BT 2020 — Numerical Methods for Biology
Jan–May 2018
Quiz Questions

April 27, 2018

1. **Justify** (as briefly as possible) whether each of the following statements is true or false. If the justification is incorrect, no credit will be awarded. **Answer sub-questions in the correct order.**

   (a) The float `0xfff1234561234567` is normalised

   (b) For a linear system given by $Ax = b$, if our measurements of the values of b_i are 99.0% accurate, then our estimates of x will have a relative error of at most 1%

   (c) The matrix $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ is in echelon form

   (d) The conditioning of a problem depends on the algorithm used to solve it

   (e) The ∞-norm of $[3 \quad 9 \quad -2 \quad -11]$ is 9.0

   (f) The single-precision floats 0x00000000 and 0x80000000 represent the same number

   (g) Consider a program that uses a Monte Carlo method to estimate π. For this program, the estimate of π from $10,000$ iterations is guaranteed to be closer to the real value of π than after $9,000$ iterations.

   (h) The vector $[-1 \quad 0 \quad 1/\sqrt{2}]^T$ is not an eigenvector of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

   (i) If $\mathbf{A} = \mathbf{QR}$ gives the QR decomposition of $\mathbf{A}$, then the eigenvalues of $\mathbf{QR}$ and $\mathbf{RQ}$ are the same

   (j) The product of three orthogonal matrices is not always orthogonal

2. Why is pivoting important in Gaussian elimination?

3. Given an example of a biological problem, where echelon forms are relevant.

4. What is the floating-point double (hex) representation of the number 2.025?

5. IEEE also has a 128-bit notation where $|e| = 15$. What is the value of the smallest normalised floating point number that can be represented in 128 bits?

6. On June 4, 1996 an unmanned Ariane 5 rocket launched by the European Space Agency exploded just forty seconds after its lift-off from Kourou, French Guiana. The rocket was on its first voyage, after a decade of development costing $7 billion. The destroyed rocket and its cargo were valued at $500 million. A board of inquiry investigated the causes of the explosion and in two weeks

issued a report. It turned out that the cause of the failure was a software error in the inertial reference system. What was nature of the floating-point error that led to this failure?

7. Perform a Cholesky decomposition of the matrix

$$A = \begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix}$$

8. Perform Gauss elimination on the following system and solve for x:

$$A = \begin{bmatrix} -1 & -2 & 1 & 1 \\ 1 & 2 & 4 & -3 \\ 3 & 3 & -8 & 2 \\ -1 & 1 & 6 & -3 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 5 \\ -7 \\ 7 \end{bmatrix}$$

9. Indicate the correct **answer(s)** for the following questions with multiple choices and **explain your answer**. Partial credit will not be awarded.

   (a) Which of the following are possible sequences of random bits from a uniform random generator?

   i. $[0, 0, 0, 0, 0, 0, 0, 0]$
   ii. $[0, 1, 1, 1, 1, 0, 1, 0]$
   iii. $[1, 1, 1, 1, 0, 0, 0, 0]$
   iv. $[0, 1, 1, 0, 0, 1, 0, 1]$
   v. $[0, 1, 0, 1, 0, 1, 0, 1]$

   (b) Which of the following are hyperparameters for genetic algorithms?

   i. Cooling schedule
   ii. Population size
   iii. Number of generations
   iv. Initial temperature

   (c) A 7×7 matrix with all real entries can have __ complex eigenvalues

   i. 6
   ii. 1
   iii. 3
   iv. 0

10. State the Metropolis criterion in simulated annealing. Why is it important?

11. Write the Maclaurin series expansion of $ln(1 + x)$ until the 5^{th} term. Write down a generic expression as a function of n and x, where n is the number of terms and ranges from 1 to infinity. Give the range of x in which the expansion is valid.

12. Compute the eigenvalues of the matrix: $J = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

13. Outline the steps to fit a curve $y = ax^2 + bx - 5$ to the points $(2, 1)$, $(6, 49)$, $(18, 481)$.

14. Illustrate how you will use the power iteration method to find an eigenvalue of the matrix $\begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}^T$, starting with the vector $u_0 = [2 \quad 1]^T$.

15. Find a matrix H such that it will reflect the vector $\begin{bmatrix} 1 & 1 & 4 & 8 \end{bmatrix}^T$, such that the last *two* elements of the vector are zeroed out.

16. Which of the following expressions evaluate to **zero** in MATLAB (on a 64-bit computer)? **Explain your reasoning for each case.**

    (a) `sqrt(3)-(3/sqrt(3))`

    (b) `(1+eps(1)/2)-1`

    (c) `10^400-10^400`

    (d) `0.125 + 0.375 - 0.5`

17. What is the floating-point double (hex) representation of the number 2^{52}?

18. What is the floating-point double (hex) representation of the number 1.025?

19. For single precision, what is the value of the smallest normalised float that can represented ($|s| = 1, |e| = 8, |f| = 23$)

20. How will you represent NaN in single precision?

21. What is a Householder reflection?

22. Explain how you will use computer simulations to determine the probability of rolling a 10 with three fair dice (write a small piece of code).

23. Compute the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 2 \\ 1 & 6 \end{bmatrix}$

24. Compute $\begin{bmatrix} 5 & 2 \\ 1 & 6 \end{bmatrix}^{25}$