CHAPTER EIGHT

The Yield Curve

"The timing looks right for a Fed easing and this bodes well for short average-life MBS," read the research report in Susan's hands. Yes, she had heard this story many times before but it always seemed to be built on some sort of spurious reasoning. If only she could develop some way to value the impact on MBS cashflows. Her single security calculators were just fine for yields and even vectors of prepayments, but they provided no guidance to judge the effect of a reshaped yield curve.

■ INTRODUCTION

In several previous chapters, we have concentrated on building tools related to both MBS and the yield curve. These tools have consisted of cashflow generators and methods of risk, and yield curve analysis. In this chapter, we bring some of these tools together to analyze the relationship between the yield curve and MBS value. The exercises in this chapter develop an understanding of how the entire curve relates to the MBS spread over Treasuries and how to relate curve reshaping to changes in value.

■ SHAPE OF THE CURVE

MBS have a fundamental relationship to the yield curve for several reasons. Monthly cashflows make the value of an MBS susceptible to changes in rates along the entire curve. While this is essentially true for all coupon-bearing securities, MBS will generally be affected more because cashflows are more "front loaded" than securities such as Treasury bonds. For example, changes in the short end of the yield curve are generally going to have a greater impact on the change in value for a 30-year MBS than for a 30-year Treasury bond.

The relationship between MBS cashflows and interest rates is well known. This has been the central focus of discussions related to prepayments. We will extend this relationship further to understand how expectations about forward rates influence MBS cashflow for purposes of valuation.

To get some feel for the impact of curve shape on prepayments and valuation, consider the upward sloping yield curve. An upward sloping yield curve indicates that the markets value securities as if rates are expected to increase in the future. If we used this rising rate scenario to drive a prepayment model, we would be slowing down the expected prepayment rates compared to an assumption of rates remaining constant at today's rates. Under the forward rate expectations, investors in premiums and IOs would benefit, while holders of discounts, POs, and inverse floaters would see an erosion of value. Forward rate expectations are built into valuation tools such as OAS, so it is important for an investor to understand the relationship between forward rates, cashflows, and valuation.

SPREAD TO THE BENCHMARK

MBS are essentially a spread product. That is, investors calibrate the relative advantage of an MBS to the Treasury yield curve to derive some measure of value. The Treasury curve serves as a useful benchmark because it both represents the risk-free rate of interest and has observable yields extending out to 30 years. Since other markets are compared to Treasuries, such as corporates and agencies, we can use the Treasury curve as a means to calibrate intermarket relative value.

The most common measure of spread compares the difference in yield between two securities. When comparing a five-year corporate issue to a five-year Treasury note, the comparison is somewhat straightforward. However, it makes no sense to compare the yield of a 30-year MBS to that of a 30-year Treasury. Although the two securities share a similar final maturity, the pattern and timing of cashflows do not make for a reasonable comparison.

The appropriate benchmark Treasury is chosen not based on the final maturity of an MBS but rather on its weighted average life (WAL).[1] The weighted average life is used as a proxy for the maturity of an MBS as it represents the average time to receipt of principal. In some cases, the Treasury benchmark is chosen by rounding the WAL of the particular MBS down to the closest on-the-run Treasury.

Spreads to Benchmark Treasury

Assume that the Treasury yield curve has the following rates:

[1] In some cases the benchmark spread may be compared to a Treasury of similar cashflow duration. In this way a benchmark is found that relates to the timing of both principal and interest.

Table 8-1
Sample Yield Curve

Maturity	Yield
1	4.50
2	5.00
3	5.75
5	6.00
10	6.60
30	7.00

We determine MBS spreads to a specific benchmark Treasury for the following securities:

Table 8-2
Spreads to Benchmark Treasury

	Yield	WAL	Benchmark Treasury	Spread (bps)
MBS 1	6.00	2.4	2	100
MBS 2	6.70	4.4	3	95
MBS 3	7.10	7.0	5	110
MBS 4	7.75	12.0	10	115

Exercise 8-1
Suppose you decide to relax the prepayment speeds somewhat on the MBS in the example above, just enough to push the WALs out a bit. Determine the appropriate benchmark by rounding (up or down) to the closest Treasury. Then compute the spread to the benchmark. For simplicity we'll hold the yields of the MBS constant.

Work area

	Yield	WAL	Benchmark Treasury	Spread (bps)
MBS 1	6.00	2.6		
MBS 2	6.70	4.6		
MBS 3	7.10	7.6		
MBS 4	7.75	12.8		

Spread to Interpolated Average Life

As the previous example and exercise illustrates, the interaction of prepayment assumption and choice of appropriate Treasury benchmark may be somewhat skewed. A small change in the prepayment assumption could lead to a very different picture regarding the spread of an MBS. In an upward sloping yield curve, MBS will be marketed with as much cashflow pushed to the front end of the yield curve as possible. This leads to an increase in the marketed spread of the security.

An alternative to the arbitrary choice of a benchmark is the use of the interpolated average-life Treasury. The average life of a coupon-bearing Treasury security is the same as its maturity. Consequently, we can compare the spread of an MBS with a 3.6-year average life by interpolating between the three- and five-year Treasury securities.

Using the securities in Table 8-2, we have interpolated the yield of the Treasury security with the same average life. Spreads are calculated relative to the interpolated Treasury instead of the specific benchmark.

Table 8-3

Spreads to Interpolated Treasury

	Yield	WAL	Benchmark Yield	Benchmark Spread
MBS 1	6.00	2.4	5.30	70
MBS 2	6.70	4.4	5.93	77
MBS 3	7.10	7.0	6.24	86
MBS 4	7.75	12.0	6.64	111

The greatest effect can be seen for the shorter MBS. This is consistent with the effects of the first exercise, which showed how spreads moved when we changed the benchmark security.

Exercise 8-2

Solve for the benchmark yield and interpolated spread for the securities in the table below, as shown in Table 8-3.

Work area

	Yield	WAL	Benchmark Yield	Benchmark Spread
MBS 1	6.00	2.6		
MBS 2	6.70	4.6		
MBS 3	7.10	7.6		
MBS 4	7.75	12.8		

■ SPREAD TO THE CURVE

Taking the spread to the interpolated average-life Treasury is a reasonable improvement over rounding to a specific Treasury but it still has its limitations. Most importantly, taking a spread to a point on the yield curve ignores the shape of the curve and the relationship to MBS valuation.

To point this out, let's recall the equation, shown in Chapter 3, used to calculate the yield of an MBS, as seen again in Equation 8-1.

$$\text{Price} = \frac{\text{Cashflow}_{T_1}}{(1+\text{Yield}/1200)^{T_1}} + \ldots + \frac{\text{Cashflow}_{T_{WAM}}}{(1+\text{Yield}/1200)^{T_{WAM}}}$$

$$\text{Price} = \sum_{i=1}^{WAM} \frac{\text{Cashflow}_{T_i}}{(1+\text{Yield}/1200)^{T_i}}$$

Equation 8-1
Price and Yield for an MBS

We solve for the yield that equates the present value of the cashflows to the current price (including accrued interest). The subscript T again refers to the time from settlement, including actual delay days.

Yield is a useful measure to capture the expected return from holding a security until its final cashflow but it makes a fairly significant assumption. Namely, solving for yield assumes that we can continually re-invest monthly cashflows at the yield of the security. This can be a fairly aggressive assumption for long dated securities with cashflows spread out along the yield curve. A more reasonable assumption would be to consider a constant re-investment rate above the Treasury curve. That is, solving for a constant spread instead of solving for a specific yield.

The spread we're going to solve for is a specified number of basis points above the zero coupon Treasury curve. This would lead to the following modification of Equation 8-1.

$$\text{Price} = \frac{\text{Cashflow}_{T_1}}{(1+S_{T_1}+\text{sprd})^{T_1}} + \ldots + \frac{\text{Cashflow}_{T_{WAM}}}{(1+S_{T_{WAM}}+\text{sprd})^{T_{WAM}}}$$

$$\text{Price} = \sum_{i=1}^{WAM} \frac{\text{Cashflow}_{T_i}}{(1+S_{T_i}+\text{sprd})^{T_i}}$$

Equation 8-2
Constant Spread over Zero Curve

In Equation 8-2, the S terms refer to the spot rates corresponding to the maturity of the MBS cashflow. That is, we discount the MBS cashflow occurring in month 24 by the 24-month spot rate. We solve for the constant spread over the spot rates that equates the present value of the cashflows to the current price of the MBS. The spot rates in Formula 8-2 have been converted to monthly equivalents for use with MBS.

Solving for Spot Rates and Spread

In Chapter 4, we developed the method for deriving the spot curve given a par curve. An example can be seen in Table 8-4.

Table 8-4
Par and Spot Yield Curves

Maturity	Par Rate	Spot Rate
1	5.0	5.000
2	6.5	6.549
3	8.0	8.168
4	9.0	9.294
5	9.5	9.862

In order to illustrate the calculation of spread, we will assume a security with the following annual cashflows, based on a 9.5% coupon, a price of par, and an arbitrary amortization schedule shown in Table 8-5.

Table 8-5
Cashflows for a 9.5% Bond

Year	Balance	Principal	Interest	Cashflow
0	100			−100
1	90	10	9.50	19.50
2	75	15	8.55	23.55
3	55	20	7.13	27.13
4	30	25	5.23	30.23
5	0	30	2.85	32.85

Solving for the constant spread would require us to solve the following equation:

Equation 8-3
Solving for Spread to the Curve

$$100 = \frac{19.5}{(1+.05+sprd)} + \frac{23.55}{(1+.06549+sprd)^2} + \ldots + \frac{32.85}{(1+.09862+sprd)^5}$$

We could solve the equation using a trial and error method or by using some more sophisticated techniques. Using the "Goal Seek" feature of Excel, we could solve the equation by finding the value of the spread that makes the sum of the cashflows minus the starting price of 100 equal to zero.

The results of solving for the spread and other measures are shown in Table 8-6, below:

Table 8-6
Spread Measures

WAL	3.5
Yield	9.50
Spread to WAL	100
Spread to Spot Curve	87

The yield of the security equals the coupon if the starting price equals par. The spread to the interpolated average life security equals 100 basis points. However, after adjusting to the entire curve, our spread equals 87 basis points. The spread to the zero curve is lower than to the weighted average life because of the curve's relative shape and the distribution of cashflows. The marginal rate of change in the spot curve increases relative to the par curve.

The cashflows and price from Table 8-5 have been modified as shown in Table 8-7:

Table 8-7
Modified Cashflows

Year	Balance	Principal	Interest	Cashflow
0	100			−102
1	70	30	9.50	39.50
2	45	25	6.65	31.65
3	25	20	4.28	24.28
4	10	15	2.38	17.38
5	0	10	0.95	10.95

Based on these values, the following yield and WAL were calculated:

WAL	2.50
Yield	8.55

Exercise 8-3a

What is the spread to the interpolated average life par yield curve?

Exercise 8-3b

Using a spreadsheet, you can solve for the spread to the curve using a numerical procedure. Otherwise, solve for the spread by interpolating through the following table:

Spread	Price Minus Discounted Cashflows
0	1.697
50	.621
75	.091
100	–.435

To solve for the spread to the curve find the point where the price minus the discounted cashflows goes to 0. (Hint: The spread will be between 75 and 100 basis points).

EFFECT OF CHANGING THE CURVE SHAPE

Now that we have constructed a method for calculating the spread to the curve, we can apply this measure to judge the impact of a curve reshaping. We can change the shape of the par curve and then revalue the security at the same constant spread to the curve. In Table 8-8, the change in the curve will come from a steepening. Rates on the short end are going to decline while we will hold rates on the long end constant.

Table 8-8
Revised Yield Curve

Maturity	Par Rate	Spot Rate
1	3.0	3.000
2	5.0	5.051
3	8.0	8.317
4	9.0	9.425
5	9.5	9.975

Table 8-9
Valuing Cashflows at Initial Zero Spread of 87 Basis Points

Year	Balance	Principal	Interest	Cashflow	Discount Factor	Discounted Cashflow
0	100			–100.00	1.0000	–100.00
1	90	10	9.50	19.50	0.9627	18.77
2	75	15	8.55	23.55	0.8913	20.99
3	55	20	7.13	27.13	0.7682	20.84
4	30	25	5.23	30.23	0.6757	20.42
5	0	30	2.85	32.85	0.5976	19.63

In Table 8-9, we have added two new columns: the discount factor and the discounted cashflow. The discount factors relate to the terms in Equation 8-2. For example, the discount factors can be computed as follows:

$$\text{Discount Factor}_1 = \frac{1}{(1+.03+.0087)} = 0.9627$$

$$\text{Discount Factor}_5 = \frac{1}{(1+.09975+.0087)^5} = 0.5976$$

Equation 8-4
Solving for Discount Factors

To determine the discounted cashflow, we multiply the discount factor times the cashflow. We can then sum the discounted cashflows to get the present value of the security. This is equivalent to using Equation 8-2, except that we have assumed annual, not monthly cashflows. After the steepening of the yield curve, the present value of the cashflows now equals $100.65. The effect of short rates falling was a rise in price.

The standard method of solving for present value as shown in Equation 8-1, does not reflect a change in the shape of the yield curve on value. Neither the numerator containing the cashflows nor the denominator containing the yield are allowed to change, so there is no way to approximate the change in value.

After shifting the price to $100.65 we recompute the following measures of spread:

Table 8-10
Spread Measures

WAL	3.5
Yield	9.27
Spread to WAL	77
Spread to Spot Curve	87

Exercise 8-4

Why did the measures of yield and spread to the WAL benchmark change after the yield curve steepened? Why does the WAL remain constant? Why does the spread to the spot curve now exceed the spread to the average life benchmark?

Exercise 8-5

Impact of an Inverted Curve

We can use the same methods developed in Table 8-4 to judge the impact of an inverted curve on value. Now we will assume the following curve shape:

Maturity	Par Rate	Spot Rate
1	11.0	11.000
2	10.0	9.950
3	9.5	9.420
4	9.0	8.874
5	8.0	7.746

We will use the same spread of 87 basis points to the curve to compute the value. In the table below, calculate the discount factors and discounted cashflows:

Work area

Year	Balance	Principal	Interest	Cashflow	Discount Factor	Discounted Cashflow
0	100			−100	1.000	−100
1	90	10	9.500	19.500		
2	75	15	8.550	23.550		
3	55	20	7.125	27.125		
4	30	25	5.225	30.225		
5	0	30	2.850	32.850		

What is the updated present value of the discounted cashflows?

FORWARD RATE EFFECTS

In our previous analysis, we did not examine the impact of expected forward rates on MBS value. Because of prepayments, MBS cashflows depend upon the level of interest rates. We can now expand the analysis to develop some analytical measures of forward rates on cashflows and value.

Prepayment rates are driven by mortgage current coupon rates. Current coupon rates can be proxied by an intermediate Treasury plus a spread. A typical proxy is the 10-year Treasury rate. When creating a prepayment estimate, we can assume that the 10-year Treasury rate remains constant at current levels. Alternatively, we could assume that prepayment rates will be influenced by the forward 10-year rates.

To determine the forward 10-year rates we only need to observe the current par yield curve. From that we can derive the spot curve and from the spot curve we can calculate forward rates. That is, we could derive the projected 10-year rate in 2 years, in 3 years, and so on. (This procedure was described in Chapter 4.) Using these projected rates, we can calculate prepayment rates and solve for cashflows and spread.

Forward Rates

To keep matters somewhat simple, we will assume that our prepayments will be influenced by the one-year forward rates. Based on the starting term structure we can calculate the following implied one-year forward rates:

Table 8-11
Implied Forward Rates

Maturity	Par Rate	Spot Rate	One-Year Forward Rate
1	5.0	5.00	8.12
2	6.5	6.55	11.48
3	8.0	8.17	12.74
4	9.0	9.29	12.16
5	9.5	9.86	

The forward rate of 11.48% corresponds to the one-year rate starting at the end of year 2 and going until the end of year 3.

In order to demonstrate the combined impact of higher forward rates and slower prepayments, we will slow down the principal paydown used in the previous example.

Table 8-12
Cashflows with Slower Amortization

Year	Balance	Principal	Interest	Cashflow	Discount Factor	Discounted Cashflow
0	100			−100.00	1.00	−100.00
1	95	5	9.50	14.50	0.95	13.74
2	85	10	9.03	19.03	0.87	16.59
3	70	15	8.08	23.08	0.78	17.96
4	50	20	6.65	26.65	0.69	18.32
5	0	50	4.75	54.75	0.61	33.39

Using the same starting price we will recompute spreads to the curve and to the average-life benchmark.

The slowing of prepayments has shifted the average life to four years. The extension has reduced both the spread to the curve and to the average-life benchmark. Now, though, the spread to the curve exceeds the spread to a point on the curve. Once we extend the cashflows in this example, the average-life point of the MBS moves to a relatively flatter section of the curve. It's relatively more steep to the left of the average life than to the right of the average life.

Table 8-13
Spread Measures

WAL	4
Yield	9.50
WAL Spd	50
CRV Spd	53

Table 8-14
Change in Par and Spot Rates

	Par	Spot
Yr 1 - Yr 2	1.50	1.55
Yr 2 - Yr 3	1.50	1.62
Yr 3 - Yr 4	1.00	1.12
Yr 4 - Yr 5	0.50	0.57

Table 8-14 shows the change in par and spot rates going across the yield curve based upon the curves shown in Table 8-11. For example, as we go from year 1 to year 2, the par curve increases by 150 basis points while the spot curve increases by 155 basis points. Our security now has an average life of four years, and the spot curve is relatively more steep going from years 3 to 4 than the par curve. This increased steepness helps to raise the spread to the curve relative to the WAL benchmark.

Now that we have computed the spread based on implied forward rates, we have two measures of spread to the curve: the zero curve spread and forward curve spread.

Table 8-15
More Spread Measures

Spread Measure	Spread (bps)
Zero Curve	87
Forward Curve	53
Forward Effect	34

By applying the forward rates to project the prepayment rates, we extend the cashflows along the curve. This negatively impacts the relative value of the security, reducing its spread by 34 basis points.

Forward-Rate Effects with an Inverted Curve

Let's consider the interaction of forward rates with a reshaped yield curve. We will use the inverted curve from Exercise 8-5. After computing the spot rates we compute the implied one-period forward rates as shown in Table 8-16 below:

Table 8-16
One-Year Forward Rates

Maturity	Par Rate	Spot Rate	One-Year Forward Rate
1	11.0	11.00	8.91
2	10.0	9.95	8.37
3	9.5	9.42	7.25
4	9.0	8.87	3.35
5	8.0	7.75	

Table 8-16 shows that for an inverted curve, the forward rates are lower than the spot rates. To judge the impact of the new forward rates on prepayments we will compare the forward rates in the inverted curve to those of the base case in the table below:

Table 8-17
Inverted Curve Forward Rates

Year	Base Curve Forward Rates	Inverted Curve Forward Rates
1	8.12	8.91
2	11.48	8.37
3	12.74	7.25
4	12.16	3.35

Exercise 8-6

What should happen to the projected prepayment rates in the inverted curve?

Exercise 8-7

Valuing Cashflows in the Inverted Curve

To solve for the spread, we have computed the discounted cashflows shown in the table below at different spreads to the curve and compared their sum to the current price.

Cashflows

Year	Balance	Principal	Interest	Cashflow
0	100			-100.00
1	90	10	9.50	19.50
2	70	20	8.55	28.55
3	45	25	6.65	31.65
4	5	40	4.28	44.28
5	0	5	0.48	5.48

Values for the cashflows above are shown in the table below:

Spread	Price Minus Discounted Cashflows
-50	1.929
0	0.625
50	-0.654

By using interpolation, solve for the spread so that the price minus the discounted present cashflows equals zero. In addition, what is the forward rate effect? What is the intuition behind the relatively large forward effect? Round the spread to the nearest basis point.

Exercise 8-8 (Advanced)

Taking the information from Exercise 8-5, compute the yield and spread to the benchmark WAL. Compare the spread to the curve and WAL spread. What accounts for the large difference?

Work area

Yield	
WAL Spread	
Curve Spread	

Exercise 8-9 (Advanced)

Derive the forward rates from the steepening yield curve in Table 8-8. What will be the impact on prepayments relative to the forward rates derived in Table 8-11? How will the changing prepayment rates relate to the change in the discounting of cashflows?

Work area

Years Forward	One-Year Maturity Forward Rates
1	
2	
3	
4	

Exercise 8-10 (Advanced)

In Exercise 8-4, we held the spread constant. Let's assume that with the yield curve steepening, the price only rose to 100.25. What is the revised yield, spread to the curve, and spread to the WAL benchmark?

Work area

Yield	
WAL Spread	
Curve Spread	

■ REVIEW QUESTIONS

What is the forward effect on IOs when the yield curve is upward sloping?

What features of a bond will lead to the greatest forward effect?

What are some of the advantages of using a forward spread for an inverse floater rather than the static yield spread?

■ ANSWERS TO EXERCISES

8-1

	Yield	WAL	Benchmark Treasury	Spread (bps)
MBS 1	6.00	2.6	3	25
MBS 2	6.70	4.6	5	70
MBS 3	7.10	7.6	10	50
MBS 4	7.75	12.8	10	115

8-2

	Yield	WAL	Benchmark Yield	Benchmark Spread
MBS 1	6.00	2.6	5.45	55
MBS 2	6.70	4.6	5.95	75
MBS 3	7.10	7.6	6.31	79
MBS 4	7.75	12.8	6.66	109

8-3a

WAL spread is 130 basis points.

8-3b

The spread to the curve is 79 basis points.

8-4

Early cashflows discounted at lower rates cause the price to go up, which decreases the yield and spread to the benchmark. The WAL remains constant because the timing of the cashflows does not change in this example. Incorporating prepayments that are sensitive to interest rate changes would change the cashflows. The spread to the spot curve was kept constant. However, the increased price resulted in a lower yield, which in turn resulted in a lower spread to the benchmark since the benchmark did not change.

8-5

Year	Balance	Principal	Interest	Cashflow	Discount Factor	Discounted Cashflow
0	100			−100	1.0000	−100
1	90	10	9.500	19.500	0.8939	17.43
2	75	15	8.550	23.550	0.8143	19.18
3	55	20	7.125	27.125	0.7454	20.22
4	30	25	5.225	30.225	0.6894	20.84
5	0	30	2.850	32.850	0.6615	21.73

The present value is $99.39.

8-6

Projected prepayments should rise in the third and fourth years as forward rates predict falling interest rates.

8-7

The spread is 24 basis points. The forward effect is 63 basis points. The large forward-rate effect is due to the significantly increased early cashflows being discounted at high rates.

8-8

Yield	9.72
WAL Spread	47 basis points
Curve Spread	87 basis points

When the curve was inverted, the benchmark WAL yield rose. This depressed the WAL spread. With a WAL of 3.5 years, a significant portion of the MBS cashflow is concentrated at the sector of the curve where the yields are low.

8-9

Years Forward	One-Year Maturity Forward Rates
1	7.143
2	15.157
3	12.818
4	12.203

In Table 8-8's curve, prepayments will be slower than Table 8-11 in two periods forward because the implied forward rate is higher (15.157 versus 11.48). The forward rates for three and four periods forward are comparable for both curves. However, since the curve in Table 8-11 is generally increasing over time, prepayments for that curve would be slowest in three and four periods forward. The changing prepayments will be reflected in the price of the bond.

8-10

Yield	9.41
WAL Spread	91 basis points
Curve Spread	102 basis points

CHAPTER NINE

Option-Adjusted Spread

Bob, the salesman from Bulls & Bears, had a favorite saying: "There are no bad bonds, only bad prices." He would use this phrase when trying to sell some securities that had some fairly ugly warts. There was the time he showed the PAC II with the paydown profile that looked like the humps of a camel. Susan generally felt there were some good opportunities in broken PACs and recombinations, but she needed a tool that could help quantify both the prepayment and yield curve risk.

She had heard about the OAS models but had stayed somewhat away from them. Everyone seemed to have a different model and they always gave her different results. However, even the corporate bond portfolio managers were talking about OAS for callable bonds. Susan realized that if the corporate managers could learn about OAS then this could not be too difficult. It was time to put on the rocket scientist garb again.

■ INTRODUCTION

Investing in mortgage-backed securities entails a fair degree of uncertainty. The future economic and interest rate environment is unknown and therefore the future prepayment rates and future security cashflows are unknown. Option-adjusted spread analysis is one method of dealing with the multitude of possible investment outcomes and summarizing the performance of the security.

Option-adjusted spread (OAS) analysis is designed to quantify the impact of the dynamics of prepayments on the value of the security. The method represents a simulation of the performance of the security under a set of possible interest rate environments. The interest rate environments must satisfy certain conditions in order to allow the analysis to produce reliable relative value analysis.

As with all analysis, option-adjusted spread has certain limitations. The strengths and weaknesses of the approach can be best understood through a detailed understanding of the process.

MONTE CARLO METHOD

The customary method of evaluating mortgage-backed securities is the Monte Carlo method. This method of valuation involves simulation of the performance of the bond under a specific set of conditions. Monte Carlo methods can be viewed as an extension of the valuation techniques described in Chapter 7.

First, let's review the four-step mortgage evaluation process and then apply that process to the Monte Carlo OAS evaluation. In the first phase, the environment is determined. For OAS, that means determining the rate paths to be used in the analysis. In the prepayment phase, a prepayment model, such as the one described in Chapter 5, is used to project the prepayment rates. In the cashflow phase, the cashflows of the security are calculated. For passthroughs, the process is similar to the analysis in Chapter 3 and for CMOs the methods of Chapter 6 are used. In the final phase, analysis, the cashflows are used to calculate various measures of risk and value, the most important of which is the option-adjusted spread (OAS). Other measures, which provide further insight into the value and risk of the security, can also be computed. These measures include option cost, effective duration, and convexity.

Table 9-1
Four-Step Process

Environment	Prepayment	Cashflow	Analysis
Create interest rate paths consistent with term structure	Generate prepayment forecasts using model	Produce cashflows for MBS or CMOs	Calculate OAS, option cost, effective duration, convexity

Pathwise versus Lattice Models

In Chapter 4, we demonstrated the valuation of fixed-income securities with embedded options using a binomial valuation method. For reasons that we will describe below, MBS are not easily evaluated in a lattice-based model. Instead, MBS valuation is usually conducted in a "pathwise" manner. By pathwise, we mean that an entire interest rate path is generated starting now and continuing until the maturity of the security. This path may be a single interest rate or a set of rates representing the full yield curve. Each path represents just one possible path for interest rates to take. A large number of paths are used to simulate the entire range of possible outcomes.

In the following example we show the connection between pathwise valuation and lattice-based valuation. We demonstrate that there is a direct link between the two approaches and that pathwise valuation and binomial valuation can produce the same results. We will use the same binomial trees as shown in Chapter 4.

Figure 9-1
The Binomial Tree Created in Chapter 4

Creating a pathwise analysis:

Step 1 The binomial tree specifies all of the possible interest rate states over time. We need to establish a list of all possible interest rate paths through the tree. All of the paths start at the 5% rate. From there the path can either go up to 8.5% or down to 5.7%. Note that the down rate is actually higher than the initial rate. This is still called the down path. From either of these points, the rate path can go either up or down. Thus there are 2×2 or four possible paths. Generally there are 2^n possible paths, where n is the number of steps. For a mortgage valuation with 360 months of cashflows there could be 2^{360} or over 1×10^{100} possible paths! The fastest computers couldn't process that many paths in a reasonable time. Even if each path took 1/100th of a second, we could only calculate about 1×10^{11} paths before the mortgage matured! Table 9-2 lists the four possible paths and the interest rates for each path.

Table 9-2
Possible Interest Rate Paths

		Year	
Path	1	2	3
1		up	up
2		up	down
3		down	up
4		down	down
1	5.000	8.488	11.007
2	5.000	8.488	7.378
3	5.000	5.690	7.378
4	5.000	5.690	4.946

Step 2 Determine the cashflows of the bond for each path. Assume the bond matures at the end of year 3 and has an annual coupon of 7.5%. Since the bond is option-free, the cashflows are the same for each path. The bond receives a coupon of 7.5 in years 1 and 2 and receives principal of 100 and coupon of 7.5 in year 3.

Step 3 Value a fixed-rate bond using pathwise valuation. For path 1, start at the last period. The cashflow at the end of period 3 is 107.5. That cashflow is discounted back one period at 11.007%, resulting in a value of 96.8411. That value plus the 7.5 coupon received in period 2 is discounted at 8.488% to produce a value of 96.1174. That value plus the 7.5 coupon received in period 1 is discounted at 5% to produce a value of 98.7403. That value, 98.7403 represents the value of the security for the first path. It is the value of the security if you were certain that rates would follow path 1 (up, up; 5.0%, 8.5%, 11%) The same calculation can be repeated for each of the other three paths as shown in Table 9-3.

Table 9-3

Discounted Cashflows for All Paths

Path	Year 1	2	3
Cashflow	7.5	7.5	107.5
1	98.740	96.177	96.841
2	101.613	99.194	100.114
3	104.115	101.820	100.114
4	106.205	104.016	102.434
Average	102.668		

Step 4 The results of the valuation for each path are then averaged to produce an expected value for the bond. The average value is 102.668. Note that this is the exact same value as achieved in Chapter 4, Figure 4-5, using the binomial tree.

Evaluating Embedded Options

The evaluation of embedded options using pathwise valuation requires a different type of decision rule than lattice-based valuation. In lattice-based models, the decision to exercise is based on a comparison of the value of pursuing the strategy of exercise versus the strategy of deferring exercise. The strategy with the lowest cost is selected. In a pathwise valuation, it is not possible to fully evaluate both strategies, since only a single possible rate path is analyzed at a time. Therefore it is necessary to develop a decision rule that relies on information about coupon and current interest rate levels rather than prices.

The callable bond from Chapter 4, Figure 4-6, can also be evaluated using pathwise valuation. That bond has a coupon of 7.5%, a maturity of three years, and is callable at par at the end of year one. Here additional work is required. It is necessary to determine when the call option will be exercised.

Step 1 is the same as above.

Step 2 Working forward through time, determine when the bond will be called. To do this we need a decision rule. Assume that the bond will be called when the discount rate is less than 7.5% after the first year. Therefore the bond is called at the end of year 1 in paths 3 and 4. Table 9-4 shows the cashflows of the bond.

Table 9-4
Callable Bond Cashflow

	Year		
Path	1	2	3
1	7.5	7.5	107.5
2	7.5	7.5	107.5
3	107.5	0	0
4	107.5	0	0

Step 3 Value the bond as above, calculating the present value for each path. See Figure 9-5.

Table 9-5
Callable Bond Valuation

	Year		
Path	1	2	3
1	98.740	96.177	96.841
2	101.613	99.194	100.114
3	102.381	0	0
4	102.381	0	0
Average	101.279		

Step 4 Calculate the average value as above. Note that the average value is 101.279. This is the same value as in Chapter 4, Figure 4-6.

This analysis has shown that binomial valuation and pathwise valuation can give the same results for both callable and noncallable bonds.

Exercise 9-1

Evaluate an option-free bond with pathwise valuation using the assumptions of Exercise 4-6.

Work area

Interest Rate Paths

Path	Year 1	Year 2	Year 3	Year 4
1	0.0500	0.0694	0.1159	0.1703
2				
3				
4				
5				
6				
7				
8				

Coupon 8%

Discounted Cashflows

Path	Year 1	Year 2	Year 3	Year 4
1	94.7785	91.5174	89.8657	92.2840
2				
3				
4				
5				
6				
7				
8				
Average				

Exercise 9-2

Evaluate a bond with an embedded option using the assumptions of Exercise 4-7: Compare valuation using pathwise and lattice-based methods.

Work area

Coupon 8%

Callable at par after third year

Path	Year 1	Year 2	Year 3	Year 4
1	94.7785			
2				
3				
4				
5				
6				
7				
8				
Average				

Coupon 8%

Putable at 98 after year 3

Path	Year 1	Year 2	Year 3	Year 4
1	99.3404	96.3074	94.9880	Put @ 98
2				
3				
4				
5				
6				
7				
8				
Average				

■ REVIEW QUESTION

When will there be a difference between pathwise- and lattice-based evaluation? What are the advantages of each method? Can you think of a hybrid method, which has the advantages of both methods?

Issues in Path-Based Models

Using a path-based, rather than a lattice-based model creates opportunities and challenges for the analyst. One of the most demanding features of a lattice-based model is that the paths must "recombine." That is, the point reached following an up and then a down move, should be the same point as when following a down and then an up move. If the lattice does not have this feature then it does not recombine. A nonrecombining lattice does not have the computational advantages of a recombining lattice. In fact, it becomes just another form of pathwise analysis. Once freed from the constraints of recombination, the analyst is quite free in how interest rates change from one period to another.

Most lattice-based models follow either a normal or log-normal interest rate process. In a normal rate process, interest rate changes from period to period are additive. That is, rates change by a set amount for each step on the lattice. For example both the up move and down move might equal 25 basis points. It is easy to see that such a rate process will recombine.

Exercise 9-3

In a normal rate process, the standard deviation of the interest rates grows with the square root of time. If the initial interest rate is 9%, the yield curve is flat and the assumed volatility is 1.5% per year, at what time in the future will the probability of negative rates be greater than 2.5%? (Hint: In a normal distribution, there is about a 95% probability of events within two standard deviations of the mean.)

In a log-normal rate process, the interest rate changes from period to period are multiplicative. That is, rates change by a set percentage for each step on the lattice. For example, rates may increase by a factor of 1.20 for an up move or decrease by the reciprocal or 1/1.20 for a down move.

Exercise 9-4a

Show that a multiplicative rate process will result in a recombining lattice.

Exercise 9-4b

For a log normal process, if the step size is 50 basis points for an up rate move when the interest rate is 10%, what is the step size for a down rate move?

Exercise 9-4c

What is the step size in basis points when the interest rate is 5%? at 15%? at 1%?

Work area

	5%	15%	1%
Up			
Down			

Exercise 9-4d

What is the probability of rates less than 0?

Since the use of pathwise analysis eliminates the need for recombining paths, it is no longer necessary to restrict the interest rate process to those solutions for which recombining trees can be built. Some possible variations included are interest rate paths where the interest rate is proportional to the square root of the rate and interest rate paths where the rate process has a tendency to drift back toward some predetermined level (mean reversion).

"Path-Dependent" Cashflows and the Binomial Tree

Mortgage cashflows have the feature that the cashflow today depends on prior interest rate levels and prepayment levels. For path-dependent cashflows, it is necessary to say something about the history of rates leading up to today, not just the current level of rates or the prepayment history of the security, in order to calculate cashflow for this month.

The path-dependent nature of mortgage-backed securities arises from prepayment characteristics and structural characteristics. The prepayment characteristic that plays the largest role in creating path-dependent cashflows is burnout. Borrowers who did not prepay at the first opportunity to do so may face different costs of refinancing than other borrowers. Thus, past prepayments (and therefore past interest rate levels) affect our forecasts of future prepayments. Some OAS models are able to overcome this path-dependent aspect of mortgage valuation by splitting the mortgage pool into discrete segments with varying refinancing thresholds. Each segment can then be evaluated in a nonpath-dependent fashion. This approach requires that the prepayment estimates be constructed in a particular fashion.[1]

The second source of path dependency is not as easily overcome. This is the path dependence that arises from CMO structures. For many structures, future cashflows depend on past performance, particularly structures with scheduled and support classes. The past history of prepayments will affect future distribution of cashflows. For these structures it is extremely difficult to segment the CMO into pieces that are independent of each other and, thus, avoid path dependence.

Example: Path Dependence and the Binomial Tree

Suppose that at a particular time, say year 3, a bond can be called. The call price depends on prior interest rate levels. If interest rates have fallen below 7% then the bond can be called at par; otherwise the bond must be called at 102.

Binomial pricing requires that pricing begin from the maturity of the bond, and move backward in time. It also requires that a single price be assigned to each node.

In this example, it is unclear what price to place on the bond at node X. There are paths leading to node X where the interest rate has fallen below 7%. Therefore, the price of the bond should be 100; however, there are other paths leading to that point where the interest rate never fell below 7%. Thus it is necessary to assign both 100 and 102 to that point. Since both prices cannot be used to step back from node X to the earlier nodes, the binomial tree method cannot be used to value this bond without using a much more sophisticated approach.

[1]Davidson, A. S., M. D. Herskovitz and L. D. Van Drunen, "The Refinancing Threshold Pricing Model: An Economic Approach to Valuing MBS," *Journal of Real Estate Economics and Finance*, 1, 2, 117-130.

■ REVIEW QUESTIONS

Why can path-dependent features that are linked to balance or amount outstanding and interest rate levels be evaluated with a binomial tree?

Are ARMs path-dependent securities? Why?

Monte Carlo Analysis and Sampling

As shown above, pathwise valuation is more readily adaptable for path-dependent securities such as mortgages. However, pathwise analysis has the disadvantage that it is much more time consuming. In a lattice-based model, the number of calculations grows roughly proportionally to the square of the number of periods. This can be seen by counting the nodes. For a one-period binomial there are three nodes. For a two-period binomial there are six nodes. The number of nodes equals $(n + 1) \times (n + 2)/2$. The number of calculations is roughly proportional to the number of nodes. For a pathwise analysis, the number of calculations is roughly proportional to the number of paths. For a one-period binomial there are two paths (one up, one down) For a two-period model there are four paths and for a three-period model there are eight paths. However, the number of paths begins to grow quickly as the number of paths equals 2^n. Thus for a 10-period model, the number of nodes in a binomial model is 66 and the number of paths is 1,024. For greater numbers of periods, the number of paths soon becomes unmanageable.

Exercise 9-5

How many nodes and how many paths for a 30-period model?

In order to utilize the advantages of path-based modeling without bearing the computational cost of calculating billions of paths, analysts have developed a method of randomly sampling from all available paths. These methods are called Monte Carlo evaluation. A Monte Carlo method involves randomly selecting paths in order to approximate the results of a full pathwise evaluation.

One way of generating interest rate levels is to use a precalculated binomial tree to determine the rate levels at each time period. The Monte Carlo process then involves selecting whether to go up or down from the previous node. The interest rate level is then read off of the precalculated tree. Another method is to create a random interest rate path based on an equation that describes how interest rates change over time. If the second method is used then it is necessary to adjust these interest rate paths to ensure proper pricing of the Treasury curve.

Regardless of which method is chosen, analysts seek to find ways of choosing paths so that the paths are representative, while at the same time require the fewest number of paths. These methods are generally referred to as variance reduction methods. A commonly used method is the use of antithetical paths. In this method, for each path that is generated, an additional path is created that moves in exactly the opposite fashion. For example if one path began with the sequence "up, up, down," the antithetical path would begin "down, down, up." This technique serves to produce more accurate results with fewer paths. Many other techniques are possible.

■ REVIEW QUESTION

How does the antithetical path method work to reduce variance? Does it ensure accurate mean interest rates? Accurate cross-sectional volatilities? What are the features of a good variance reduction technique?

■ SIMPLE OAS ANALYSIS

In this section, we will construct a very simple Monte Carlo model to calculate the option-adjusted spread of a security. The calculations will follow the usual four-step process outlined below in Table 9-6.

Table 9-6

Monte Carlo within the Four-Step Process

Environment	Prepayment	Cashflow	Analysis
Create interest rate paths consistent with current Treasury rates	Generate prepayment forecasts using model	Produce cashflows for MBS or CMOs	Calculate OAS, option cost, effective duration, convexity
Step 1. Use additive interest rate process	Step 3. Calculate spread of mortgage to current rate	Step 5. Calculate interest and principal cashflows	Step 6. Calculate the price for each path
Step 2. Coin toss process to choose paths	Step 4. Look up prepayment rate in table		Step 7. Calculate average price and OAS

Table 9-7 provides the template for the calculation for each path. The OAS analysis will consist of a set of similar calculations, one for each path. Below, we go through the step by step process for each path.

Table 9-7

OAS Calculation Template

Period	0	1	2	3	4
Coin			Heads	Tails	Heads
Rate		9%	11%	9%	11%
Prepay		40%	5%	40%	
Ending Balance	100	60.00	57.00	34.20	0.00
Interest		11.00	6.60	6.27	3.76
Total Cash Flow		51.00	9.60	29.07	37.96
Present Value	100.62	59.68	57.24	33.89	

Bond Assumptions	
Coupon	11
Maturity	4 years
Vol.	2%
OAS	100

Prepayments	
Spread	**Prepay**
–5%	5%
–1%	5%
0%	5%
1%	15%
2%	40%
3%	60%
4%	70%

Step 1 Choose an interest rate process. Here we are using an additive interest rate process. The initial rate is 9% and the step size is 2% per year.

Step 2 Choose random numbers. Here we use a coin toss method. Toss a coin once for each time period. Record the results as heads or tails. Heads means an up rate path; tails means a down rate path. Our coin produced a result of heads, tails, heads. This produces an interest rate path of 9%, 11%, 9%, 11%.

Step 3 The prepayment model for this example is a look-up table with the input equal to the spread between the mortgage coupon and the discount rate. The mortgage coupon equals 11%. For each period calculate the spread.

Step 4 Determine the prepayment rate from the table. The spreads are 2%, 0, 2%, 0. The prepayment rates are 40%, 5% 40%. The final prepayment rate does not matter, because the bond matures in year 4.

Step 5 Calculate interest and principal cashflows. The interest cashflows equal the coupon times the beginning balance. The principal cashflows equal the prepayment rate times the ending balance from the previous period. The balances are 100, 60, 57, 34.2, and 0. The interest cashflows are 11, 6.6, 6.27, and 3.76.

Step 6 Calculate the price for this path assuming an OAS of 100 basis points. The discounting process is accomplished by starting from the last cashflow and then discounting backward. The discount rate is calculated by adding the OAS (in percent) to the current interest rate. The cashflow in year 4 is 37.962 and the discount rate is 12% (11% + 1%). The present value of that cashflow in year 3 is 33.89. The sum of the present value of the year 4 cashflow and the cashflow from year 3 is 62.96. The present value is 57.24 using a 10% discount rate. The process is repeated for year 2 and then for year 1. The calculated present value is 100.62 for this rate path. This process can be represented in general by Equation 9-1.

Equation 9-1
Path Present Value

$$PV_i = \sum_{j=1}^{WAM}\left(\frac{\text{cashflow}_{ij}}{\prod_{k=1}^{j}\left(1+r_{ik}+OAS\right)}\right)$$

Where cashflow_{ij} is the cashflow of the bond in path i for period j and r_{ik} is the discount rate for path i for period k.

Exercise 9-6

Repeat Steps 1-6 for the antithetical path Tails, Heads, Tails.

Work area

Period	0	1	2	3	4
Coin			Tails	Heads	Tails
Rate		9%			
Prepay					
Ending Balance	100				0
Interest		11			
Total Cash Flow					
Present Value					

Bond Assumptions	
Coupon	11
Maturity	4 years
Vol.	2%
OAS	100

Prepayments	
Spread	Prepay
–5%	5%
–1%	5%
0%	5%
1%	15%
2%	40%
3%	60%
4%	70%

Table 9-8
Price Given OAS

Path	Price
up up up	97.54
up up down	98.86
up down up	100.62
up down down	101.54
down up up	102.49
down up down	102.79
down down up	103.26
down down down	103.43
Average	101.32

Step 7 Calculate the OAS. Since there are three periods in which the rates can change, there are eight possible paths. Assuming a 100 basis

point OAS, the values for each path are as shown is Table 9-8. The average value is 101.32. This calculation is shown in Equation 9-2. The OAS level can be calculated for a given price through a trial and error method. Table 9-9 shows the average values for various OAS levels. If the market price equals 101.32 then the correct OAS is 100.

Equation 9-2
OAS Model Price

$$\text{Model Price} = \frac{1}{N}\sum_{i=1}^{N} PV_i$$

Vary the OAS in Equation 9-1 until Model Price equals market price.

Table 9-9
OAS and Price

OAS	Price
50	102.26
75	101.78
100	101.32
125	100.85

Option Cost

Option cost represents the difference between the yield spread to the spot curve and the option-adjusted spread of the instrument. Option cost is calculated using the following steps.

Step 1. Assume constant interest rates. Note that because the yield curve is flat, stable rates equal the forward rates.

Step 2. Follow steps 3 through 6 above.

Step 3. Calculate the OAS and spread for that one path.

Step 4. Subtract the OAS from the spread calculated in the previous step. That is the option cost.

Exercise 9-7

Calculate the option cost for this security assuming a price of 101.32.

Work area

Price	
Yield	
Spread	
OAS	
Option Cost	

Option cost represents the value of the options embedded in the security. Theoretically it represents the cost to dynamically hedge the prepayment risk of the security. Practically it represents the additional yield that an investor should require for a security with option risk.

Exercise 9-8a (Advanced)

Calculate the price of a bond as in Table 9-7 and 9-8. Use the binomial tree in Exercise 4-5a in Chapter 4 for the interest rate paths. Assume a bond coupon of 9%, an OAS of 100bp, and the prepayment speeds used in Table 9-7. (Round down the spread difference to determine the correct prepayment speed.)

Exercise 9-8b (Advanced)

Calculate the option cost assuming prepayments based on stable rates (5% short rate) but discounting with spot rates.

Exercise 9-8c (Advanced)

Calculate the option cost assuming prepayment rates and discounting based on forward rates.

APPLYING OAS WITH SAMPLE RESULTS

Once the OAS model is built and the ability to calculate every conceivable spread and risk measure is at your fingertips, what does all this information actually tell you? Table 9-10 presents the analytical results for various coupons in the GNMA 30-year pass-through market. The prices are actual market prices as of 25 August 1995, and the calculations were generated using Michael Herskovitz, Inc., and Andrew Davidson & Co., Inc., OAS and prepayment models. The prepayment forecast and average life under stable rates are shown on the right hand side of the table under "Base PSA" and "Base WAL."

Table 9-10

GNMA 30-Year Analysis for 8/25/95

					MARKET Spread Settlement 08/25/1995		SPREAD TO		
Agency	Mat	CPN	WAM	AGE	Price	Yield	WAL TRSY	Zero Curve	Fwrd Curve
GNMA	30	6.0	342	18	92-26	7.18	81	75	75
GNMA	30	6.5	343	17	95-16	7.26	89	84	84
GNMA	30	7.0	343	17	98-5	7.37	101	99	96
GNMA	30	7.5	344	16	100-14	7.49	120	113	113
GNMA	30	8.0	347	13	102-16	7.56	142	127	128
GNMA	30	8.5	350	10	103-30	7.58	147	137	145
GNMA	30	9.0	349	11	105-0	7.48	148	134	151
GNMA	30	9.5	339	21	106-6	6.99	101	89	116
GNMA	30	10.0	296	64	108-28	6.35	37	30	64

					OPTIONS ANALYSIS			RISK		PREPAYMENT			
Agency	Mat	CPN	WAM	AGE	OAS	Option Cost	Fwd Curve Effect	Dur	Convex	Base PSA	FWD PSA	Base WAL	FWD WAL
GNMA	30	6.0	342	18	71	4	0	6.2	0.2	99	99	10.5	10.5
GNMA	30	6.5	343	17	75	9	0	5.8	–0.2	99	99	10.7	10.7
GNMA	30	7.0	343	17	76	20	3	5.3	–0.7	116	103	9.9	10.6
GNMA	30	7.5	344	16	79	34	0	4.6	–1.2	135	128	9.2	9.5
GNMA	30	8.0	347	13	79	49	–1	3.8	–1.5	185	138	7.6	9.3
GNMA	30	8.5	350	10	86	59	–8	3.1	–1.6	274	190	5.7	7.6
GNMA	30	9.0	349	11	92	59	–17	2.4	–1.5	381	275	4.2	5.6
GNMA	30	9.5	339	21	65	51	–27	1.2	–1.2	476	382	3.0	3.8
GNMA	30	10.0	296	64	22	42	–34	0.7	–0.9	469	400	2.9	3.5

Based on Michael Herskovitz, Inc., and Andrew Davidson & Co., Inc., OAS model and prepayment model.

Spread Measures

Three different yield measures are calculated. The first, spread to WAL Treasury, represents the yield advantage over a comparable average life Treasury. On the current coupon GNMA 7.0, the yield is 101 basis points over a 9.9-year Treasury. While this is a simple and easily understood measure, it does not capture the different yield spreads earned on the earlier and later cashflows.

The spread to the zero curve incorporates the shape of the curve and the timing of the cashflows. The earlier cashflows are measured against the shorter end of the curve, while the later cashflows are compared to longer maturity Treasuries. This measure is calculated by considering an MBS as a portfolio of bullet bonds. This means each period's cashflow is evaluated as a separate bond. The yield on each cashflow is measured against the appropriate Treasury, and the spread which produces the market price is calculated as was shown in Equation 8-2. The zero spread on all the GNMA coupons is lower than the WAL spread. This is primarily because the lower yield spreads earned on the longer dated cashflows have a greater effect than the higher spreads from the earlier cashflows. The flatter the yield curve, the less difference will be seen between the WAL and zero spread measures.

The spread to the forward curve incorporates an arbitrage-free framework in addition to the curve shape effect. The implied forward curve is used to generate prepayment forecasts as well as to discount the cashflows. Under a positively sloped yield curve, the prepayment projections will generally be slower than under flat rates. This is because a positively sloped yield curve predicts higher future interest rates. Higher interest rates generally lead to slower prepayment rates as homeowners have less of an incentive to refinance. This can be seen by comparing the "Base PSA" column with the "FWD PSA" column on the right-hand side of the table. The corresponding average lives are shown under "BASE WAL" and "FWD WAL."

The forward curve has the greatest impact on high premiums. On GNMA 10s, the spread to the forward curve is 34 basis points higher than the spread to the zero curve. This is due to the lower forward PSA of 366 versus 428 under stable rates. For discounts and low premiums, the zero and forward spreads are virtually the same.

OAS Analysis

After calculating the spread to the forward curve, one more major component of value must be implicitly evaluated in order to arrive at an OAS estimation. This is the option cost that reflects the loss in value resulting from varying interest rates and prepayments. The option cost, calculated using a Monte Carlo model, is the difference between the OAS and the spread to the forward curve. The OAS values on most of the GNMA coupons are between 70-80 basis point range.

The medium premium coupons, such as the 8.5s and 9s with an option cost of 59 basis points, show the highest OAS. These coupons have the highest volatility of prepayments. As the coupon reaches 10, the option cost decreases, reflecting burned-out loans.

The OAS level of an MBS can be decomposed as follows:

Equation 9-3
OAS Decomposition

$$\text{OAS} = \text{WAL spread} - \text{curve shape effect} - \text{forward effect} - \text{option cost}$$

where the curve shape effect is the difference between the WAL spread and the zero spread, the forward effect is the difference between the zero spread and the forward spread, and the option cost is the difference between the forward spread and the OAS.

Risk

The two risk measures calculated using the OAS model are duration and convexity. These numbers provide an indication of possible price volatility given interest rate changes. Low premium passthroughs have the highest negative convexity due to their proximity to the refinancing threshold point. The convexity of GNMA 8s is calculated to be −1.5, indicating an additional price change of 0.75% under a 100 bp rate shift. Therefore, although GNMA 8s may have an effective duration of 3.8 years, it is likely to behave as an approximately 4.55-year duration instrument if rates rise 100 basis points. Discounts exhibit much lower negative convexity due to the lower volatility of prepayments. The high premiums that are more burned-out also have a lower negative convexity.

These risk numbers can be used to calculate hedge ratios. A hedge ratio represents the amount of instrument B needed to hedge the price volatility of instrument A. A hedge ratio is calculated by Equation 9-4.

Equation 9-4
Hedging Bond A with Bond B

$$\text{Hedge Ratio} = \frac{\text{effdur}_A \times \text{price}_A}{\text{effdur}_B \times \text{price}_B}$$

Exercise 9-9

Assume the effective duration of the 10-year Treasury is 7.45 and its price is 102.75. Calculate how many 10 year Treasuries are needed to hedge 10 million face amount of GNMA 7s, using the table below. Do the same thing with GNMA 8s and 9s. How many GNMA 7s should be used to hedge GNMA 9s?

Work area

Instrument	Duration	Price
10-Yr Treasury	7.45	102.27
GNMA 7.0	5.30	98.16
GNMA 8.0	3.80	102.50
GNMA 9.0	2.40	105.00

Treasury Hedge for $10MM GNMAs:		
	Ratio	Amount($MM)
GNMA 7.0		
GNMA 8.0		
GNMA 9.0		

GNMA 7 Hedge for $10MM GNMA 9s:		
	Ratio	Amount($MM)
GNMA 7.0		

These hedge ratios are only valid for relatively small movements in the yield curve. After that, convexity effects would require rebalancing. Also, these hedge ratios assume parallel shifts in the yield curve.

The different aspects of risk and value in MBS can be quantified with various analytical measures. No one measure fully describes all the risks of MBS. A full understanding of MBS requires knowledge of all the dimensions of risk and value comparison.

■ REVIEW QUESTIONS

What is the best interest rate process to use?

What are the advantages and disadvantages of basing the Monte Carlo analysis on a lattice?

Which features of a Monte Carlo OAS model have the greatest impact on the results?

If two firms are offering the same security and firm A says the OAS is 20 basis points higher than firm B, would it be better to buy from firm A? What if firm A has a higher price than firm B?

If you want a low-risk, high-return bond, should you buy bond A with an OAS of 350 and an effective duration of 15 or bond B with an OAS of 90 and an effective duration of 3?

If you buy a CMO with a high OAS, and hedge it with treasuries based on the CMO's effective duration, are you guaranteed a profit?

ANSWERS TO EXERCISES

9-1

Interest Rate Paths

Path	Year 1	Year 2	Year 3	Year 4
1	0.0500	0.0694	0.1159	0.1703
2	0.0500	0.0694	0.1159	0.1262
3	0.0500	0.0694	0.0859	0.1262
4	0.0500	0.0694	0.0859	0.0935
5	0.0500	0.0514	0.0859	0.1262
6	0.0500	0.0514	0.0859	0.0935
7	0.0500	0.0514	0.0636	0.0935
8	0.0500	0.0514	0.0636	0.0692

Coupon 8%

Discounted Cashflows

Path	Year 1	Year 2	Year 3	Year 4
1	94.7785	91.5174	89.8657	92.2840
2	97.6652	94.5484	93.1070	95.9009
3	99.9596	96.9576	95.6833	95.9009
4	102.3117	99.4273	98.3243	98.7688
5	101.5387	98.6156	95.6833	95.9009
6	103.9310	101.1275	98.3243	98.7688
7	105.7950	103.0847	100.3821	98.7688
8	107.7007	105.0857	102.4859	101.0064
Average	101.7101			

9-2

Coupon 8%

Callable at par after third year

Path	Year 1	Year 2	Year 3	Year 4
1	94.7785	91.5174	89.8657	92.2840
2	97.6652	94.5484	93.1070	95.9009
3	99.9596	96.9576	95.6833	95.9009
4	102.3117	99.4273	98.3243	98.7688
5	101.5387	98.6156	95.6833	95.9009
6	103.9310	101.1275	98.3243	98.7688
7	105.7950	103.0847	100.3821	98.7688
8	106.8436	104.1857	101.5396	called @ 100
Average	101.6029			

Coupon 8%

Putable after year 3 at 98

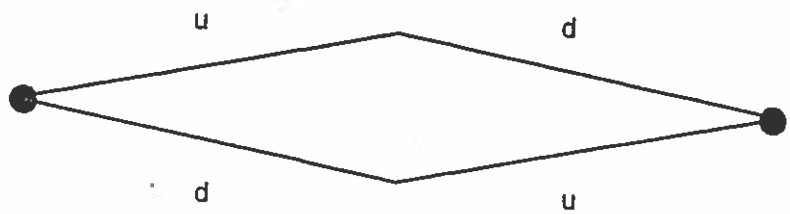
Path	Year 1	Year 2	Year 3	Year 4
1	99.3404	96.3074	94.9880	put @ 98
2	99.3404	96.3074	94.9880	put @ 98
3	101.6812	98.7653	97.6163	put @ 98
4	102.3117	99.4273	98.3243	98.7688
5	103.2897	100.4542	97.6163	put @ 98
6	103.9310	101.1275	98.3243	98.7688
7	105.7950	103.0847	100.3821	98.7688
8	107.7007	105.0857	102.4859	101.0064
Average	102.9238			

9-3

Let T=Time

$2 \times 1.5\% \times T^{.5} = 9\%$, $T^{.5} = 3$, $T = 9$

9-4a



$$u \times d = d \times u$$

$$1.2 \times \frac{1}{1.2} = \frac{1}{1.2} \times 1.2$$

$$= 1$$

9-4b

The step size for a down rate move is 47.62bp.

9-4c

	5%	15%	1%
Up	25.0	75.0	5.0
Down	23.8	71.4	4.8

9-4d

There is a zero probability of rates less than zero.

9-5

There are 496 nodes and 1,073,741,824 paths.

9-6

Period	0	1	2	3	4
Coin			Tails	Heads	Tails
Rate		9%	7%	9%	7%
Prepay		40%	70%	40%	
Ending Balance	100	60.00	18.00	10.80	0.00
Interest		11.00	6.60	1.98	1.19
Total Cash Flow		51.00	48.60	9.18	11.99
Present Value	102.79	62.07	18.44	11.10	

9-7

Price	101.32
Yield	10.275%
Spread	127.5bp
OAS	100bp
Option Cost	27.5bp

9-8a

The price is 101.85.

9-8b

The option cost assuming prepayments based on stable rates is 49.2 bp.

9-8c

The option cost assuming prepayments based on forward rates is 13.8 bp.

9-9

Instrument	Duration	Price
10 Yr Treasury	7.45	102.27
GNMA 7.0	5.30	98.16
GNMA 8.0	3.80	102.50
GNMA 9.0	2.40	105.00

Treasury Hedge for $10MM GNMAs		
	Ratio	Amount($MM)
GNMA 7.0	0.68	6.83
GNMA 8.0	0.51	5.11
GNMA 9.0	0.33	3.31

GNMA 7 Hedge for $10MM GNMA 9s		
	Ratio	Amount($MM)
GNMA 7.0	0.48	4.84

CHAPTER TEN

Regulatory Measures

Susan's persistence and hard work were paying off. On a risk-adjusted basis, she was outperforming her peers. Susan was getting impatient, and she wanted to take on more responsibility. Bob, the salesman from Bulls & Bears, told Susan that the Bodyguard, a large life insurance company, was looking for an assistant CIO to cover the debt markets. Susan was going to try and get this job. She knew the markets and the bonds. As she soon realized, the insurance industry had developed their own investment rules. Knowing OAS was not going to be enough. She now had to enter the world of regulatory guidelines.

INTRODUCTION

The mortgage market has been successful because of its ability through CMO creation to adapt itself to a variety of investor appetites. Each type of investor faces a unique set of investment requirements through regulatory structure and investment objectives. Regulatory considerations have a large impact upon the type of mortgage products that can be purchased. Regulation of MBS is a tricky process. Ideally, regulations should guard against inappropriate risk taking either through ignorance or by intent. Judging what is "inappropriate," however, is a difficult task. Creating regulations that guard without making unnecessary restrictions on the sophisticated and capable investment manager is even more difficult. Two widely used regulatory measures, the FFIEC Test and FLUX, are discussed in this chapter.

FFIEC TEST

As part of developing policies of safety and soundness, the Federal Financial Institutions Examining Council (FFIEC, pronounced "fifec") has developed a set of guidelines to determine the riskiness of MBS. Depository institutions may hold "risky" securities but the securities must be reported as part of the trading account based on market value.

Alternatively, the securities may be booked as assets held for sale marked at the lower of cost or market.

As part of the test for CMO suitability, the FFIEC has instituted three testing criteria, as listed in Table 10-1. Any bond that fails one or more of the three FFIEC tests falls into the "risky" category.

Table 10-1
FFIEC Risk Tests

Base Cash Average Life	Average life of the CMO cannot exceed 10 years
Average-Life Sensitivity	CMO cannot extend more than four years or shorten more than six years based on an instantaneous +/– 300 basis point shift of the yield curve.
Price Sensitivity	The value of the CMO cannot change by more than 17% for a yield curve change of +/– 300 basis points or any incremental 100 basis point shift up to 300 basis points.

The FFIEC test covers many of the major risk categories which have been described in the workbook. It incorporates the notions of cashflow stability, as well as duration and convexity. For many institutions, considering these factors is an important step in the measurement and control of risk.

As a risk measurement tool, FFIEC has several limitations. Some of the faults of the test are listed below:

- The FFIEC tests do not consider the impacts of changing the shape of the yield curve. Key rate duration and twist measures could be used to examine this type of risk.
- Instantaneous interest rate shifts may be too conservative. Looking at volatility of this magnitude may not be realistic and could lead to disqualification of securities that are not truly risky.
- The tests do not consider the impact of interest rate paths. Whipsaw scenarios may cause a security to be vulnerable to cashflow variability as the rules of the CMO structures change.

Despite its faults, the FFIEC test does lead portfolio managers to carefully consider the risk profiles of securities they intend on purchasing.

Exercise 10-1

Using the tables below, determine whether the two bonds pass or fail the FFIEC tests.

Bond 1	FNR 1993-244 A
Date	1/12/96
Collateral	100% FNCL 8
WAC	8.472
WAM	313
Price	93-19

	Security		Treasury Spread		Security		WAL Change		Price Change	
Shift	PSA	WAL	Mat.	BP	Yield	Price	Actual	Max	Actual	Max
-300	632	1.19	1.19	62	2.749	96-27	-0.01	-6.0	3.5%	17%
-200	587	1.19	1.19	62	3.749	95-24	-0.01	-6.0	2.3%	17%
-100	515	1.19	1.19	62	4.749	94-21	-0.01	-6.0	1.2%	17%
0	335	1.20	1.20	62	5.750	93-19	n/a		0.0%	
+100	200	1.24	1.24	62	6.751	92-10	0.04	+4.0	-1.4%	17%
+200	150	1.39	1.39	62	7.755	90-07	0.19	+4.0	-3.6%	17%
+300	129	1.52	1.52	62	8.758	88-03	0.32	+4.0	-5.9%	17%

Bond 2	FNR 1994-45 D
Date	1/12/96
Collateral	100% FNCL 7
WAC	7.484
WAM	324
Price	92-25

	Security		Treasury Spread		Security		WAL Change		Price Change	
Shift	PSA	WAL	Mat.	BP	Yield	Price	Actual	Max	Actual	Max
-300	875	2.71	2.71	135	3.565	107-07	-7.98	-6.0	11.9%	17%
-200	800	3.07	3.07	135	4.597	105-04	-7.62	-6.0	9.8%	17%
-100	440	6.30	6.30	135	5.861	103-09	-4.40	-6.0	7.8%	17%
0	185	10.70	10.70	135	7.108	92-25	n/a		0.0%	
+100	140	13.26	13.26	135	8.160	87-05	2.56	+4.0	-9.0%	17%
+200	115	15.07	15.07	135	9.197	78-26	4.38	+4.0	-17.7%	17%
+300	103	16.04	16.04	135	10.217	71-20	5.35	+4.0	25.2%	17%

Source: Bloomberg Financial Markets, 1996.

THE FLOW UNCERTAINTY INDEX

The FLUX (FLow Uncertainty IndeX) model was designed by a committee formed by the NAIC (National Association of Insurance Commissioners) to provide an objective measure of relative variability of cashflows of individual CMO tranches over a range of possible interest rate scenarios. FLUX was created to assist insurance industry regulators in prioritizing investment portfolios for review and analysis. Andrew Davidson was responsible for the development of the FLUX methodology.

Measure of Cashflow Variability

FLUX measures the variability of the cashflows of CMO bonds under various interest rate assumptions. The measure has two components: a present value measure and a timing measure. FLUX is a measure of cashflow variability *not* a measure of market risk. FLUX does not measure risk associated with duration or spread changes. For example, a long Treasury bond (noncallable) would have a zero FLUX since its cashflows are certain. While FLUX is not a measure of value, it can be used by portfolio managers to determine the relative riskiness of CMO bonds.

Investors in CMOs face the risk that their investments will not perform as expected. While many total rate of return investors are primarily concerned about the market volatility of their investments, portfolio investors are ultimately more concerned about the cashflows generated by their investments. While there are many measures of bond value, such as yield and OAS, and many measures of price and return volatility including tools like duration, convexity and total return, there is little available to measure cashflow variation. Average life variability provides some indication of cashflow risk, but it does not capture coupon effects or the impact of changing cashflow patterns. Even measures like OAS duration are not truly measures of cashflow volatility. OAS duration takes prepayment forecasts into account, but OAS does not measure the risk of prepayments differing from the model assumptions.

Calculations

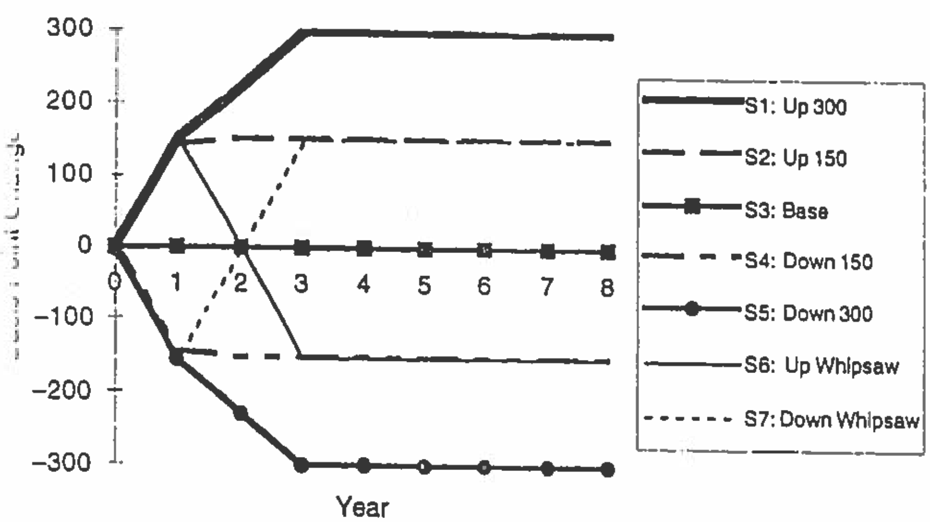
FLUX is specifically a measure of cashflow volatility. The FLUX measure is calculated based on the cashflows of the security. To calculate the measure it is necessary to calculate the cashflows of the bond in a base case and then to compare those cashflows to the cashflows of the same bond under different prepayment assumptions. In this way, the riskiness of the bond is calculated relative to its own cashflows. A present value measure and timing measure are calculated for each scenario. These measures are then summed for each scenario. The root mean square (RMS) of the scenario scores is the FLUX of the bond. Root mean square (RMS) is a technique from statistics that captures both the mean and dispersion of a group of numbers. It is calculated by squaring each number, taking the average of those squares, and then taking the square root, hence root mean square.

Floating rate CMOs require a slightly modified form of the FLUX calculations, which allows for cashflow volatility associated with coupon adjustments to the index. IOs and Inverse Floaters are evaluated using the calculations presented in this chapter.

Scenario Choices

While there are many ways of choosing scenarios, it is convenient to generate prepayment assumptions based on interest rate scenarios. The NAIC committee originally chose scenarios using a trinomial technique developed by Tom Ho of GAT. The scenarios that contribute the most to the FLUX measure were chosen. The current scenarios are shown in Figure 10-1. The base case assumes that interest rates remain constant. Under the six scenarios, interest rates move by increments of 150 basis points to nodes at 1, 2, and 3 years. Scenarios are: rates move up or down by 150 basis points, rates move up or down by 300 basis points, and two whipsaw scenarios. The committee plans to review the scenarios periodically to stay current with market conditions and developing CMO structures. The Public Securities Association (PSA) provides median dealer month-by-month prepayment speeds for various collateral types and coupons, for each scenario, once a year to be used in calculating FLUX scores.

Figure 10-1
FLUX Scenarios



Present Value Component

The present value component is the magnitude of the negative percent changes in the present value in each scenario relative to the base scenario.

This component captures the risk of premium bonds prepaying sooner than expected or discount bonds prepaying later than expected. IOs and some POs receive high scores on this component. The timing component is calculated based on a period by period comparison of the cumulative present value of the cashflows in each scenario relative to the base case. This component measures potential reinvestment risk or asset-liability mismatch risk. Support bonds receive high scores on this measure.

The measure requires a discount rate and a volatility rate as inputs, which are announced annually by the NAIC. The discount rate should reflect current market rates on noncallable, high-quality securities, while the volatility should reflect the expected annual volatility of interest rates. It should be similar to the step size for the interest rate scenarios.

FLUX FORMULAS

1. *Cumulative Present Value* is the sum of the present values of cashflows for each month for each scenario.

$$CPV_{s,m} = \frac{\text{Principal}_{s,m} + \text{Interest}_{s,m}}{(1+\text{r})^m} + \text{CPV}_{s,(m-1)}$$

where m = the month
r = the monthly interest rate in decimal, set by the NAIC
s = the scenario

2. The *Present Value Score* is the percent decrease in present value from the base case scenario and is calculated for each scenario by dividing the difference in the total cumulative present values by the total cumulative present value of the base case.

$$\Delta PV_s = \frac{MAX\left(0,\ CPV_{base,M} - CPV_{s,M}\right)}{CPV_{base,M}}$$

where M = total number of months

3. *Absolute Scaled Differences* are calculated by taking the ratio of the cumulative present value of the monthly cashflows for each scenario and the cumulative present value of the total cashflows for each scenario. This ratio for the base case is also calculated and subtracted from the first ratio. Then an absolute value is taken of the difference.

$$ASD_{s,m} = \left| \frac{CPV_{s,m}}{CPV_{s,M}} - \frac{CPV_{base,m}}{CPV_{base,M}} \right|$$

where $CPV_{s,M}$ = cumulative present value of scenario cashflows
base = base case

4. The *Timing Score* is the sum for each scenario of the Absolute Scaled Differences for each month each multiplied by yield volatility.

$$T\%_s = \sum_{m=1}^{M} ASD_{s,m} \times V$$

where V = monthly market volatility in decimal (given)

5. *Scenario FLUX* is the sum of the PV Score and the Timing Score for each scenario.

$$FLUX_s = \Delta PV\%_s + T\%_s$$

6. *FLUX for a CMO* is a root mean squared of the Scenario FLUX scores. This is calculated by taking the square root of the mean of the squares of the Scenario FLUX scores.

$$FLUX = \sqrt{\frac{1}{S}\sum_{s=1}^{S} FLUX_s^2}$$

where S = number of interest rate scenarios

Table 10-2 shows an example of the FLUX calculations, assuming a 6% discount rate and a 1.5% volatility (150 bp yield volatility). The example is shown using annual cashflows, while real FLUX uses monthly cashflows. In the base case, the bond pays off fully in year 3 with a coupon of 8%. In scenario 1, half the principal pays down in year 2. In scenario 2, the bond extends to four years.

The cumulative present value (CPV) represents the present value today of that year's cashflow plus the previous CPV. For example, the CPV in year 2 in the base case is given by:

$$14.667 = 8/(1+.06)^2 + 7.547$$

The CPV in year 4 is equal to the present value of the bond's cashflows. The absolute scaled differences (ASD) are calculated by taking the difference between the CPV for the bond in a given year and scenario divided by the present value for that scenario and the scaled CPV of the base case. For example, the ASD for year 2 in Scenario 1 is given by:

$$0.427 = 59.167/104.506 - 14.667/105.346$$

The timing measure equals the sum of the absolute values of the ASDs times the volatility:

$$0.64\% = (0.001 + 0.427) \times 1.5\%$$

The scenario FLUX score is the sum of the timing measure and the PV measure:

$$1.44\% = 0.64\% + 0.80\%$$

The bond's FLUX is the root mean square of the scenario FLUXs:

$$1.3\% = \sqrt{0.5 \times (1.442\%^2 + 1.202\%^2)}$$

Table 10-2

FLUX Calculation Example

BASE	1	2	3	4
Principal	0	0	100	0
Interest	8	8	8	0
CPV	7.547	14.667	105.346	105.346

Interest Rate	6.0
Volatility	1.5%
Total CPV	105.346

Scenario 1	1	2	3	4
Principal	0	50	50	0
Interest	8	8	4	
CPV	7.547	59.167	104.506	104.506
ASD	0.001	0.427	0.000	0.000

Total CPV	T%
104.506	0.64%
Change PV%	FLUX
0.80%	1.44%

Scenario 2	1	2	3	4
Principal	0	0	0	100
Interest	8	8	8	8
CPV	7.547	14.667	21.384	106.930
ASD	0.001	0.002	0.800	0.000

Total CPV	T%
106.930	1.20%
Change PV%	FLUX
0.00%	1.20%

FLUX	1.30%

OUTPUT TABLE

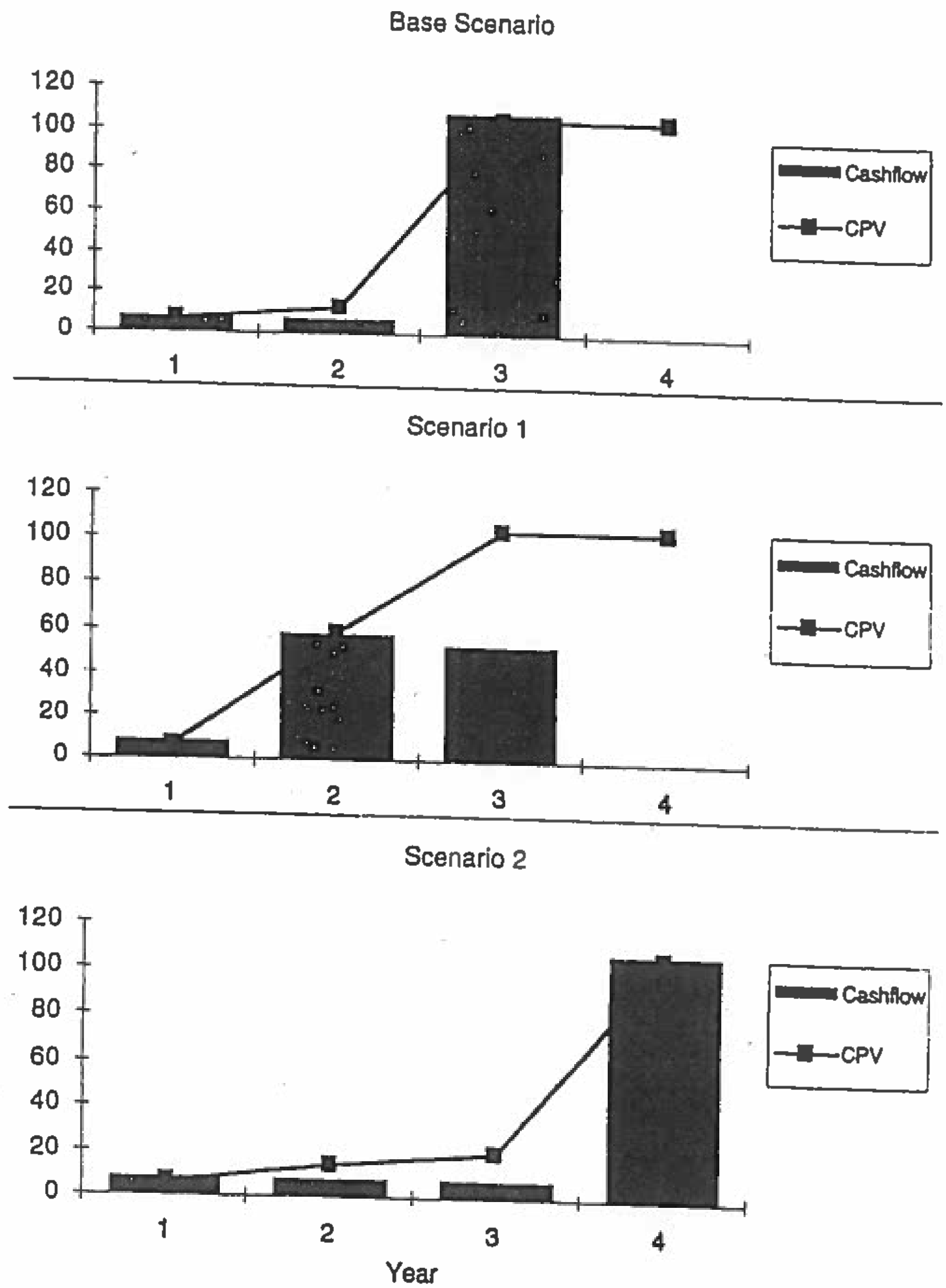
	BASE	Scenario 1	Scenario 2	RMS
Present Value	105.35	0.80%	0.00%	0.60%
Timing		0.64%	1.20%	1.00%
Total		1.44%	1.20%	1.30%

The present value measure differs from other similar measures because the discount rate is held constant for each scenario. Effective duration and OAS measures involve changing the discount rate for each scenario. Here we keep the discount rate constant because we want to measure the potential for change in cashflows, not the potential for changing value.

The Timing Measure

The timing measure is a bit harder to understand, since cumulative present value and absolute scaled differences are new concepts. Figure 10-2 shows the cashflows of the bond under the three scenarios and the cumulative present value. From the graphs, it is easy to see that the cashflows change from scenario to scenario. The amount of cashflow changes only slightly, while the timing of the cashflows changes significantly. The timing calculation provides a numerical measure of how much the timing has changed. The CPV indicates how fast the cashflows are received. The difference in the CPVs reflects how much faster or slower the cashflows are received in the scenario relative to the base case. Cashflows received sooner than expected are subject to reinvestment risk. Cashflows received later than expected expose the investor to additional borrowing costs. The volatility factor helps provide an estimate of the potential impact of these differences in timing.

Figure 10-2
Cashflows for each Scenario



FLUX is a measure developed for the insurance regulators as a screening tool. It is a tool designed specifically to address cashflow risk of individual bonds. FLUX is another tool that investors can use to assess the risk characteristics of CMOs.

The CMO Cashflow Variability Project was headed by Chris Anderson of Merrill Lynch. Dave Hall of ITT/Hartford, Michael Siegal of Goldman Sachs, and Max Bublitz of Conseco headed subcommittees. Lutheran Brotherhood, First Boston, BARRA, Chalke/Intex, Bloomberg, and GAT were all deeply involved in the testing of model formulations and parameters. Numerous other people and firms have also been involved with the project.

Exercise 10-2

Perform a FLUX calculation based upon the following data:

BASE	1	2	3	4
Principal	0	0	100	0
Interest	7.5	7.5	7.5	0
CPV				

Interest Rate	7.5
Volatility	10.0%
Total CPV	

Scenario 1	1	2	3	4
Principal	75	25	0	0
Interest	7.5	7.5	0	0
CPV				
ASD				

Total CPV	T%

Change PV%	FLUX

Scenario 2	1	2	3	4
Principal	0	0	0	100
Interest	7.5	7.5	7.5	7.5
CPV				
ASD				