : digest DDT. → carbon source.

→ anthropological.

② Antibiotic resistance → Not cool! → antibiotic resistance in hospitals - UK

→ MRSA - methicillin resistant staphylococcus aureus.

13/3/19

→ Evolution happens in population.
→ genetically heritable. - →

Applications
→ any optimisation problem, ppl usually go with non-evol. algo
→ if that doesn't work, evol. algo is set to work.

Defn:
Search procedure that probablistically applies search operator to a set of pts. in search space

↓ no longer a single pt (i.e, simulated annealing)

Biological Evolution:
→ Lamark & others → bad theory! but ✗. computing.
→ Darwin & Wallace → survival of the fittest
→ Mendellian genetics → genotype - phenotype mapping

"Species transmute over a period of time - passing on phenotypic changen"

→ Genetic algos are blackboxes that converge poorly. But okay!

(4) → Deep neural networks are overfitting → mathematical sin !!

(5) → India's 10k genome sequencing project.

(6) ______

→ Evolutionary Algo (EA)

→ Genetic Algorithm (GA) → bit representation

→ Evol. Programming (EP)

→ Evol. Strategies (ES) → real no. representation

→ Genetic Programming.

→ Evolvable software

→ Evolvable hardware (Adrian Thomson)

Key Terms

(i) Population

(ii) Chromosome / Individual

- a small change in genotype – Mutation. (iii)
- exchange of genetic material – Recombination (iv)
- ability to survive & reproduce – Fitness (v)
- ability to survive a screen test – Selection. (vi)

______

Mathematical rep:

10000 generation = x

100 = n

x_{00} x_{10} x_{20} x_{x0}
x_{01}
x_{02}
x_{03}
x_{0n}

→ POPULATION

0 : x^{th} generation

n : population size

1	1	0	1	1	0	0	1	0	1	1	0	1

INDIVIDUAL

↓

1	1	0	0	1	0	0	1	0	1	1	0	1

MUTATION – Point

∴ ~~how~~ which bit changes? → random!

& ~~no.~~ what is of mutation rate? → hyperparameter!

∴ $N\mu = 10$ → 10 mutations per generation.

(μ → mutation rate)

RECOMBINATION within children → reproduce

→ population

10101|011 → 10101110

10000|110 → 10000011

higher the $\boxed{\text{FITNESS}}$ → lower the COST

∴ strategies : if cost $f^n = f$ & fitness $= g$.

$g = -f$ / $g = -|f|$ / $g = \frac{1}{f}$ / etc....

→ Probability ~~of choosing based on fitn~~ of $\boxed{\text{selection}}$ is based on $\boxed{\text{fitness}}$ } Fitness proportionate

→ Tournament selection : Pick k individuals @ random and choose the fittest from these. (repeat it 100 times)

→ Random selection. → completely!

→ Elitest selection (Hill climbing)

↓

~~order them~~ is but might end up in local minima.

→ all these are basically strategies to choose the parameter vector.

But....

In every context, ~~does~~ what are these bits? & does this rep. allow mutations?

→ If we are looking @ Delivery man's schedule : 1 2 3 4 5 6 7 8 9 10

↓ mutate

1 2 4 4 5 6 7 8 9 10

↓

2 parcels to ④th guy.

∴ we'd opt for swapping. that is how we'd define mutation.

1 2 3 5 4 6 7 8 9 10

$\boxed{\text{NASA Antenna Design – Evol. Algo}}$ ← Reading

How do we find representation based on problems?

Evolution Strategies :

> strategy parameter.

> all parameters evolve.

> self - adaptation.

> real numbers.

> self -adaptation : genotype adapts to alter the evolutionary

④
→ Field Programmable Gate Array (FPGA).
(a set of n×n gates that can be assigned as any gate while in the field).

→ Representation paradigms.
① simple binary chromosome.
② trees & complex data structure.
③ Cartesian Genetic Programming.

they are programmed to do such computations

→ [computationally heavy problems are dumped onto GPU(s).

IMPLEMENTING EAs.

Operators:
① macro-mutation.
② hybrid operators.
③ Operators for permutation
...

Selection: (PTB)

Application:
① scheduling
② Biology → phylogenetic trees.
→ protein folding.
→ clustering μarray data
→ identifying coding regions
③ Electric circuit design.

When to use EAs?
① when we know nothing about search space.
② often useful.
③ no reason how it is any better than GA?

Challenges: ① Black-box behaviour

Singular Value Decomposition

18/3/19. ROOT FINDIN
Defn: Given alge
Classification of m
finding approximate soln which includes lot of iterat
• (x+2)(x−3
roots : r =
Sim
Analytical met
however, when
∴ Graphical m
→ Sometime
Application
Hypositting
Numerical
Open meth

[ATHI SIR takes over]

18/3/19. ROOT FINDING PROBLEMS - SOLUTIONS of NON LINEAR EQN

Def^n: Given algebraic eq^n $f(x)$, we find $x(s)$ $\rightarrow$ r such that $f(r) = 0$

Classification of methods: ① analytical soln.

② graphical methods $\rightarrow$ useful for initial guesses.

finding approximate soln which includes lot of iteration. $\leftarrow$ ③ numerical methods

$\rightarrow$ bracketing methods.

$\rightarrow$ open methods

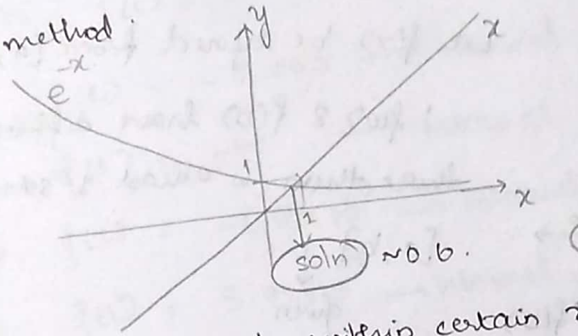
$(x+2)(x-3)^2(x+1) = f(x)$.

roots: $\underbrace{r = -2, -1}_{\text{simple roots}}$ and $r = 3 \rightarrow$ with multiplicity $= 2$.

Analytical method: $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

however, when $f(x) = x - e^{-x}$; $\nexists$ method X.

$\therefore$ graphical method:



root $\in [0, 1]$

bracketing method

$\rightarrow$ sometimes, we find soln within certain ranges. we set boundaries.

Application: Binding affinity: $(10^{-6} - 10^{-9})$

how strong 2 molecules are bound.

Hypersetting: dimension intermixing for soln finding

Numerical Methods:

① Bisection ② Newton's ③ Secants

④ False Position ⑤ Muller's ⑥ Bairstow

Open method $\rightarrow$ starts with 1/more initial guesses.

in each iteration, new root is obtained

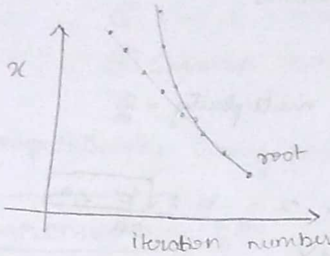
not ~~better than~~ bracketing method

Convergence Notation

Let $x_1, x_2, \ldots$ converge to x.

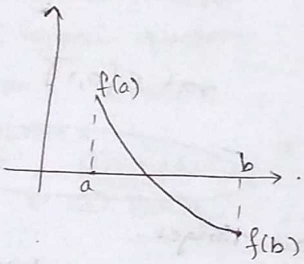
Linear convergence : $\frac{|x_{n+1} - x|}{|x_n - x|} \le C$ → the relative error of x_n converges.

Convergence of order p : $\frac{|x_{n+1} - x|}{|x_n - x|^p} \le C$ [quadratic $p=2$]



linear is slower than quadratic

Intermediate Value Theorem :



Let $f(x)$ be defined from $[a, b]$.

∵ $f(a)$ & $f(b)$ have different signs there there is atleast 1 soln within $[a, b]$.

↓ given

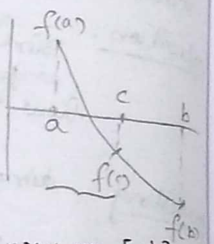
Bisection algorithm :

loop

(i) compute . $c = \frac{a+b}{2}$ and $f(c)$. . if $f(c) = 0$ → break.

(ii) check $f(a) \cdot f(c) < 0$ → new interval: $[a, c]$

$f(a) \cdot f(c) > 0$ → new interval : $[c, b]$.

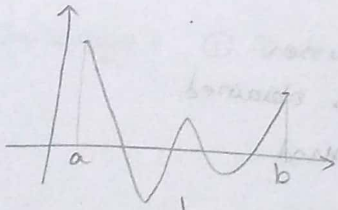


end loop.

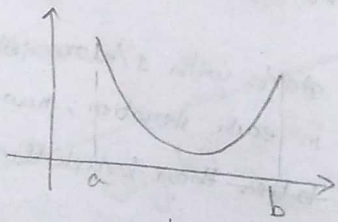
→ assumptions : (i) $f(x)$ is continuous on $[a,b]$

(ii) $f(a) \cdot f(b) < 0$

Problem :



↓ you gotta break the intervals.



↓ no real zeroes

Q. $f(x) = x^3 - 3x + 1$

$f(0) = 1$. $f(2) = 3$ -

Let $c = \frac{0+2}{2} = 1$.

$[0, 1]$

$c = \frac{1}{2}$. $f(c) = \frac{?}{8}$ =

$[0, 0.5]$

$c = 0.25$ $f(c) = 0$

~~$c = 0.75$, $f(c)$~~

$c = 0.375$, $f(c)$ =

$c = 0.3125$, $f(c)$ =

$c = 0.34375$, $f(c)$

$c = 0.36$, $f(c)$

$c = 0.352$, $f(c)$

$c = 0.348$, $f(c)$

$c = 0.346$, $f(c)$

pliss

$c = 0.347$, $f(c)$

$c = 0.3475$, $f(c)$

$c = 0.34725$, $f($

How / when do we

→ pre-set n

→ fixed erro

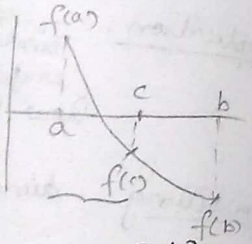
→ fixed prec

...lative error, on ...
...ges

...adratic $p=2$]

quadratic

...rom $[a, b]$.
different signs
1 soln within



...uous on $[a,b]$

8. $f(x) = x^3 - 3x + 1$ in the interval $[0, 2]$

$f(0) = 1$, $f(2) = 8 - 6 + 1 = 3$ → assumptions are not satisfied.

Let $c = \frac{0+2}{2} = 1$. $f(1) = 1 - 3 + 1 = -1$

$[0, 1]$

$c = \frac{1}{2}$, $f(c) = \frac{1}{8} = 0.125 - 1.5 + 1 = -0.375$

$[0, 0.5]$

$c = 0.25$ $f(c) = 0.2656$ → ∴ interval $[0.25, 0.5]$

~~c = 0.75, f(c) = -0.828 → interval: 0.5,~~

$c = 0.375$, $f(c) = -0.072$ → ∴ interval $[0.25, 0.375]$

$c = 0.3125$, $f(c) = 0.093$ → interval: $[0.3125, 0.375]$

$c = 0.34375$, $f(c) = +ve$ (9.9×10^{-3}) → interval: $[0.34375, 0.375]$

$c = 0.36$, $f(c) = -0.033$ → interval: $[0.34375, 0.36]$

$c = 0.352$, $f(c) = -ve$ → interval: $[0.34375, 0.352]$

$c = 0.348$, $f(c) = -1.5 \times 10^{-3}$ → interval: $[0.34375, 0.348]$

$c = 0.346$, $f(c) = 3.75 \times 10^{-3}$ → interval: $[0.346, 0.348]$

pliss ... $\boxed{\text{Ans } 0.347}$

$c = 0.347$, $f(c) = +ve$ → interval $[0.347, 0.348]$

$c = 0.3475$, $f(c) = -ve$ $[-5.4 \times 10^{-4}]$ → interval $[0.347, 0.3475]$

$c = 0.34725$, $f(c) = 1.2 \times 10^{-4}$ → close to 0.

How/when do we stop?

→ pre-set number of iteration

→ fixed error rate

→ fixed precision

$x \quad f(x) = x - \cos x$

Q $f(x) = \cos(x)$ absolute error < 0.2 in [0.5, 0.9]

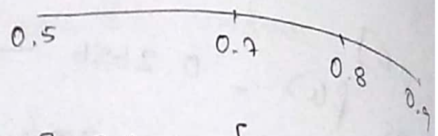
~~f(0.5) =~~ (1) Assuming ~~go~~ degree s.

~~$f(a) = \cos 0.5 = 0.99996$; $\cos(0.9) = 0.99987$.~~

radians :

$f(a) = -0.3776$, $f(0.9) = 0.2784$. $\rightarrow c = 0.7$.

$f(c) = -0.0648$ error : 0.2 ✓. [0.7, 0.9]



$c' = 0.8$.

$f(c) = 0.1033$ ~~$\rightarrow$ inter $x - \cos x = 0.8 - 0.1033$~~ [0.7, 0.8]

note: $\boxed{\text{apparent error @ every stage} \rightarrow \sim \frac{b-a}{2}}$.

$c' = 0.75$.

$f(c) =$

Bisection method.

→ slow to converge → no derivative req.

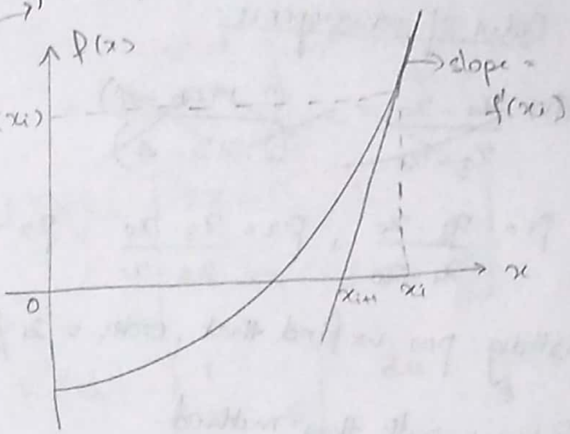
→ good intermediate approx. may be discarded

→ bound (proper) is required.

Newton - Raphson method. { Assumption,

① $f(x)$ is continuous & first derivative is known.

② initial guess x_0, such that $f'(x_0) \neq 0$



$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

$$\Rightarrow x_i - x_{i+1} = \frac{f(x_i)}{f'(x_i)}$$ that's why

$$\Rightarrow \boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

Q. $f(x) = x^3 - 2x^2 + x - 3$, $x_0 = 4$

$f'(x) = 3x^2 - 4x + 1$.

• $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f(4) = 64 - 32 + 4 - 3 = 33$.

$f'(4) = 48 - 16 + 1 = 33$. $\Rightarrow x_1 = 4 - 1 = 3$.

$\frac{48 \; 9}{39}$

• $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$f(3) = 27 - 18 + \cancel{3} - \cancel{3} = 9$.

$f'(3) = 27 - 12 + 1 = 15 + 1 = 16$. $\Rightarrow x_2 = 3 - \frac{9}{16} = \frac{39}{16}$

$= 2.4375$.

• $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$f\left(\frac{39}{16}\right) = \cancel{2.4375}$. 2.0369 $f'\left(\frac{39}{16}\right) = 9.0742$.

$x_3 = 2.4375 - \frac{2.0369}{9.0742} = 2.213$.

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$f(2.213) = 0.256$.

$f'(2.213) = 6.8404$ $\Rightarrow x_4 = 2.213 - \frac{0.256}{6.8404}$

$\underline{x_4 = 2.1756}$.

$f(2.1756) = 0.0065$

$f'(2.1756) = 6.4969$ $\Rightarrow x_5 = 2.1756 - \frac{0.0065}{6.4969}$

$\underline{x_5 = 2.1746}$

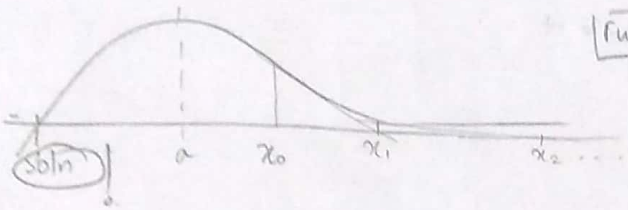
Order of convergence:

(i) $\cancel{\dfrac{x_4 - x_0}{x_3 - x_0}} = \cancel{\dfrac{(2.1756 - 4)}{(2.213 - 4)}} =$

$p_1 = \dfrac{x_2 - x_0}{x_1 - x_0}$, $p_2 = \dfrac{x_3 - x_0}{x_2 - x_0}$, $p_3 = \dfrac{x_4 - x_0}{x_3 - x_0}$, ...

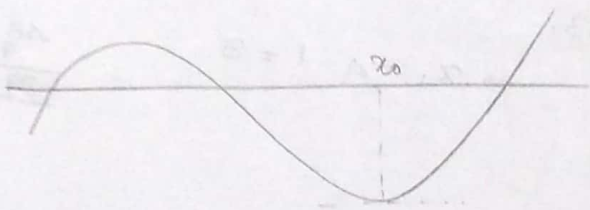
plotting p(s) we find that, order = 2 // usually Quadratic convergence

Problems with this method.



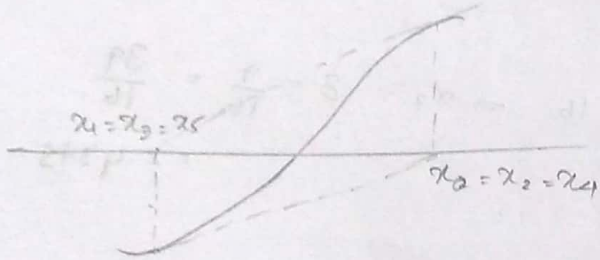
$\boxed{\text{runaway}}$ problem.

$\therefore x_0$ should be less than a.



$f'(x_0) = 0$

if given x_0 is this, we cannot proceed.



$\boxed{\text{Cycle}}$

algorithm cycles b/w 2 values x_0 & x_1.

Problems:

$f'(x_i)$ not available & or

too complex to solve analytically

→ For systems of non-linear eqⁿ:

X : vector
F(X) : matrix.

initial guess = X_0 of $F(X) = 0$

$$x_{k+1} = X_k - [F'(X_k)]^{-1} F(X_k).$$

where
$$F(X) = \begin{bmatrix} f_1(x_1, x_2, \dots) \\ f_2(x_1, x_2, \dots) \\ \vdots \end{bmatrix} \qquad F'(X) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots \\ \frac{\partial f_2}{\partial x_1} & \dots & \\ \vdots & \vdots & \vdots \end{bmatrix}$$

Example:
$$y + x^2 - 0.5 - x = 0.$$
$$x^2 - 5xy - y = 0.$$

initial guess : $x = 1$, $y = 0$.

$$X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\} \qquad F'(X) = \begin{bmatrix} (2x-1) & 1 \\ 2x - 5y & (-5x-1) \end{bmatrix}$$

$$F(X_0) = \begin{bmatrix} f_1(X) = -0.5 \\ f_2(X) = 1 \end{bmatrix} \qquad F'(X_0) = \begin{bmatrix} 1 & 1 \\ 2 & -6 \end{bmatrix}$$

det = −6 − 2 = −8

$$[F'(X_0)]^{-1} = \frac{1}{-8}\begin{bmatrix} -6 & -1 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow X_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{8}\begin{bmatrix} -6 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} -0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{8}\begin{bmatrix} \cancel{1.25}\ 2 \\ 0 \end{bmatrix}$$

$$X_1 = \cancel{\begin{bmatrix} 1.214 \\ 0 \end{bmatrix}} \quad \begin{bmatrix} 1.25 \\ 0.25 \end{bmatrix}$$

$$\cancel{F(x_1) = \begin{bmatrix} -0.2397 \\ 1.474 \end{bmatrix}} \qquad \cancel{F'(x_1) = \begin{bmatrix} 1.428 & 1 \\ 2.428 & \end{bmatrix}}$$

$$F(x_1) = \begin{bmatrix} 0.0625 \\ -0.25 \end{bmatrix} \qquad F'(x_1) = \begin{bmatrix} 1.5 & 1 \\ 1.25 & -7.25 \end{bmatrix}$$

det = −12.125

$$X_2 = \begin{bmatrix} 1.25 \\ 0.25 \end{bmatrix} + \frac{1}{12.125}\begin{bmatrix} 1.5 & 1 \\ 1.25 & -7.25 \end{bmatrix}\begin{bmatrix} 0.0625 \\ -0.25 \end{bmatrix}$$

$$\begin{bmatrix} 1.25 \\ 0.25 \end{bmatrix} + \frac{1}{12.125}\begin{bmatrix} -0.15625 \\ \cancel{2.89}\ 2.594 \end{bmatrix} = \cancel{\begin{bmatrix} 1.234 \\ 0.406 \end{bmatrix}} \quad \begin{bmatrix} 1.23 \\ 0.2163 \end{bmatrix}$$

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$$F(X_2) = \begin{bmatrix} -4.1\times10^{-3} \\ -0.01005 \end{bmatrix} \qquad F'(X_2) = \begin{bmatrix} 1.46 & 1 \\ 1.395 & -2.15 \end{bmatrix}$$

det = -11.834

$$[F'(X_2)]^{-1}F(X_2) = \frac{1}{-11.834} \cancel{\begin{bmatrix} 1.46 & 1 \\ 1.395 & -2.15 \end{bmatrix}} \begin{bmatrix} -2.15 & -1 \\ -1.395 & 1.46 \end{bmatrix} \begin{bmatrix} -4.1\times10^{-3} \\ -0.01005 \end{bmatrix}$$

$$= \begin{bmatrix} 3.326\times10^{-3} \\ -0.757\times10^{-3} \end{bmatrix}$$

$$\Rightarrow X_3 = \begin{bmatrix} 1.23 \\ 0.213 \end{bmatrix} + \begin{bmatrix} 3.326\times10^{-3} \\ -0.759\times10^{-3} \end{bmatrix} = \begin{bmatrix} 1.233 \\ 0.2122 \end{bmatrix}$$

$\boxed{20/3/19}$

Q. $y + x^2 - 1 - x = 0$

$x^2 - 2y^2 - y = 0$

$$F'(x) = \begin{bmatrix} (2x-1) & 1 \\ 2x & (-4y-1) \end{bmatrix}$$

$x = 0, y = 0.$

$$X_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad F(X_0) = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad F'(X_0) = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

det = 1.

$$[F'(X_0)]^{-1} = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + - \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$F(X_1) = \begin{bmatrix} y - x + 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad F'(X_1) = \begin{bmatrix} -3 & 1 \\ -2 & -1 \end{bmatrix}$$ det $= 3 + 2 = 5$

$$[F'(X_1)]^{-1} \times F(X_1) = \frac{1}{5} \begin{bmatrix} -1 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

$-1 + \frac{2}{5}$

$$X_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \frac{1}{5} \begin{bmatrix} +2 \\ +1 \end{bmatrix} = \begin{bmatrix} -3/5 \\ 1/5 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$F(X_2) = \begin{bmatrix} +4/25 + 9/25 - \ldots \\ 9/25 - 2/25 - \ldots \end{bmatrix}$$

$$F'(X_2) = \begin{bmatrix} -11/5 & 1 \\ -6/5 & -9/5 \ldots \end{bmatrix}$$

$$X_3 = \frac{1}{5} \begin{bmatrix} -3 \\ 1 \end{bmatrix} - \frac{25}{12\ldots}$$

$$= \begin{bmatrix} -3/5 \\ 1/5 \end{bmatrix} + \frac{1}{12\ldots} \begin{bmatrix} +4\ldots \\ +3\ldots \end{bmatrix}$$

$$X_3 = \begin{bmatrix} -0.6 \\ 0.2 \end{bmatrix} - \frac{25}{12\ldots}$$

$$X_4 = \begin{bmatrix} -0.5257\ldots \\ 0.1980\ldots \end{bmatrix}$$

SECANT Method.

→ Secant method

$f'(x) \simeq \frac{f(x+h) - \ldots}{h}$

if x_i & x_{i-1} are ...

$f'(x_i) = \frac{f(x_i) - \ldots}{(x_i - \ldots)}$

$x_{i+1} = x_i - \frac{f(\ldots}{f(\ldots}$

∴ we are discuss...

$$f(x_2) = \begin{bmatrix} \cancel{+4/5} + 9/25 \cancel{- + 3/5} \\ \cancel{8/25} - 2/25 \cancel{- 6/25} \end{bmatrix} = \begin{bmatrix} -4/25 \\ \cancel{238/25} \end{bmatrix}$$

$$f'(x_2) = \begin{bmatrix} -11/5 & 1 \\ -6/5 & -9/5 \end{bmatrix} \qquad [F'(x^2)]^{-1} = \frac{25}{129}\begin{bmatrix} -9/5 & -1 \\ 6/5 & -11/5 \end{bmatrix} \quad \left\{ \det = \frac{99}{25} + \frac{30}{25} = \frac{129}{25} \right.$$

$$x_3 = \frac{1}{5}\begin{bmatrix} -3 \\ 1 \end{bmatrix} - \frac{25}{129}\begin{bmatrix} -9/5 & -6/5 \\ 6/5 & -11/5 \end{bmatrix}\begin{bmatrix} \cancel{4/25} \\ \cancel{28}/25 \end{bmatrix} \qquad \cdot \frac{-36}{5} \quad -38$$

$$= \begin{bmatrix} \cancel{3/5} \\ 1/5 \end{bmatrix} \cancel{+ \frac{1}{129}\begin{bmatrix} +45.2 \\ +38.8 \end{bmatrix}} \quad \cancel{-0.6} \quad 0.2$$

$$x_3 = \begin{bmatrix} -0.6 \\ 0.2 \end{bmatrix} - \frac{25}{129}\begin{bmatrix} \cancel{-28}\ 46/5 \\ 2/5 \end{bmatrix} = \begin{bmatrix} -0.528 \\ 0.203 \end{bmatrix} //$$

$$\ldots \quad x_4 = \begin{bmatrix} -0.5257 \\ 0.1980 \end{bmatrix} \qquad x_5 = \begin{bmatrix} -0.5257 \\ 0.1980 \end{bmatrix} \;!!!$$

SECANT Method.

→ Secant method → Examples → Convergence Analysis.

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}.$$

if x_i & x_{i-1} are 2 initial points:

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{(x_i - x_{i-1})} \Rightarrow x_i \cancel{- x_{i+1}} = \frac{f(x_i) - f(x_{i-1})}{f'(x_i)} + x_{i-1}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{\frac{f(x_i) - f(x_{i-1})}{(x_i - x_{i-1})}} = x_i - f(x_i) \cdot \frac{(x_i - x_{i-1})}{f(x_i) - f(x_i)}$$

∴ we are discretising the function!.

$f(x) = x^2 \pm 2x + 0.5 \qquad x_0 = 0, \; x_1 = 1$

$$x_{i+1} = x_i - f(x_i)\frac{(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

x_0	$f(x_0)$
0	0.5
1 ~~0.5~~	-0.5
0.5	-0.25
0	0.5
1/3	0.056
0.372	-0.105

Soln:

$f(x_0) = 0.5 \qquad f(x_1) = 1 - 2 + 0.5 = -0.5$

$x_2 = 1 \cancel{x_1} + (+0.5)\left[\frac{\cancel{1-0}}{\cancel{-0.5-0.5}}\right]^{-1} = \boxed{0.5 = x_2}$

$f(x_2) = 0.25 - 1 + 0.5 = -0.25$

$x_3 = 0.5 + (+0.25)\left[\frac{0.5 - 1}{-0.25 + 0.5}\right] = 0.5 + 0.\cancel{25}\left[\frac{-0.5}{0.\cancel{25}}\right] = 0$

$x_4 = 0 + -(0.5)\left[\frac{0 - 0.5}{0.5 + 0.25}\right] = \frac{+0.25}{0.75} = \frac{1}{3}$

$f(x_4) = \frac{1}{9} - \frac{2}{3} + 0.5 = 0.056$

$x_5 = 0.3333 - (0.05\cancel{6})^{56}\frac{[0.333 - 0]}{[0.05\cancel{6}6 - 0.5]}$

$x_5 = \cancel{0.33} \quad \cancel{0.372} \quad \boxed{0.2916}$ $\xrightarrow{x}$ to get the other root, one should start from another set of x_0 & x_1.

$f(x_5) = \cancel{-0.105} \quad 1.8 \times 10^{-3} = 0.0018$

~~Modified~~ $x^2 - 2x + 0.5 \qquad \sqrt{D} = \sqrt{4 - 4(0.5)} = \sqrt{2}$

$x = \frac{+2 \pm \sqrt{2}}{2} = \frac{\sqrt{2} \pm 1}{\sqrt{2}} = 1 \pm \frac{1}{\sqrt{2}}$

roots: $x = 1 + \frac{1}{\sqrt{2}}, \quad x = 1 - \frac{1}{\sqrt{2}}$

$= 1.707 \qquad = +0.29.$

Modified Secant method:

→ difference b/w 2 x_i are given (δ)

$$f'(x_i) = \frac{f(x_i + \delta x_i) - f(x_i)}{\delta x_i}$$

$$\rightarrow x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta x_i) - f(x_i)}.$$

but how to select δ?
else it will diverge!!

then have 4 decimal digits

$\boxed{error = -0.001}$

$f(x) = x^5 + x^3 + 3.$ $x_0 = -1$, $x_1 = -1.1$

x	$f(x)$
-1	1
-1.1	0.0585
-1.1062	-0.01
-1.105	0.0031
-1.105	0.001

$f(-1) = -1 - 1 + 3 = 1$

$f(-1.1) = 0.0585$

$$x_2 = -1.1 - (0.0585)\left[\frac{+0.1}{+0.9415}\right]$$

$$x_3 = -1.1062 + (+0.01)\left[\frac{+0.0062}{+0.0685}\right]$$

$x_3 = -1.105$

$f(x_3) = 0.007$

$$x_4 = -1.105 - (0.007)\left[\frac{0.001}{0.017}\right] = -1.105$$

$\Delta f(x) = 0.0001.$