Chapter Five

Modeling Prepayments

Today some members of the Bulls & Bears MBS research group were coming down to discuss their latest prepayment model. Like the soap companies, the research groups were always promising something "new and improved." The sophisticated statistics indicated that these guys did spend time on the models. But there always seemed to be a gap keeping Susan from putting the models into the simple analytics that she had been building. Maybe the researchers could give a few pointers on some simple modeling techniques. If this went well, she told Bob, the salesman, she'd pick up the tab for lunch.

■ INTRODUCTION

Prepayments are the primary feature of mortgage-backed securities that distinguishes them from all other bonds. Understanding and forecasting prepayments is essential to successful evaluation of mortgage-backed securities as investments. Prepayments represent the actions of individuals and, therefore, forecasting prepayments requires forecasting the aggregate impact of these individual decisions. In this chapter, we look at actual prepayment data. Thorough analysis of the data is required before we even begin the modeling process. We need to understand the limitations of the data while we are identifying the key factors that drive prepayments. This chapter shows how prepayment analysts evaluate data and create prepayment models. In this chapter, we will develop a simple prepayment model that demonstrates several of the principles of prepayment modeling. This model is not intended for actual use in any real investment analysis.

■ SOURCES OF PREPAYMENTS

Prepayments result from four types of events: moving, refinancing, debt retirement, and defaults. Defaults are not actually prepayments; however, most mortgage-backed securities have credit guarantees that transform borrower defaults into prepayments to investors. In order to better understand mortgage-backed securities, it is necessary to understand the conditions under which borrowers will either move, refinance, or default. Because prepayments reflect the actions of individuals, who are driven by a wide variety of economic and social forces, exact predictions are not possible. Prepayment forecasting instead relies on statistical and economic analysis to develop an indication of potential prepayment activity.

■ PURPOSE OF MODELS

Evaluation of the investment characteristics of mortgage-backed securities requires estimates of prepayment rates. These estimates can take various forms, from a single assumption chosen based on experience to complex models that take into account loan level details.

These forecasts, regardless of their source, are used to understand the performance characteristics of mortgage-backed securities and to determine appropriate valuation of different investments.

Prepayment forecasters face a fundamental problem. They seek to estimate future events in a changing world. For example, new loan types are constantly being created and the loan origination process is continually evolving. Still, the primary guide to future prepayments is past prepayments. Thus forecasters develop models that seek to explain prior prepayments. They hope that this information will provide valuable insights into future prepayments. Since the economic and social environment is constantly changing and prepayments are affected by a host of factors, it is unlikely that any historically based analysis will completely reflect future prepayments.

Investing based on prepayment models is a little like driving while looking through the rearview mirror. It may be hard to stay on the road, but it's better than driving with your eyes closed.

Two features that characterize good models are that they are robust and parsimonious. Robust models have the feature that they provide good forecasts under a variety of conditions. That is, they do not need to be continually adjusted to reflect changing environments. If the models need to be changed frequently, then they probably will not provide accurate forecasts of future prepayments. Parsimonious means that the models are as simple as possible. Parsimonious models capture the major variables that affect prepayments using the fewest number of parameters. Parsimonious models have the advantage that they do not "over fit" the data. Using complex models with many parameters, it is possible to set the parameters so that the model provides an excellent fit to historical data, but will not provide accurate projections. The added variables may reflect spurious one-time correlations rather than real long-term relationships. Parsimonious models are also easier to incorporate into valuation tools. In the following pages, we will review the major parameters to be considered and demonstrate how they can be combined to create a prepayment model.

PREPAYMENT DATA

The primary consideration when using prepayment data is that you can only evaluate the data that history has provided. In forecasting prepayments, it would be useful to have historical data that demonstrates the prepayment characteristics of loans under a variety of interest rate and economic conditions. Unfortunately, analysis is limited by the actual data that history provides. Figure 5-1 shows the production of new GNMA 30-year loans by year and by coupon. These graphs are contour graphs. They are like looking at mountain ranges from above. From the chart, we can see that there is only a limited range of coupons produced each year. Thus while we might be interested in how GNMA 8s originated in 1989 prepaid in 1992, there is no data available. For other loan types the amount of data might be even more limited. Figure 5-2 shows the loan originations for conventional seven-year balloon loans. Here we can see that there is even less data available to analyze.

Figure 5-1
New GNMA 30-Year Production by Year and Coupon ($ Billion)

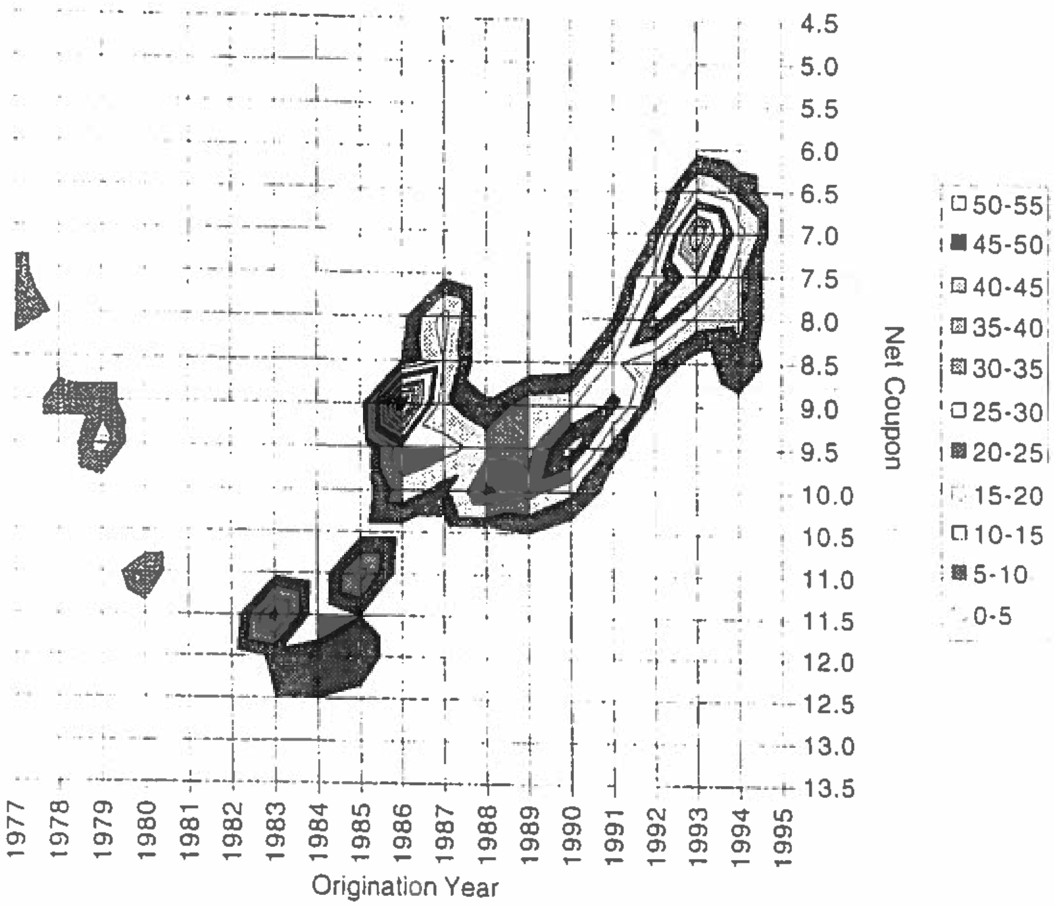


Figure 5-2

Loan Originations for Conventional Seven-Year Balloons ($ Billion)

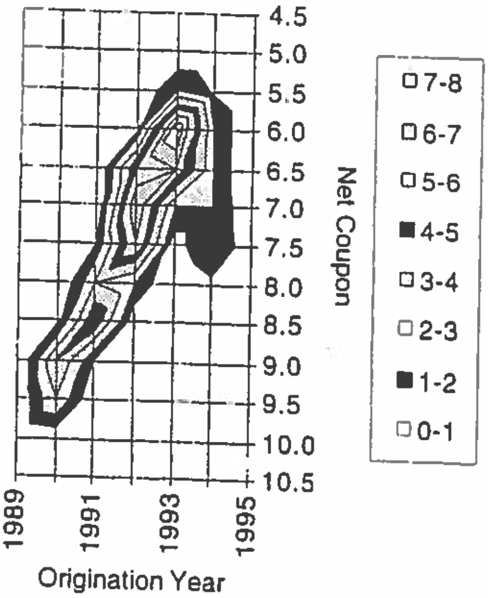
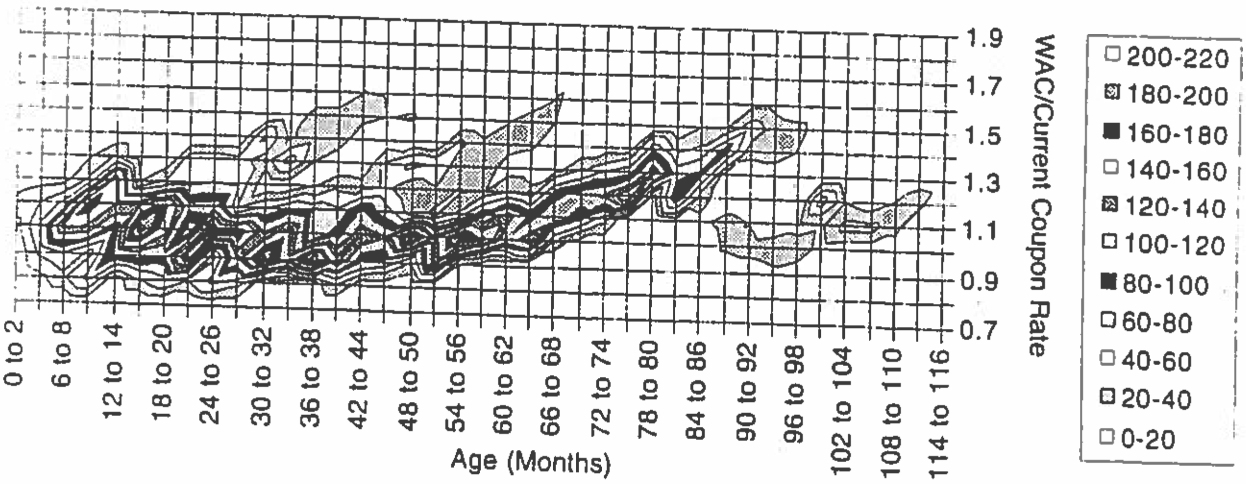


Figure 5-3

Figure 5-1 Data Transformed to Show WAC/CC and Loan Age



Two of the most important factors in determining prepayment behavior are the interest rate of the loans relative to current market rates and the age of the loan. Figure 5-3 shows the data of Figure 5-1 transformed to show the various combinations of loan coupon relative to the current coupon rate and loan age. The interest rate effect can be represented by comparing the interest rate on the loan to the interest rate on loans currently being originated. The yield on MBS selling just below par, called the current coupon yield, is a good proxy for current mortgage levels. In our analysis we divide the weighted average gross coupons on the loans by the current coupon yield as a measure of the interest rate effect. Loans with higher ratios generally have a greater incentive to prepay. For the GNMA data, we can see that there is a fairly wide range of data available for different combinations of coupon ratio and loan age. However, even for this data set there are limitations.

Exercise 5-1

For what range of loan age is there no data for loans with coupon ratio of 1.1? of 0.9?

Exercise 5-2

What is the highest ratio achieved for loans with ages less than 29 months?

Figure 5-4

Balloon Seven-Year by Age and WAC/CC ($ Billion, PSA categories)

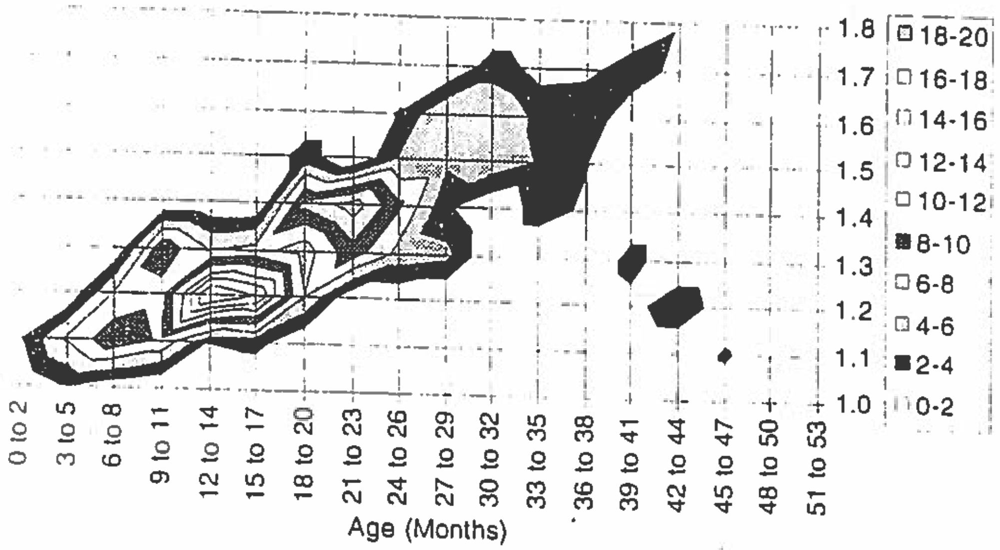


Figure 5-4 shows a similar analysis for the balloon loans in Figure 5-2. Forecasts for data points between existing data points involve interpolation. Forecasts for data points outside the range of existing data points require extrapolation. Generally interpolation is more accurate than extrapolation.

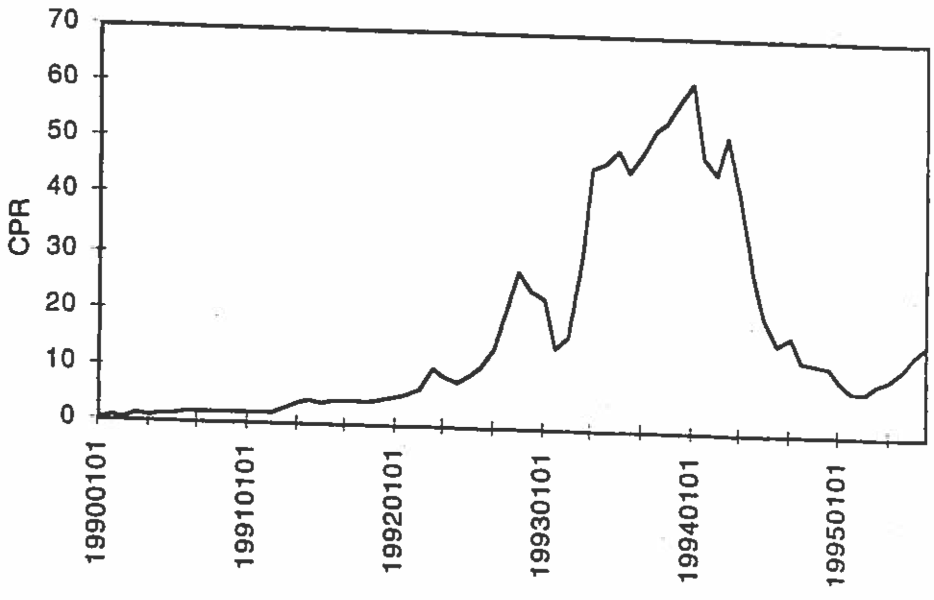
Exercise 5-3

Identify on Figures 5-3 and 5-4 the areas of interpolation and the areas of extrapolation. What does this indicate about the relative reliability of forecasts for GNMA 30-year loans and conventional seven-year balloons? (Keep in mind that the maximum age for the balloon loans is 94 months.)

Viewing prepayment data graphically can often provide insight into prepayment behavior and facilitate prepayment modeling and forecasting. There are two basic types of prepayment graphs that can be examined. The first type is longitudinal or historical data. In this analysis we look at the prepayments on a group of loans or pools over time. This is also sometimes referred to as static pool analysis, because the pool of loans is held constant over time. Figure 5-5 is an example of longitudinal prepayment behavior.

Figure 5-5

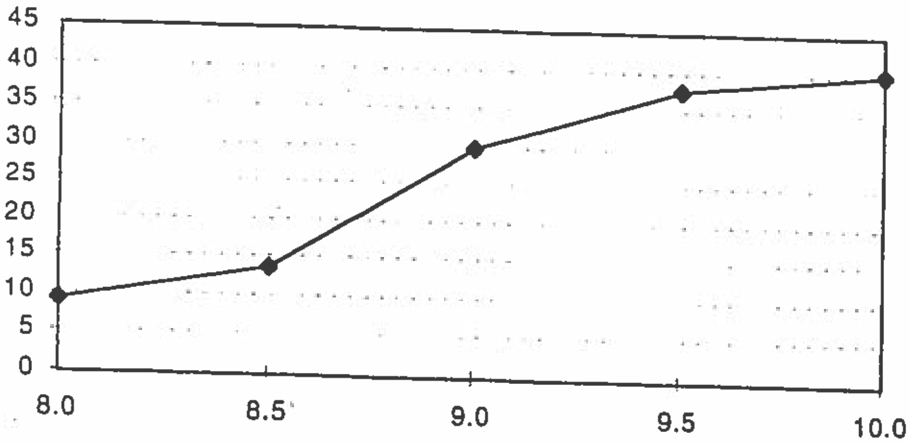
Longitudinal Data: Prepayments Over Time for 1989 Originated GNMA 9s



The second type is cross-sectional data. This involves looking at the prepayment rate on different pools of loans, with the data separated by a common characteristic. The most common form of cross sectional data is prepayments by coupon for a given time period. Figure 5-6 shows the one-month prepayment rates on various coupons in the month of April 1993.

Figure 5-6

Cross-Sectional Data: CPR of GNMA 1989 Originations by Coupon



The two graphs produced above are based on the data in Table 5-1. This data is provided so you can learn about prepayment modeling. We do not recommend that any extensive analysis be conducted on this data because it is a relatively small data set. This data should only be used for the exercises in this chapter. The data represents one-month prepayment rates for various GNMA 30-year pools, expressed as CPRs. All of the loans were originated in 1989. The "origination month" indicates the average month of origination. The "current coupon" column represents the monthly average current coupon yield for GNMA 30-year MBS expressed as a bond-equivalent yield.

Table 5-1

GNMA 30-Year Prepayment Data in % CPR

Coupon Orig month	8.0 2/89	8.5 5/89	9.0 8/89	9.5 5/89	10.0 4/89	Current Coupon Yield
19900101	0.7	0.5	0.5	1.6	2.5	9.57
19900201	0.9	0.8	1.1	1.6	2.5	9.83
19900301	0.7	0.7	0.6	2.0	2.7	9.93
19900401	1.4	0.9	1.2	2.5	3.6	10.11
19900501	1.3	0.8	1.0	2.5	3.4	10.10

19900601	1.8	1.4	1.4	2.9	4.2	9.78
19900701	1.4	1.4	1.6	3.4	4.4	9.67
19900801	1.2	1.8	1.8	3.1	4.7	9.83
19900901	2.2	1.7	1.9	3.6	5.4	9.91
19901001	1.7	1.6	1.7	3.3	4.3	9.90
19901101	3.0	4.2	2.0	3.3	4.9	9.59
19901201	1.8	1.7	1.7	2.9	4.5	9.24
19910101	1.8	1.9	2.0	3.7	4.9	9.17
19910201	1.9	1.7	1.8	2.6	4.5	8.86
19910301	1.5	2.2	1.9	3.2	5.2	9.13
19910401	2.5	2.9	2.8	4.4	6.7	9.06
19910501	3.5	7.0	3.6	5.4	8.9	9.07
19910601	3.9	4.1	4.3	6.1	9.7	9.26
19910701	3.6	3.2	4.0	6.5	9.7	9.15
19910801	3.2	3.3	4.1	6.4	8.6	8.73
19910901	3.0	4.2	4.2	6.4	8.6	8.48
19911001	4.4	3.2	4.1	5.7	7.9	8.31
19911101	4.1	3.4	4.4	6.9	11.9	8.22
19911201	3.9	4.2	4.6	8.0	16.2	7.82
19920101	3.9	4.2	5.3	10.2	20.9	7.80
19920201	4.4	5.3	5.9	12.6	23.5	8.15
19920301	4.3	4.7	6.5	18.9	30.0	8.38
19920401	6.1	6.2	10.2	24.0	39.6	8.26
19920501	7.3	5.6	9.0	20.3	33.6	8.14
19920601	7.3	7.1	8.2	16.4	27.3	7.96
19920701	7.7	6.5	9.4	18.0	26.3	7.48
19920801	7.4	7.6	10.7	19.2	26.5	7.29
19920901	7.1	7.0	14.2	25.9	29.2	7.30
19921001	6.3	9.1	21.6	37.5	37.9	7.61
19921101	8.0	11.0	28.0	43.6	43.6	7.88
19921201	8.7	10.4	24.5	38.7	40.2	7.68
19930101	7.9	10.4	23.1	38.5	44.5	7.42
19930201	6.3	6.0	14.9	25.3	31.0	7.11
19930301	5.8	6.0	16.8	25.6	29.6	6.89
19930401	9.4	13.7	29.9	37.7	40.0	6.90
19930501	10.8	27.5	46.4	49.4	48.9	6.96
19930601	15.1	28.6	47.3	49.9	47.3	6.81

19930701	16.4	33.2	49.5	52.8	51.4	6.61
19930801	16.1	31.4	45.7	50.0	46.8	6.52
19930901	17.6	35.6	49.2	48.3	47.5	6.21
19931001	24.2	39.4	53.4	51.9	49.3	6.22
19931101	28.6	46.5	54.9	52.5	48.8	6.69
19931201	34.5	50.6	58.9	54.3	53.3	6.68
19940101	33.8	49.9	62.0	60.5	57.6	6.53
19940201	23.7	36.5	48.6	48.9	49.3	6.80
19940301	23.6	34.1	45.8	46.3	45.2	7.46
19940401	25.4	39.0	52.6	53.8	54.6	8.07
19940501	18.3	24.8	41.6	46.7	50.7	8.25
19940601	17.5	19.0	28.5	34.7	41.8	8.13
19940701	12.5	15.7	20.9	27.1	33.5	8.34
19940801	10.8	11.9	16.1	23.0	27.3	8.27
19940901	11.0	14.0	17.3	20.9	25.0	8.47
19941001	11.7	8.6	13.2	19.5	22.2	8.80
19941101	8.1	8.8	12.9	16.7	21.3	9.00
19941201	8.8	9.4	12.5	14.7	17.7	8.91
19950101	9.5	7.3	9.9	12.8	17.6	8.82
19950201	5.9	5.1	8.0	9.7	13.8	8.49
19950301	5.8	6.4	7.8	9.1	11.7	8.22
19950401	8.2	10.6	9.5	12.2	13.2	8.11
19950501	7.8	8.4	10.6	12.6	16.1	7.69
19950601	12.0	11.4	12.2	14.8	16.0	7.34
19950701	12.2	12.4	14.9	16.3	18.7	7.41
19950801	11.4	14.6	16.6	16.9	18.5	7.63

Source: Bloomberg Financial Markets.

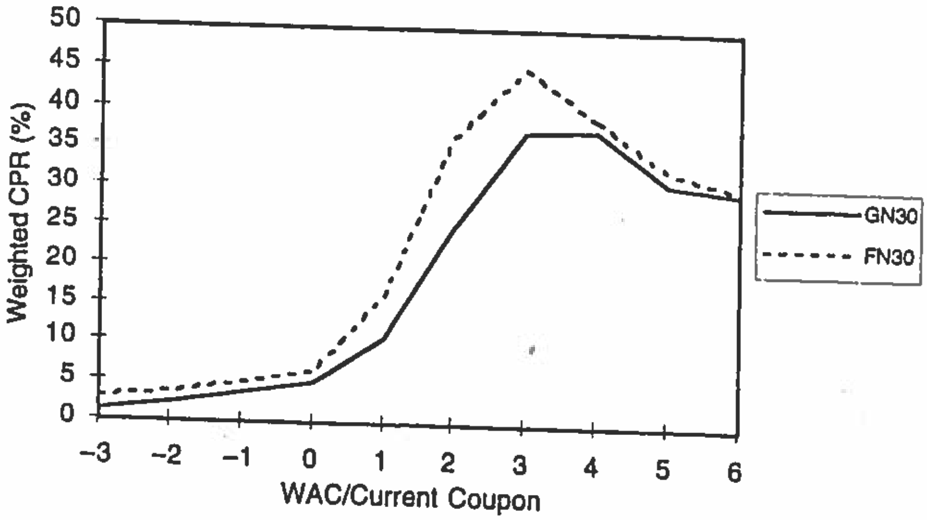
Interest Rate Effect

The primary factor that influences prepayment rates is the borrower's opportunity to refinance. This is reflected by the coupon on the borrower's loan versus the interest rate currently available in the market for new loans. We use the yield on mortgage-backed securities trading near par, sometimes called the current coupon ~~yield,~~ as a proxy for the rates available to borrowers. The relationship between the mortgage coupon and the current coupon yield can be reflected either as a difference or as a ratio. Loans where the difference between the loan coupon and the current coupon is greater than zero or where the ratio is above one have a greater incentive to refinance.

Figure 5-7 is a cross-sectional analysis of prepayment rates as a function of the coupon on the loan relative to the current coupon yield. Note that prepayment rates are relatively stable when the coupon on the mortgage is below the current coupon yield, but then increase rapidly as the incentive to refinance to a lower coupon loan increases. The prepayment rate then peaks and begins to fall off.

Figure 5-7

Cross-Sectional Data Showing the Interest Rate Effect



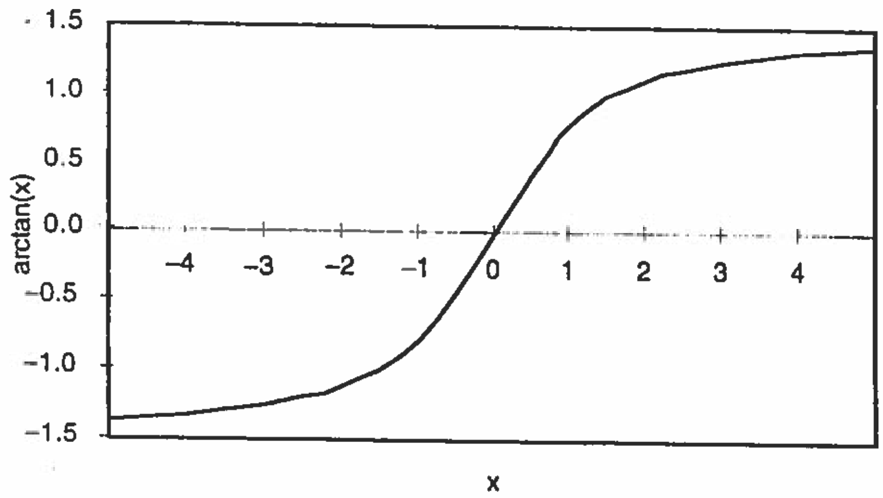
■ REVIEW QUESTION

Issues to think about: Do borrowers with very high coupon differences relative to the current coupon yield (greater than 300 basis points) have less incentive to refinance than borrowers with lower coupon differences? What accounts for the dip in the prepayment curve?

One approach to modeling prepayments is to choose a functional form that reflects the analysts' view of the relationship between the variables. The analyst then attempts to find parameter values that match the function to the data. This is normally done using sophisticated statistical packages. This process is complicated by nonlinear interactions between the factors driving prepayments. Below we will present a very simplified functional form that represents the product of functions for each of the individual factors.

The arctangent function is a convenient nonlinear representation for the shape of the prepayment curve. At one point, many analysts used this function. Now, however, it has been replaced by more complex functional forms. The arctangent function has the shape shown in Figure 5-8. Note the similarity between this shape and Figure 5-7. While the arctangent function has the same general shape as the prepayment curve, it goes through different points.

Figure 5-8
The Arctangent Function



In order to use the arctangent function for prepayment analysis, it is necessary to transform the function to match the prepayment curve. The arctangent function can be transformed by using a function of the form:

$$\text{Interest Rate Effect} = a + b \times (\arctan(c + d \times (\text{diff})))$$

Equation 5-1
Arctangent Function Transformation for Interest Rate Effect

where a, b, c, and d are constants and diff is the difference between the coupon on the loan and the current coupon yield expressed in basis points. The constants a, b, c, and d need to be chosen in order to match the arctangent curve to the prepayment data. Equation 5-1 has four constants (a, b, c, and d); therefore we need four points on the prepayment curve to solve for these values.

Exercise 5-4

Solve for a, b, c, and d in Equation 5-1. (Note: In Excel, the arctangent function is atan(x)).

Assume:

1. The maximum CPR is 50%.
2. The minimum CPR is 6%.
3. The midpoint 28% CPR occurs at diff = 200 basis points.
4. At midpoint, max slope is 6% CPR for a 10 basis point rate shift.

Using a little algebra and calculus, Equation 5-1 gives the following relationships:

1. a = (max CPR + min CPR)/2.
2. b = (max CPR – a)/(π/2).
3. d = max slope/b.
4. c = –d × midpoint diff.

Note: More advanced readers can derive these equations.

Based on these equations solve for a, b, c, and d. Note that this is not an actual prepayment function. While these numbers are similar to GNMA prepayment estimates the results are only intended as an example of how to fit the arctangent curve to prepayment data. Solve these equations using prepayment rates in percent (i.e. 50%) and rate changes in basis points (i.e. 200 bp).

Work area

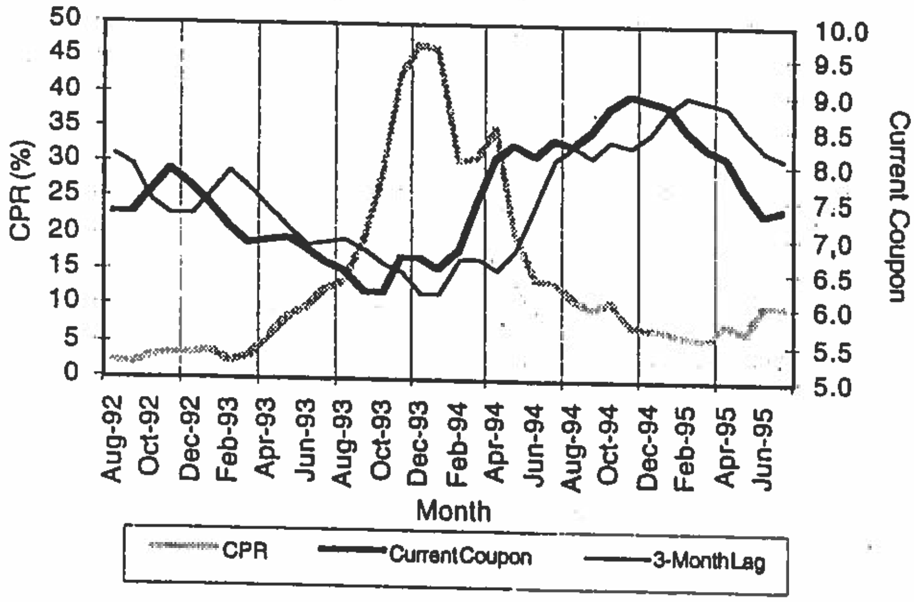
a =
b =
c =
d =

Exercise 5-5

Using the data in Table 5-1, reproduce Figure 5-7. The first step is to compute the relative coupon for each prepayment observation. Next it is necessary to place the prepayment rates in buckets for each range. For example, coupon differentials of 0.76 to 1.25 can go in the 100 basis point difference bucket. Finally an average for each bucket can be calculated.

There is usually a time lag between changes in interest rates and changes in prepayment rates. Figure 5-9 demonstrates this effect. Note how interest rates changed in September 1993 followed by a change in prepayment rates three months later. Using Figure 5-9, see if you can find another period when rates moved rapidly and the change in prepayment rates occurred later.

Figure 5-9
A Time Lag Between Interest Rate Changes and Prepayment Rates



■ REVIEW QUESTION

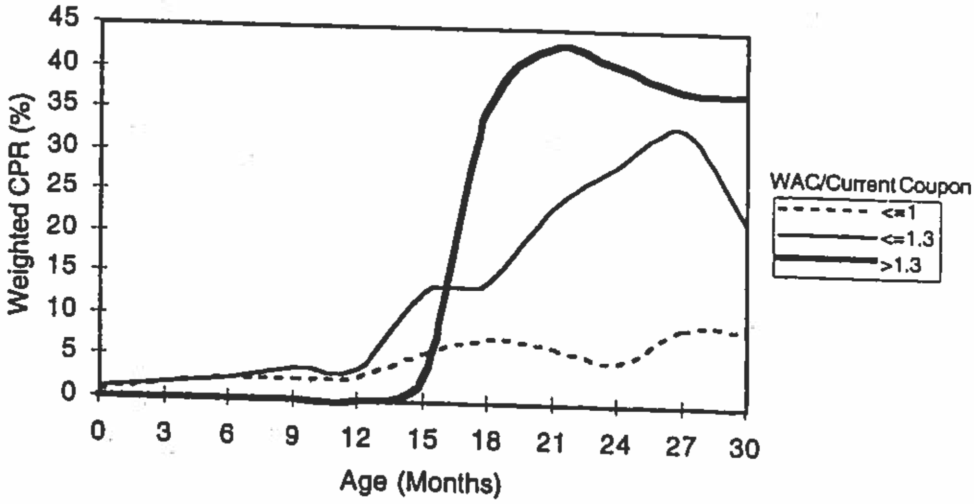
How would you change the method in Exercise 5-5 to reflect this lag effect? Many investors rely on current prepayment data as an indication of future prepayments. When is this most likely to be unreliable?

Aging

The second major component of prepayment behavior is aging. Aging reflects the observation that newer loans tend to prepay slower than older or "seasoned" loans. Figure 5-10 shows the prepayment rate of FNMA 30-year loans by loan age. The chart splits out loans by discount coupon, low premium coupons, and high premium coupons. The discount loans reach their peak prepayment speeds after 30 months. The low premiums reach their peak speed at about 27 months, while the high premiums peak in about 21 months. The PSA curve is used to approximate this effect. The graph, however, makes it clear that not all loans reach their peak speed in 30 months as assumed by the PSA curve.

Figure 5-10

Aging Effect for Current Coupon and Medium and High Premium Loans



Exercise 5-6

Graph the prepayment data in Table 5-1 by loan age. (A graph by "as of date" will approximate a graph by loan age, since all the loans were originated within a few months of each other.)

Aging can be reflected in the formula:

Equation 5-2
An Aging Formula

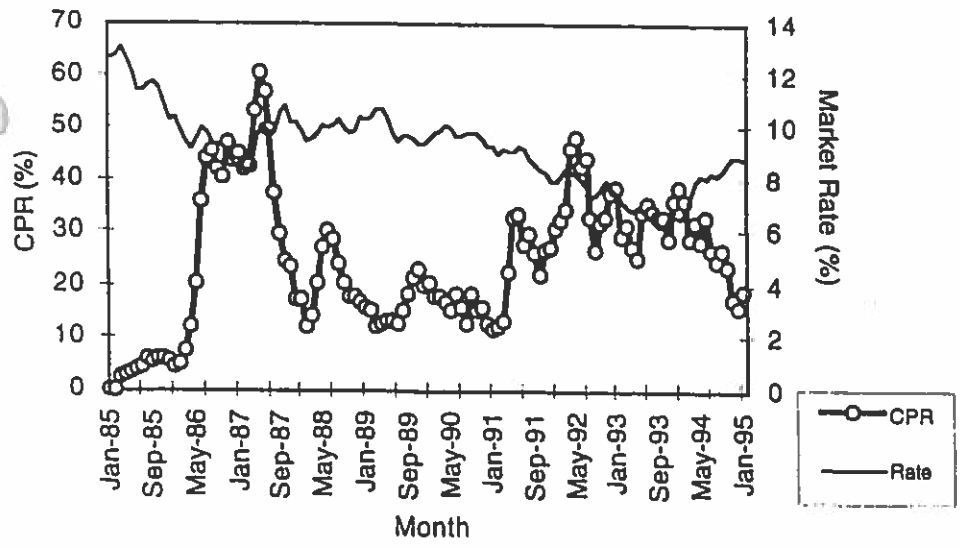
$$\text{Age \%} = \min\left[\frac{\text{age}}{e}, 1\right]$$

where age is the age of the loan in months and e is a constant. A standard assumption, consistent with the PSA model is that e = 30.

Burnout

The third factor affecting prepayments is burnout. This is perhaps the most complex of the components. Burnout reflects the observation that as interest rates drop, prepayments for a pool of loans peak and then decline. Even if rates fall further, later prepayments do not reach the earlier peaks. Figure 5-11 demonstrates this effect. This graph shows prepayments on FNMA 11.5s originated in 1985. Prepayment rates peaked near 60% CPR in the summer of 1987. They then declined rapidly, reflecting burnout and higher interest rate levels. In 1992 the burnout effect is clear. Even though rates fell below the levels of 1987, prepayments did not reach the same peak level and were unable to sustain rates over 40% CPR despite substantially lower interest rate levels.

Figure 5-11
Burnout Effect Using Longitudinal Data



A simple example of burnout is shown in Figure 5-12. Suppose a pool of 100 loans is made of 80 fast prepayers who prepay at 75% per year and 20 slow prepayers who prepay at 10% per year. During the first period, you would expect prepayments of 62% (0.80 × 75% + .20 × 10%). After one year, three quarters of the fast prepayers are gone. Only 10% of the slow prepayers have prepaid. The remaining composition is 20 fast and 18 slow. The new expected prepayment rate is about 44%.

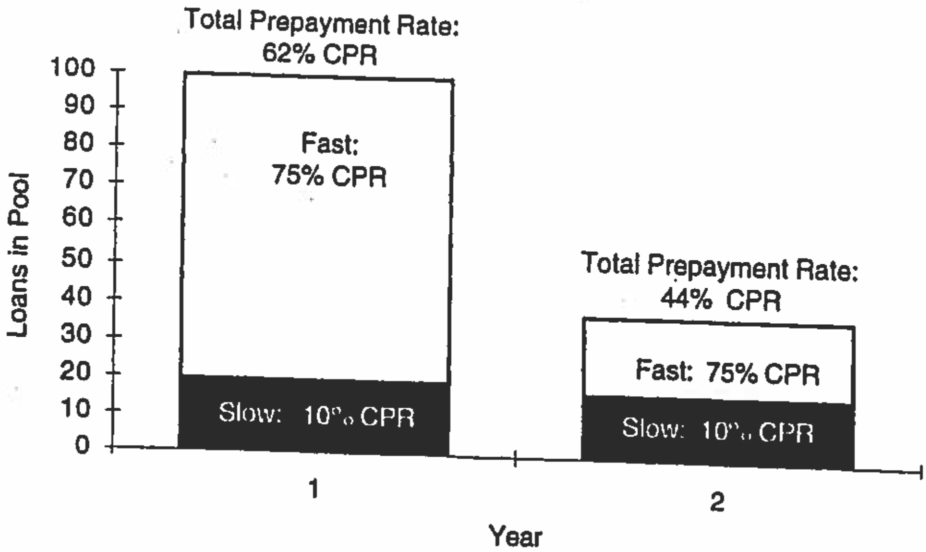
For our model we can use a simple functional form that describes burnout as a function of the pool factor.

$$\text{Burn \%} = 1 - f \times (1 - \text{factor})$$

Equation 5-3
A Burnout Formula

where f is a constant parameter of the model and "factor" is the pool factor. We will assume for our model that f = 0.7.

Figure 5-12
A Burnout Example



Exercise 5-7

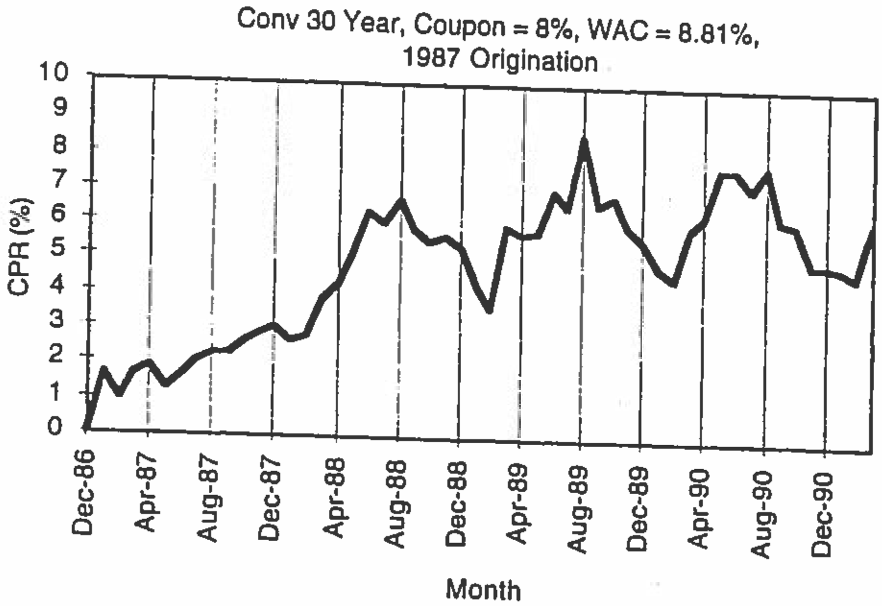
What happens to the usefulness of this equation in forecasting burnout, if the pool consists of loans that were seasoned before they were pooled? Hint: the pool factor is 1 when the pool is created, regardless of the age of the loans.

Burn % = 1

Seasonality

The fourth factor affecting prepayment rates is seasonality. One of the sources of prepayments is housing turnover. This economic activity is seasonal due to weather, school schedules, and possibly, tax considerations. The seasonality of housing turnover is clearly seen in the prepayment data. Figure 5-13 shows the prepayment rates for discount conventional loans during 1987-1990. Prepayment seasonality is reflected in the pattern showing higher prepayment rates in the late summer and lower prepayment rates in the winter.

Figure 5-13
Prepayment Seasonality



Exercise 5-8

Using the data below, calculate the average prepayment rate for each month of the year. Divide these average rates for each month by the overall average prepayment for all the data to produce monthly adjustments.

Work area

Month	89 8.0s	90 8.0s	89 8.5s	90 8.5s	Average	Factor
Jan	4.19	4.79	4.93	5.58	4.87	0.75
Feb	3.61	4.54	4.96	5.40		
Mar	5.91	5.95	6.08	6.72		
Apr	5.73	6.33	6.49	7.47		
May	5.79	7.69	6.86	8.05		
June	6.97	7.69	7.28	8.32		
July	6.56	7.15	7.46	7.96		
Aug	8.56	7.71	8.68	8.80		
Sep	6.62	6.21	7.77	7.20		
Oct	6.81	6.12	7.46	7.39		
Nov	5.98	5.00	6.67	5.71		
Dec	5.54	4.93	6.67	5.74		
Average	6.02	6.18	6.78	7.03		

In prepayment models, seasonality is often reflected as a lookup function, where the seasonal effect is found on a table:

Equation 5-4
Seasonality as a Lookup Function

$$\text{Seasonal (month)} = \text{Result from table for month}$$

Other Factors

Prepayments are also affected by a variety of other factors. Some of these include loan size, loan-to-value ratios, local employment levels, points paid at origination, and availability of other loan types. Virtually any factor that affects homeowners could have an impact on prepayments. Prepayment models can be developed to include these factors.

Building a Simple Prepayment Model

The factors and formulas that we have discussed above can be combined together to produce an overall model to describe and forecast prepayment behavior. In a sophisticated model these factors may interact with complex relationships between them. In addition, other factors may also be used in the model. Figure 5-11, shown earlier, in addition to demonstrating burnout, combines elements of all of the factors we have considered so far. See if you can identify them in the graph.

For our purposes here we will construct a simple prepayment model using Formulas 5-1 through 5-4 above. This model is *not* intended for actual analysis. The form of the model is that the effects are all multiplicative:

Equation 5-5
Simple Prepayment Model

$$\text{Interest(diff)} \times \text{Age\%(age)} \times \text{Burn\%(factor)} \times \text{Seasonal(month)}$$

Exercise 5-9

Use the parameter values established above and fill out the following table using the inputs given below. Be sure to convert the coupon difference to basis points before using Equation 5-1.

Current Coupon	WAC	Age	Factor	Month
8	9	24	0.9	Jan
7	9	50	0.8	Jul
6	9	75	0.7	Mar
8	10	50	0.6	Feb
9	7	10	0.8	Oct
7	14	120	0.4	Nov

Work area

Interest	Age	Burnout	Seasonal	Total

Table 5-2 is an example of actual forecasts for December as of December 15, 1995. These forecasts represent the median forecast of several dealers who post their prepayment forecasts on Bloomberg. Note how the forecasts follow somewhat the same pattern seen in Figure 5-7.

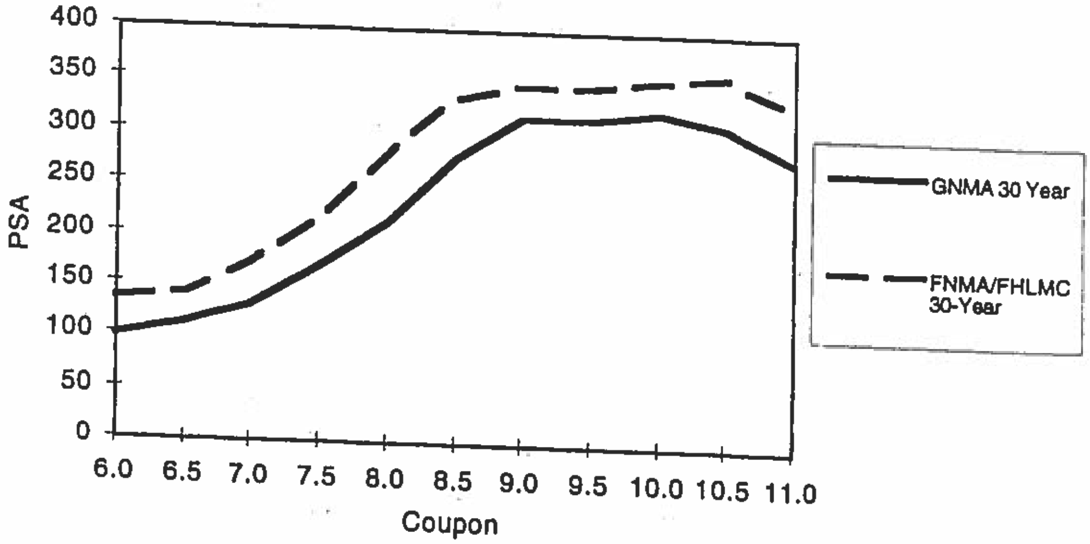
Table 5-2
Sample Forecasts from the Bloomberg

Coupon	GNMA 30-Year	FNMA/ FHLMC 30-Year
6.0	98	134
6.5	110	140
7.0	129	171
7.5	167	213
8.0	212	280
8.5	277	332
9.0	315	347
9.5	314	347
10.0	323	354
10.5	309	359
11.0	280	331

Source: Bloomberg Financial Markets.

Prepayment forecasting is a combination of economics, psychology, and statistics. The work of prepayment analysis is never complete. Innovations in mortgage products and the changing dynamics of borrowers and originators mean that every new piece of data adds to our understanding of prepayments. This exciting dynamic adds to the complexity and challenge of investing in mortgage-backed securities.

Figure 5-14
Dealer Median Prepayment Date from Table 5-2



Source: Bloomberg Financial Markets.

■ ANSWERS TO EXERCISES

5-1

Ratio of 1.1: 75-86, 111+

Ratio of 0.9: 0-2, 57+

5-2

The highest ratio is approximately 1.36.

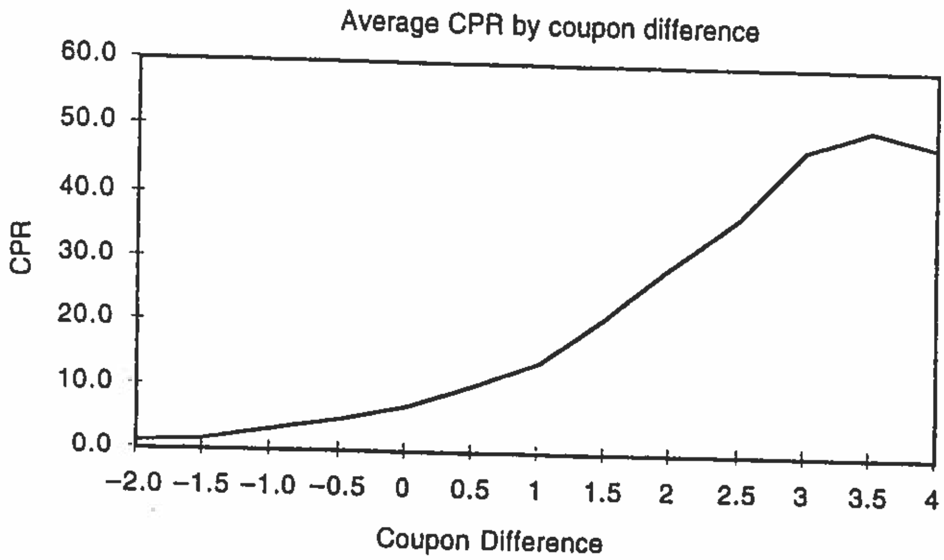
5-3

Forecasts for current coupon new loans or premium older 7-year balloons should be fairly reliable. Outside this range, say for 27-month current coupon loans, forecasts would not be very reliable since there is no data. GNMA 30-year loans have a much larger data set including some high discount to high premium loans for a variety of ages. Thus, one would expect GNMA forecasts to be more reliable.

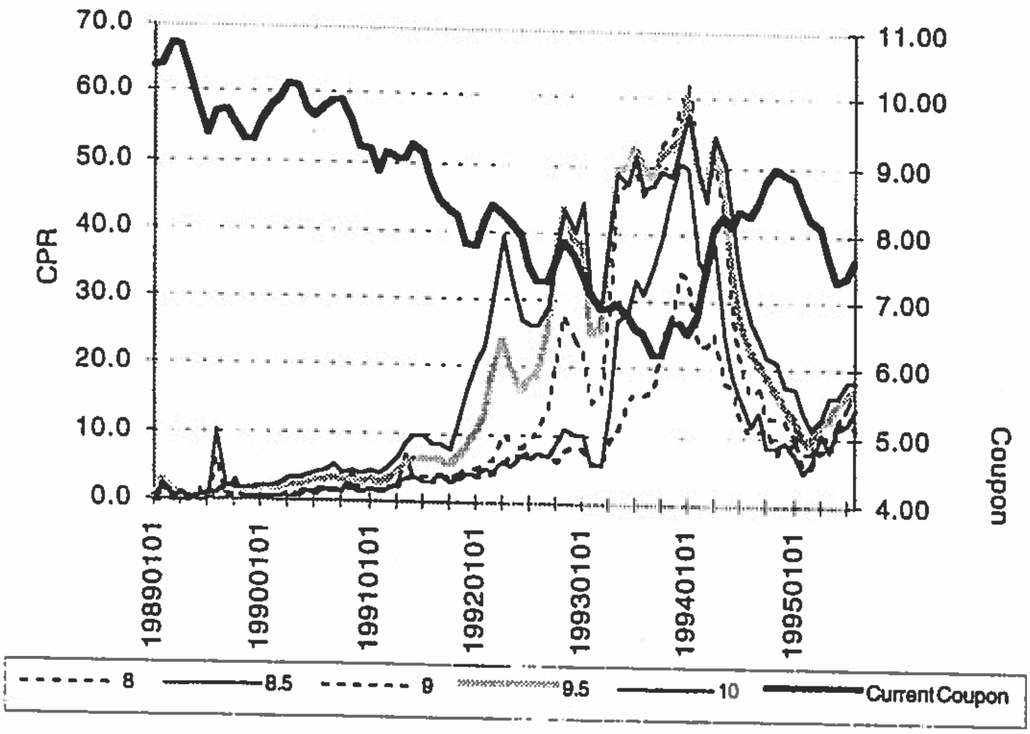
5-4

a	28.0%
b	14.0%
c	–8.571
d	0.043

5-5


Average CPR by coupon difference
60.0
50.0
40.0
30.0
20.0
10.0
0.0
CPR
−2.0 −1.5 −1.0 −0.5 0 0.5 1 1.5 2 2.5 3 3.5 4
Coupon Difference

5-6


70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
CPR
11.00
10.00
9.00
8.00
7.00
6.00
5.00
4.00
Coupon
19890101
19900101
19910101
19920101
19930101
19940101
19950101
8
8.5
9
9.5
10
Current Coupon

5-7

Generally, the pool will be more "burned-out" than the Burn% calculation indicates. The prepayment forecasts from the model will be higher than if the loans were pooled when new. There are many other variables that affect burnout other than the pool factor that should be considered in a sophisticated prepayment model.

5-8

Month	89 8.0s	90 8.0s	89 8.5s	90 8.5s	Average	Factor
Jan	4.19	4.79	4.93	5.58	4.87	0.75
Feb	3.61	4.54	4.96	5.40	4.63	0.71
Mar	5.91	5.95	6.08	6.72	6.17	0.95
Apr	5.73	6.33	6.49	7.47	6.51	1.00
May	5.79	7.69	6.86	8.05	7.10	1.09
June	6.97	7.69	7.28	8.32	7.57	1.16
July	6.56	7.15	7.46	7.96	7.28	1.12
Aug	8.56	7.71	8.68	8.80	8.44	1.30
Sep	6.62	6.21	7.77	7.20	6.95	1.07
Oct	6.81	6.12	7.46	7.39	6.95	1.07
Nov	5.98	5.00	6.67	5.71	5.84	0.90
Dec	5.54	4.93	6.67	5.74	5.72	0.88
Average	6.02	6.18	6.78	7.03	6.50	1.00

5-9

Interest	Age	Burnout	Seasonal	Total
9.2%	0.80	0.93	0.75	5.13%
28.0%	1.00	0.86	1.12	26.97%
46.8%	1.00	0.79	0.95	35.12%
28.0%	1.00	0.72	0.71	14.31%
6.8%	0.33	0.86	1.07	2.09%
49.3%	1.00	0.58	0.90	25.73%

CHAPTER SIX

CMOs, IOs, and POs, and Structuring

Like sharks on the blood trail, the Street had found out that Susan had a good bid for MBS. She was being shown lots of secondary bonds as well as new CMOs. With the secondary bonds, there seemed to be some tools available to look at risk and return. However, with the new issues, all she had were the price/yield tables. She really felt uncomfortable using these for investment decisions. It was a bit like buying shoes through a catalog. Small changes in the cut and style could make an expensive purchase uncomfortable. To make a decision she needed to try the bond on for size—manipulate the cashflows. Buying a structuring model was too expensive. Maybe she could just develop some simple tools to carve up the cashflows.

■ INTRODUCTION

Collateralized Mortgage Obligations (CMOs) facilitate the distribution of the vast amount of mortgage debt. The CMO structuring process creates bonds with a variety of investment characteristics that are different from the underlying collateral. These bonds can be distributed to investors who have a variety of investment needs. Bonds can be created with differing average lives, coupons, and prepayment sensitivities. This variety of investment products serves to expand the demand for mortgage-backed securities.

The motivation for creating CMOs is arbitrage. Wall Street dealers seek to create CMOs when the value of the CMO bonds exceeds the value of the mortgage collateral used to create the CMO plus any expenses. CMO bonds, created by Wall Street, must compete with other fixed-income securities in investor portfolios. Each CMO bond has specific investment characteristics that may be similar to other fixed-income securities. By transforming mortgages into bonds with characteristics more like other securities, the CMO market serves to link the MBS market to other fixed-income markets.

One motivating factor for the development of the CMO market was the ability to distribute comparatively uniform mortgage-backed securities to a diverse investor base. Investors were willing to pay more for bonds that met their investment needs. Another source of arbitrage was the difficulty of analyzing some high risk/high yield CMOs, creating uncertainty about fair value.

CMO creation is subject to a variety of legal and tax issues. The most important considerations are the Real Estate Mortgage Investment Conduit (REMIC) regulations. These are a set of tax rules that allow CMOs to avoid taxation at the trust level, which would otherwise make CMOs uneconomical to create. The rules distinguish between two types of securities, regular interests and residual interests. Each must meet certain requirements. The bonds that we discuss below would usually be structured as regular interests.

■ CMOS AS RULES

CMO bonds are created by distributing the cashflow of the underlying mortgage-backed securities according to a set of rules. These rules describe how principal is allocated among the various CMO bonds and how interest is allocated among the bonds. Principal and interest can be allocated using a variety of rules. While an infinite variety of rules is theoretically possible, most CMOs are created using a few standard types. For principal payments these include: sequential, pro-rata, and scheduled. For interest payments they include: fixed, floating, inverse-floating, and accrual. Most CMOs are created through combinations and layering of these structuring methods.

Constructing a CMO

In this chapter, we construct simple CMOs using principal and interest pay types described above. These exercises build on previous chapters.

The chapter contains two parallel sets of exercises. A simplified method demonstrates the main principles, but omits some of the details. The second method, a more detailed approach uses the standard mortgage cashflow calculations discussed in earlier chapters. The more detailed approach is best performed on a spreadsheet or using a programming language. Readers may perform either set of exercises or both. There are also advanced questions and exercises for which we have not provided answers. These questions are intended to provoke thought about some of the more complex issues in CMOs.

The simplified CMO model uses the following assumptions:

Annual cashflows.

Annual interest payments.

Annual prepayment as a percentage of original balance (APP).

Five-year maturity, no amortization.

No servicing fee for loans.

The spreadsheet model uses the following more realistic assumptions:

Monthly cashflows.

Monthly interest payments.

PSA model for prepayments.

30-year level pay mortgages.

Net coupon and gross coupon.

Step 1 Create the Cashflows of the Underlying Securities

The structure of any CMO is dependent on the cashflows of the underlying mortgage pool. The cashflows from Exercise 6-1 will form the basis for all the CMOs that we create in this chapter.

Exercise 6-1

Simple CMO Model:

Assume an initial balance of $100 million dollars (in millions).

Assume a security coupon of 10%.

Assume that the loan matures in five years and has no principal amortization.

Assume prepayments are stated as an annual percentage of the original balance (APP).[1] For example a 15% APP means that $15 million prepays each year.

Spreadsheet Model:

Use the cashflow model created in Chapter 3.

Assume an initial balance of $100 million.

Assume a net coupon of 10% and a gross coupon of 10.65%.

Assume a maturity of 30 years.

Exercise 6-1a (Simple CMO Model)

Calculate the cashflows of the mortgages assuming a 10% APP, 20% APP, and 30% APP. Show the principal and interest cashflows separately.

[1] APP is similar to the "ABS" method used for asset-backed securities.

Exercise 6-1b (Simple CMO Model)

Compute the average life of the mortgages for 10% APP, 20% APP and 30% APP.

Work area

Underlying Security	
Balance	100
Coupon	10
Maturity	5

APP 10%

Year	Balance	Principal	Interest	Cashflow
1	100	10	10	20
2				
3				
4				
5				
6				
Avg Life				

APP 20%

Year	Balance	Principal	Interest	Cashflow
1	100			
2				
3				
4				
5				
6				
Avg Life				

APP 30%

Year	Balance	Principal	Interest	Cashflow
1	100			
2				
3				
4				
5				
6				
Avg Life				

Exercise 6-1c (Spreadsheet Model)

Calculate the cashflows of the mortgages assuming 100% PSA, 175% PSA, and 400% PSA. Graph the cashflows and balances.

Exercise 6-1d (Spreadsheet Model)

Compute the average life of the mortgages for 100% PSA, 175% PSA, and 400% PSA.

Work area

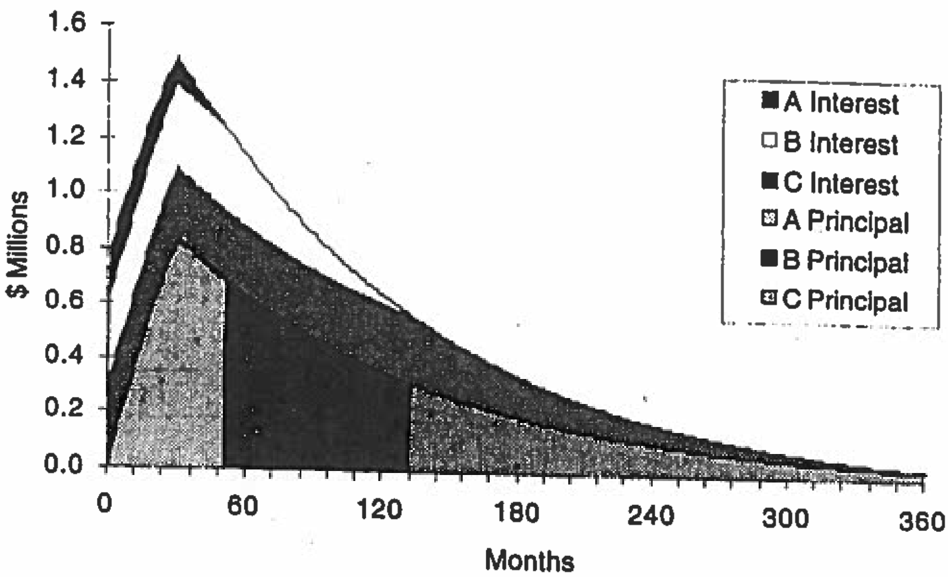
PSA	Average Life
100%	
175%	
400%	

Step 2 Create a Fixed-Rate Sequential CMO

Sequential bonds are formed using a principal payment rule. All principal payments go to the first bond until it is retired, then principal cashflows go to the next bond.

Figure 6-1

A Fixed-Rate Sequential CMO at 175% PSA



Fixed-rate is an interest payment rule. Each bond receives an interest payment each period equal to its beginning of period balance times its coupon. Even if the bond is not receiving a principal payment, it still receives interest payments. In the following exercise, there will be interest payments from the collateral that are not distributed to any bond. The cashflows for the fixed-rate sequential CMO to be built in Exercise 6-2c are shown in Figure 6-1.

Exercise 6-2

Start with the assumptions of Step 1. Assume the following bond characteristics:

Bond	Balance	Coupon
A	$30mm	7
B	$40mm	9
C	$30mm	10

Exercise 6-2a (Simple CMO Model)

Calculate the cashflows of bonds A, B, and C assuming 20% APP.

Exercise 6-2b (Simple CMO Model)

Calculate the average life of bonds A, B, and C assuming 20% APP.

Work area

Bond A

Year	Balance	Principal	Interest	Cashflow
1	30.0			
2				
3				
4				
5				
6				
Avg Life				

Bond B

Year	Balance	Principal	Interest	Cashflow
1	40.0	0	3.6	3.6
2				
3				
4				
5				
6				
Avg Life				

Bond C

Year	Balance	Principal	Interest	Cashflow
1	30.0	0		
2				
3				
4				
5				
6				
Avg Life				

Exercise 6-2c (Spreadsheet Model)

Calculate and graph the cashflows of bonds A, B, and C assuming 175% PSA.

Exercise 6-2d (Spreadsheet Model)

Calculate the average life of bonds A, B, and C assuming 175% PSA.

Work Area

Bond	Average Life
A	
B	
C	

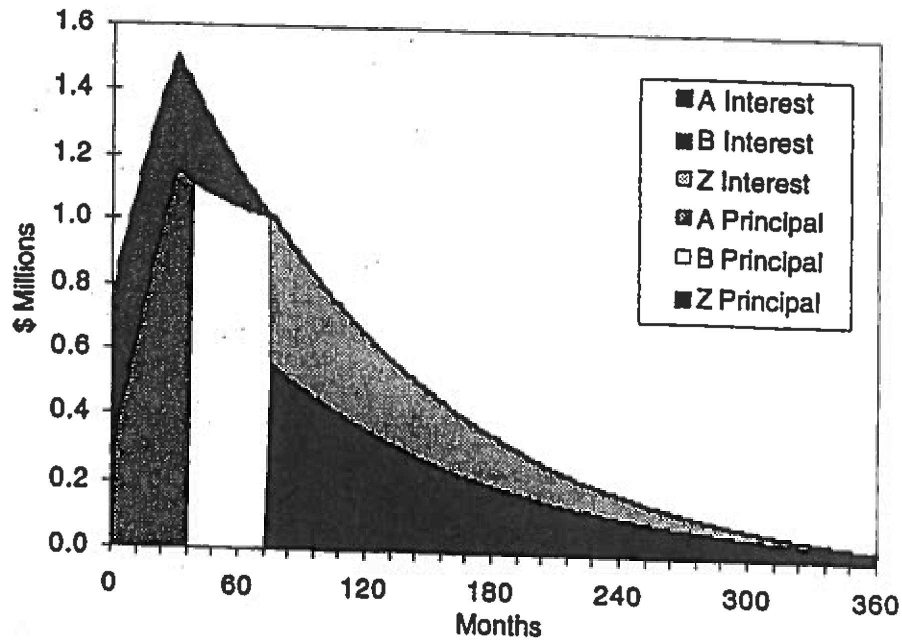
REVIEW QUESTION (Advanced)

Write principal payment and interest payment rules that would produce a bond with the cashflows equal to the difference between the collateral cashflows and the bond cashflows.

Step 3 Create a Sequential CMO with a Z-Bond

A Z-bond or accrual bond is a principal pay rule. The Z bond does not receive any cashflow until the prior bonds are completely paid down. The interest due to the Z-bond, while the prior bonds are outstanding, is added (accrues) to the principal amount due to the Z-bond. That cashflow is then added to the principal available to pay down the prior bonds. In other words, the accrual structure converts the interest accruing to the Z-bond into principal for payment to other bonds. Once the other bonds are retired, the Z-bond pays interest and principal currently. Figure 6-2 shows the cashflows of the sequential CMO with a Z-bond as calculated in Exercise 6-3c.

Figure 6-2
Sequential with a Z-bond at 175% PSA



Exercise 6-3

Replace bond C above with bond Z, an accrual bond.

Exercise 6-3a (Simple CMO Model)

Calculate the cashflows of bonds A, B, and Z assuming 20% APP.

Bond	Balance	Coupon
A	30.0	7
B	40.0	9
Z	30.0	10

Work area

Bond A

Year	Balance	Principal	Interest	Cashflow
1	30.0			
2				
3				
4				
5				
6				

Bond B

Year	Balance	Principal	Interest	Cashflow
1	40.0			
2				
3				
4				
5				
6				

Bond Z

Year	Balance	Principal	Interest	Accrual	Cashflow	Net Principal
1	30.0	0.0	3.0	–3.0	0	–3.0
2						
3						
4						
5						
6						

Exercise 6-3b (Simple CMO Model)

Calculate the average life of bonds A and B assuming 20% APP. Compare these to the average lives calculated in Step 2.

Work area

	Average Life	
Bond	**No Z**	**W/Z**
A		
B		

Exercise 6-3c (Spreadsheet Model)

Calculate and graph the cashflows of bonds A, B, and Z assuming 175% PSA.

Exercise 6-3d (Spreadsheet Model)

Calculate the average life of bonds A and B assuming 175% PSA. Compare these to the average lives calculated in Step 2.

Work area

	Average Life	
Bond	**No Z**	**W/Z**
A		
B		

Exercise 6-3e (Advanced)

Calculate the average life of the Z bond in Exercise 6-3c. Accruals create negative principal cashflows. How does this impact the calculations of average life?

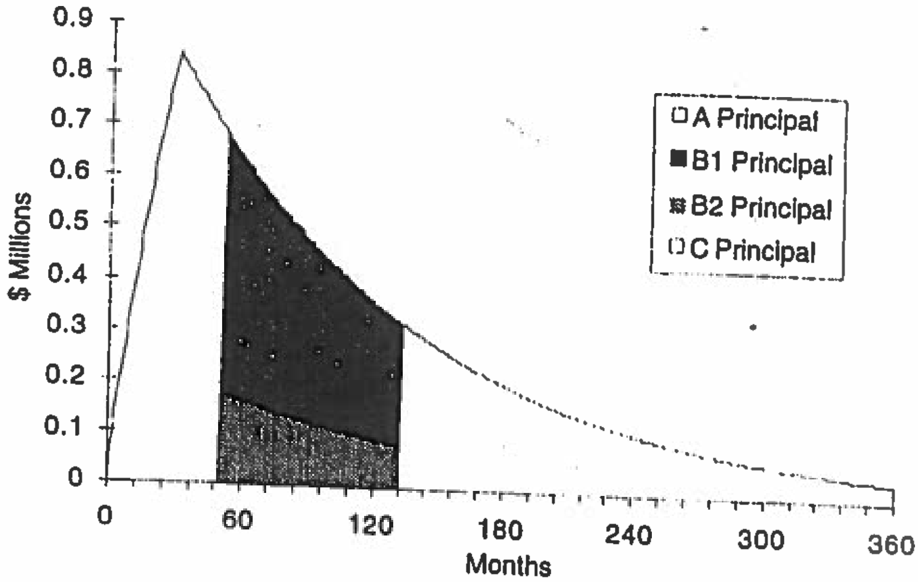
Work area

Bond	Average Life
Z	

Step 4 Create a Pro-Rata CMO

Pro-rata is a principal pay rule. It is primarily used to assign different coupons to bonds with the same principal payment characteristics. Pro-rata bonds receive principal payments in fixed proportion to each other. They can be created by splitting cashflows of a bond. Figure 6-3 shows a pro-rata bond created from Bond B of Figure 6-1 as created in Exercise 6-4c.

Figure 6-3
A Pro-Rata Bond



This is an example of the layering technique. The pro-rata structure is usually applied to a set of cashflows created through one of the other structuring methods.

The total interest payments of the pro-rata bonds must be less than or equal to the interest payments of the underlying bond. It is possible to have a pro-rata class that does not receive any principal or a class that does not receive any interest. (See IO/PO structuring in Step 7.)

Exercise 6-4

Create pro-rata bonds using Class B created in Step 2 (without the Z bond). Assume Class B1 receives 75% of the principal payments of Bond B. Assume Class B2 receives 25% of the principal payments of Bond B.

Exercise 6-4a (Simple CMO Model)

Assume that the coupon on Bond B1 is 8%. What is the maximum coupon on bond B2?

Exercise 6-4b (Simple CMO Model)

Compute and graph the cashflows of bonds B1 and B2 using the maximum coupon for bond B2. Use 20% APP.

Work area

Pro-Rata

Bond	Share	Coupon
B1	75%	8
B2	25%	

Year	Principal B1	Principal B2	Interest B1	Interest B2
1	0.00	0.00	2.40	1.20
2	7.50	2.50	2.40	1.20
3				
4				
5				
6				
Total	30.00	10.00	7.20	3.60

Exercise 6-4c

Compute and graph the cashflows of bonds B1 and B2 assuming 175% PSA.

Step 5 Create Floater/Inverse Floater Coupons

Floaters and inverse floaters are interest pay rules. Floating rate coupons are set at a margin above an index. There is also a cap and a floor on the coupon rate. A variety of indices are possible (subject to REMIC restrictions). LIBOR and Constant Maturity Treasury (CMT) indices are the most common.

Inverse floating coupons are set based on a formula so that the coupon decreases as LIBOR increases. The coupon is expressed as a base rate less the index times a multiple. The coupon has a floor that may be zero or higher.

Exercise 6-5

Assume a pro-rata structure where the floater receives 75% of the principal payments, the underlying coupon is 9%, the floater is indexed to LIBOR with a 1% margin, and the other bond is an inverse floater.

LIB + m

Exercise 6-5a

a − b × LIB

What is the maximum cap on the floater? (Hint: Minimum inverse coupon is 0.)

$\frac{75\%}{25\%} = 3\%$ 9% + 3% = 12%

12%

Exercise 6-5b

What is the maximum coupon on the inverse floater? (Hint: Minimum floater coupon is equal to the margin.)

33%

min LIB-R=0 $\frac{25}{75} = .33$.

Exercise 6-5c

What is the multiplier on the inverse floater? (Hint: If one changes the floater coupon by 1%, what happens to the inverse floater coupon?)

3

Exercise 6-5d

Write down formulas for the floater and inverse floater coupons.

Min (LIBOR + 1%, 12%)

Inv : Max (33% - 3 x LIBOR, 0)

Exercise 6-5e

Fill in the table which shows the coupons on the floater and inverse floater for various levels of LIBOR.

Work area

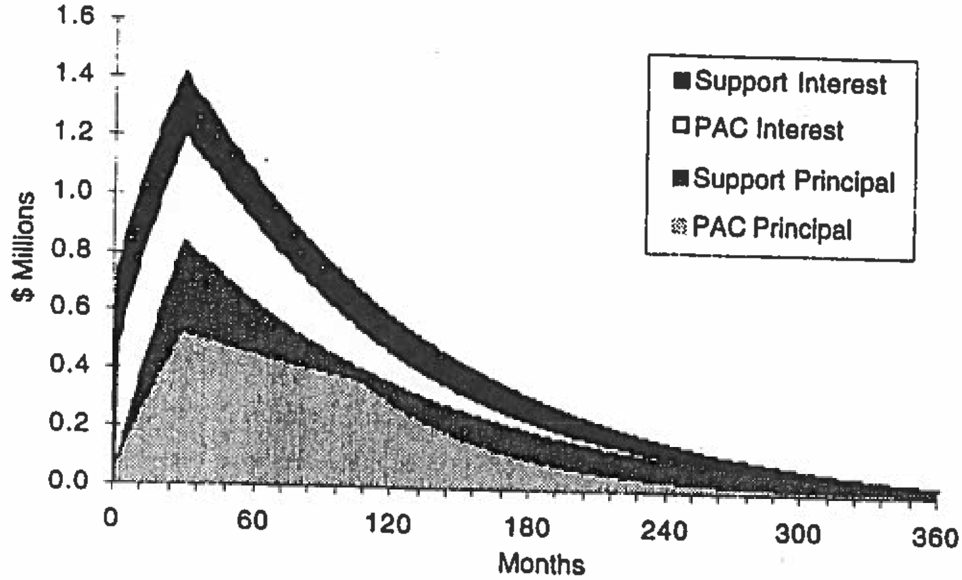
LIBOR	Floater	Inverse
3		
6	7.00	
9		6.00
12		
15		

Step 6 Create a PAC/Support CMO

Planned Amortization Class (PAC) is a principal pay rule. A support bond is always created in conjunction with the PAC class. A PAC is created so that its principal cashflows are fixed for a certain range of prepayment rates, called the PAC band.

The support bond absorbs the principal cashflows that exceed those scheduled to be paid to the PAC. The PAC structure provides for a reallocation of prepayment risk. For this exercise we will return to the unstructured cashflows of Step 1. PAC cashflows are shown in Figure 6-4 for Exercise 6-6f.

Figure 6-4
PAC Cashflows at 175%



Exercise 6-6

Exercise 6-6a (Simple CMO Model)

Using the assumptions of Step 1, calculate the principal cashflows using a 10% APP and a 30% APP. (Note: These results are available in Exercise 6-1a.)

Exercise 6-6b (Simple CMO Model)

Use the smaller of the two principal payments for each period to determine the PAC schedule.

Exercise 6-6c (Simple CMO Model)

Add these principal payments to determine the PAC bond balance. Determine the scheduled balance for each period.

Work area

Year	Principal	Bands 10%	Bands 30%	PAC Schedule	Scheduled Balance
1	20.0	10			40
2	20.0		30		
3	20.0			10	
4	20.0				
5	20.0				
6	0.0				

Year	PAC Balance	PAC Principal	Support Balance	Support Principal
1			60	
2				
3				10
4				
5				
6				

Exercise 6-6d (Simple CMO Model)

Calculate the principal cashflows of the PAC and support bonds assuming 5%, 10%, 30%, and 40% APP. Assume that principal payments will be made first to the PAC to pay down the PAC balance according to the PAC schedule. Excess principal will be used to reduce the principal balance of the support bond until its balance is reduced to zero. In the event of insufficient principal cashflow to meet the PAC schedule, the PAC has priority over the support bond for future payments until it has been paid down to its scheduled balance.

Exercise 6-6e (Simple CMO Model)

Calculate the average life of the bonds for 5%, 10%, 30%, and 40% APP.

Work area

Year	PAC Principal Cashflows APP:	5%	10%	30%	40%
1					
2					
3					
4					
5					
6					
Avg life					

Year	Support Principal Cashflows APP:	5%	10%	30%	40%
1					
2					
3					
4					
5					
6					
Avg life					

Exercise 6-6f (Spreadsheet Model)

Use 100% PSA and 300% PSA for the bands. Evaluate cashflows and average life at 75% PSA, 100% PSA, 175% PSA, 300% PSA, and 400% PSA.

Work area

PSA	Average Life PAC	Support
75%		
100%		
175%		
300%		
400%		

■ REVIEW QUESTION (Advanced)

What happens if prepayments are 100% PSA for the first five years and then 300% PSA for the remaining time? What is the average life of the PAC bond? What if prepayments are 300% PSA for the first five years and then 100% PSA for the remaining time? Effective PAC bands represent the range of speeds for which the CMO will meet its schedule. How do they change over time?

Step 7 Create an IO and a PO

Interest Only (IO) and Principal Only (PO) are interest payment rules. IOs and POs are pro-rata bonds with different coupons. The coupon on the IO is based on a "notional" principal amount. That is, it does not receive any principal payments but the interest payment is calculated by multiplying the coupon times the balance of another class.

The simplest form of an IO and PO is to strip the entire MBS. In addition, any class can be separated into an IO and PO. An IO or PO can also be stripped out of any bond to raise or lower the coupon on the remaining bond.

Exercise 6-7

Take the mortgage of Step 1. Split the cashflows into principal and interest.

Assume that for the following scenarios the mortgages will have the following yields and prepayment rates.

Scenario	Yield	APP%	PSA
–1	7	40	600
base	8	30	400
+1	9	20	250
+2	10	10	125

Exercise 6-7a (Simple CMO Model)

For each yield level calculate the cashflows of the IO and PO using the appropriate prepayment rate. For each yield level calculate the price of the IO and PO. Compare these prices with the unstripped MBS for each scenario.

Exercise 6-7b (Simple CMO Model)

Calculate the effective duration, based on 100 basis point shifts, using the 8% yield scenario as the base case. (Note: Actually, IOs and POs would tend to have different yields.) See Chapter 7 for more information about effective duration.

Work area

Principal Only

Year	Yield APP	7% 40%	8% 30%	9% 20%	10% 10%
1					10
2		40			
3			30		
4				20	
5					
6					
	Price	$88.65			
	Effective Duration				

Interest Only

Year	Yield APP	7% 40%	8% 30%	9% 20%	10% 10%
1		10			
2				8	
3			4		
4					
5					6
6					
	Price				
	Effective Duration				

	Price Comparison 7%	8%	9%	10%
PO				
IO				
MBS				

■ REVIEW QUESTIONS (Advanced)

Compare the price profile of the bond created in Exercise 6-2e with the IO. Create a 2% coupon IO strip from the PAC bond in Step 6. Compare price profiles.

Create a PO from the support bond in Step 6. Compare the price profile to the PO created in Step 7.

(Very Advanced)

Using current market levels, create a sequential structure within a PAC class and create a pro-rata floater, inverse floater within the support class. Determine spread levels at which the CMO would have greater value than the underlying MBS. Strip all bond coupons to produce bond prices below par. How much of the value of the deal is in IO and inverse floater classes?

■ ANALYZING A DEAL STRUCTURE

Most CMOs combine the principal and interest payment rules to create a multitude of different bonds from one pool of underlying cashflows. Table 6-1 shows a typical CMO bond in a typical descriptive format. This particular example was adapted from a screen on the Bloomberg.

Table 6-1
FNMA 1994-22 REMIC Summary

Dates			Underlying	
Original Amount:	540,000MM		100% FNCL	7.5% Net
Priced:	1/12/94		Orig	9/8/95
Dated:	mixed	WAC:	7.9210	7.9171
Settled:	2/28/94	WAM:	29y 3m	27y 4m
First Pay:	3/25/94	CAGE:	0y 8m	2y 3m
9/95 Amount:	479,635MM			

Prepayments			
Orig Speed:	300 PSA		
1 mo. Hist:	156 PSA		
	PAC	SUP/PAC	SUP
Original:	55.0%	24.8%	20.2%
Current:	52.2%	26.1%	21.7%

Class	Description	Cpn	Std Mty	Deal Structure Size (000)	Factor 9/95	Cur FLUX	Orig. WAL	WAL @200	Collar Band	Collar As Of
A	PAC II	5.000	3/22	243,000	0.808	1.1	3.5	2.9	357-519	9/95
B	PAC II	5.000	12/22	21,600	1.000	3.5	7.9	6.6	130-516	9/95
C	PAC II	5.000	12/23	29,700	1.000	2.9	10.4	8.8	120-482	9/95
D	PAC II	5.000	1/24	2,700	1.000	2.1	13.8	12.2	72-479	9/95
FA	FLT +	6.475	1/24	74,250	0.938	2.1	2.0	11.5	No Band	9/95
SA	INV +	3.590	1/24	23,265	0.938	30.3	2.0	11.5	No Band	9/95
SB	INV +	7.500	1/24	13,860	0.938	24.0	2.0	11.5	No Band	9/95
E	PO, SCH (22)	0.000	1/24	22,275	0.938	19.4	2.0	11.5	No Band	9/95
F	FLT, SUP	7.075	1/24	60,750	0.951	7.4	10.5	20.8	No Band	9/95
S	INV, SUP	2.406	1/24	48,600	0.951	37.4	10.5	20.8	No Band	9/95
R	R, NPR	0.000	1/24	0	1.000	—	—	—	—	—

Source: Bloomberg Financial Markets, 1995.

Exercise 6-8

Analyze the deal structure shown in Table 6-1.

Exercise 6-8a

Which bonds are the PAC classes? What relevance is the "as of" date of the collar?

Exercise 6-8b

Identify the sequential structure.

Exercise 6-8c

Identify the support bonds.

Exercise 6-8d

Identify the pro-rata bonds. How are they related? High/low coupon? Inverse/floater?

Exercise 6-8e

Why is the original WAL different from "WAL@200"?

Looking at a descriptive screen such as the one shown in Table 6-1 is not a substitute for review of the prospectus. Below we include an excerpt from the prospectus for the same deal. Note how the cash allocation rules in the prospectus confirm the analysis performed by looking at the deal summary.

Excerpt from FNMA REMIC 1994-22 Prospectus

Principal Distribution Amount

Principal will be distributed monthly on the Certificates in an amount (the "Principal Distribution Amount") equal to the aggregate distributions of the principal concurrently made on the SMBS.

On each Distribution Date, the Principal Distribution Amount will be distributed as a principal of the Classes in the following order of priority:

i. sequentially, to the A, B, C, D Classes, in that order, until the principal balances thereof are reduced to their respective Planned Balances for such Distribution Date;

ii. concurrently, to the FA, SA, SB and E Classes, in proportion to their original principal balances (or 55.5555555556%, 17.4074074074%, 10.3703703703% and 16.6666666667%, respectively), until their respective Scheduled Balances for such Distribution Date;

iii. concurrently, to the F and S Classes, in proportion to their original principal balances (or 55.5555555556% and 44.4444444444%, respectively), until the balances thereof are reduced to zero;

iv. concurrently, to the FA, SA, SB and E Classes, in the proportions set forth in clause (ii) above, without regard to the Scheduled Balances and until the principal balances thereof are reduced to zero; and

v. sequentially, to the A, B, C, and D Classes, in that order, without regard to the Planned Balances. and until the respective principal balances thereof are reduced to zero.

■ REVIEW QUESTIONS

What structuring process segments the cashflows into bonds with different average lives? What is the advantage of creating bonds with different average lives?

What cashflow, if any, is left over in Step 2? What is the advantage of having the bond coupons set below the collateral coupon? What are the disadvantages?

Where does the accrual principal on a Z bond come from?

If both bonds have the same average life, which has a higher price, an accrual bond with a yield of 10% or a zero coupon bond with a yield of 10%? Why?

With a positively sloped yield curve, how does using a Z bond affect the value of the previous classes?

If two bonds have the same average-life profile, what type of bonds are they?

How can you increase the size of the PAC bond? How does this affect the cashflows of the support bond?

What happens to the cap on the floater when the percent of floater increases?

What happens to the leverage of the inverse?

Describe an inverse floater as a bond and short-term funding.

How does increasing the slope of the inverse affect the risk of the bond?

What is the sign on the effective duration of an IO? Why?

Is the effective duration of a PO higher or lower than its cashflow duration?

Why?

■ ANSWERS TO EXERCISES

6-1a & 6-1b

Underlying Security	
Balance	100
Coupon	10
Maturity	5

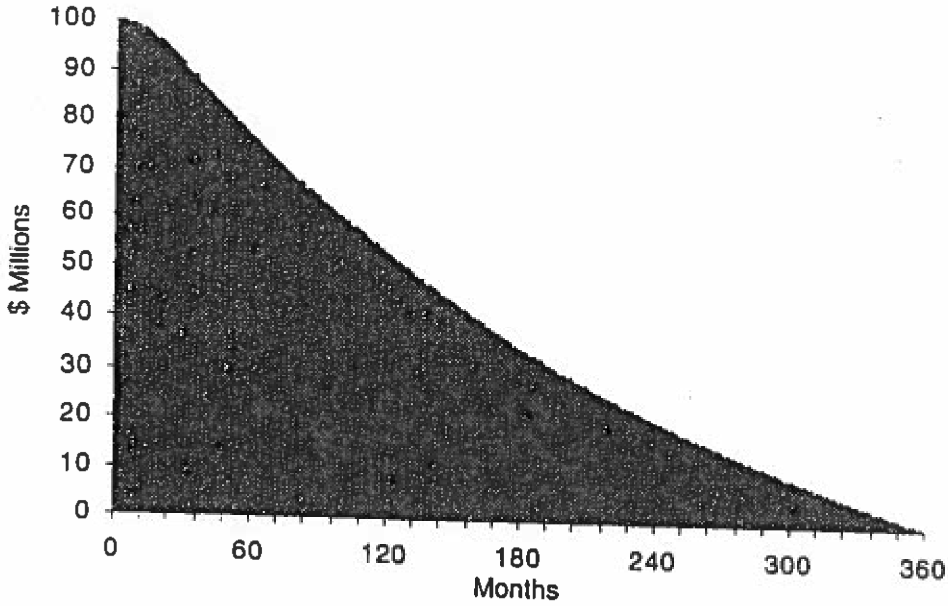
APP	10%			
Year	Balance	Principal	Interest	Cashflow
1	100	10.0	10.0	20.0
2	90	10.0	9.0	19.0
3	80	10.0	8.0	18.0
4	70	10.0	7.0	17.0
5	60	60.0	6.0	66.0
6	0	0.0	0.0	0.0
Avg Life		4.0		

APP	20%			
Year	Balance	Principal	Interest	Cashflow
1	100	20.0	10.0	30.0
2	80	20.0	8.0	28.0
3	60	20.0	6.0	26.0
4	40	20.0	4.0	24.0
5	20	20.0	2.0	22.0
6	0	0.0	0.0	0.0
Avg Life		3.0		

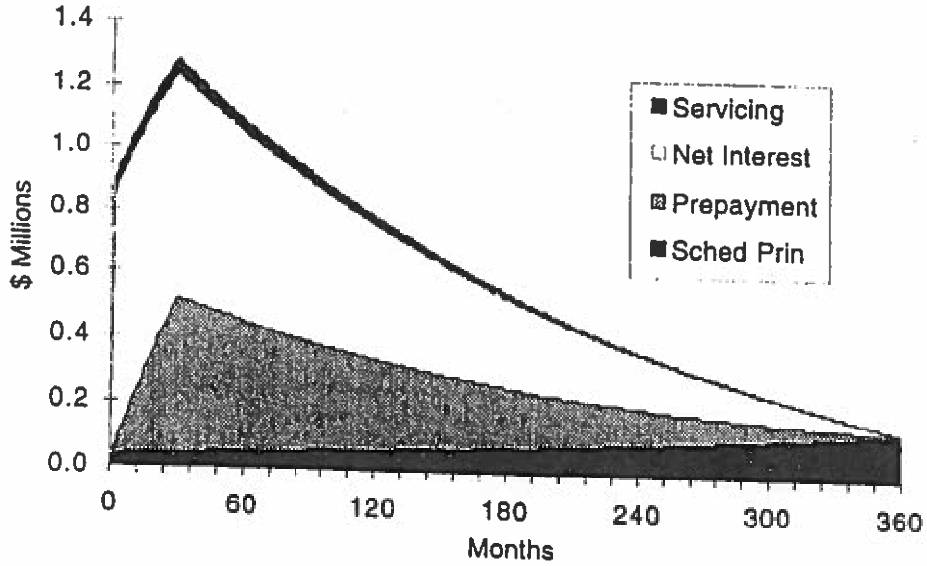
APP	30%			
Year	Balance	Principal	Interest	Cashflow
1	100	30.0	10.0	40.0
2	70	30.0	7.0	37.0
3	40	30.0	4.0	34.0
4	10	10.0	1.0	11.0
5	0	0.0	0.0	0.0
6	0	0.0	0.0	0.0
Avg Life		2.2		

6-1c

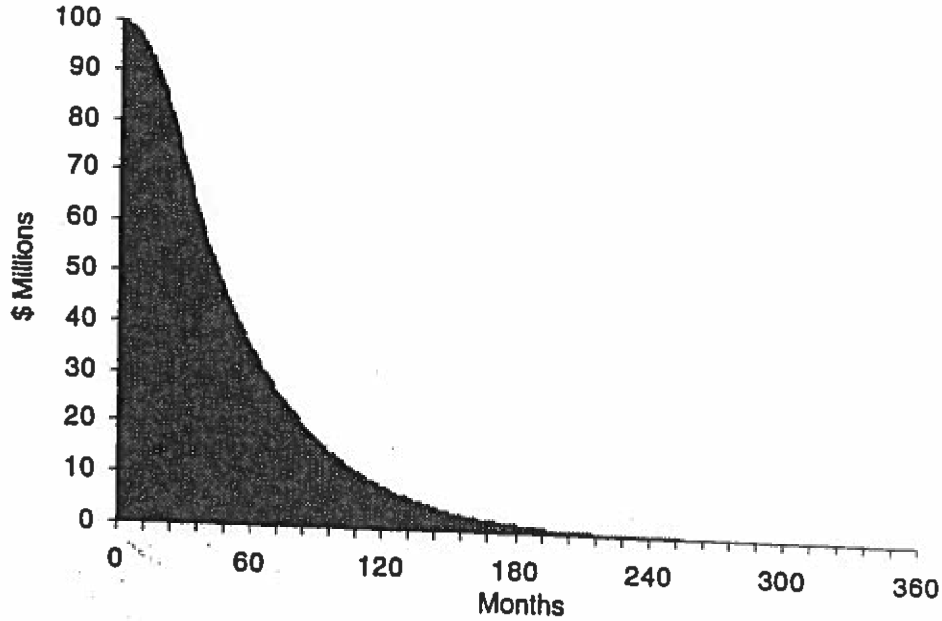
Balance 100% PSA



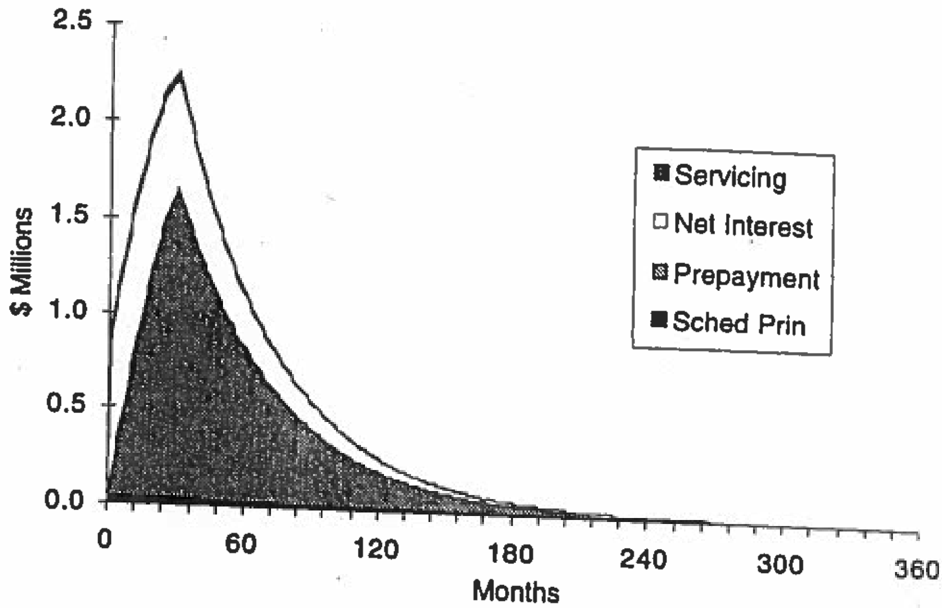
Cashflow 100% PSA



Balance 400% PSA



Cashflow 400% PSA



6-1d

PSA	Average Life
100%	12.40
175%	8.93
400%	4.75

6-2a, b

Sequential

Bond	Balance	Coupon
A	30.0	7
B	40.0	9
C	30.0	10
Total	100.0	

Bond A

Year	Balance	Principal	Interest	Cashflow
1	30.0	20.0	2.1	22.1
2	10.0	10.0	0.7	10.7
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
Avg Life		1.3		

Bond B

Year	Balance	Principal	Interest	Cashflow
1	40.0	0.0	3.6	3.6
2	40.0	10.0	3.6	13.6
3	30.0	20.0	2.7	22.7
4	10.0	10.0	0.9	10.9
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
Avg Life		3.0		

Bond C

Year	Balance	Principal	Interest	Cashflow
1	30.0	0.0	3.0	3.0
2	30.0	0.0	3.0	3.0
3	30.0	0.0	3.0	3.0
4	30.0	10.0	3.0	13.0
5	20.0	20.0	2.0	22.0
6	0.0	0.0	0.0	0.0
Avg Life		4.7		

6-2c

See Figure 6-1.

6-2d

Bond	Average Life
A	2.55
B	7.29
C	17.51

6-3a

Accrual

Bond	Balance	Coupon
A	30.0	7
B	40.0	9
Z	30.0	10

Bond A

	Balance	Principal	Interest	Cashflow
1	30.0	23.0	2.1	25.1
2	7.0	7.0	0.5	7.5
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
Avg Life		1.2		

Bond B

Balance	Principal	Interest	Cashflow
40.00	0.00	3.60	3.60
40.00	16.30	3.60	19.90
23.70	23.63	2.13	25.76
0.07	0.07	0.01	0.08
0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00
Avg Life	2.6		

Bond Z

Balance	Principal	Interest	Accrual	Cashflow	Net Principal
30.0	0.0	3.0	–3.0	0.0	–3.0
33.0	0.0	3.3	–3.3	0.0	–3.3
36.3	0.0	3.6	–3.6	0.0	–3.6
39.9	19.9	4.0	0.0	23.9	19.9
20.0	20.0	2.0	0.0	22.0	20.0
0.0	0.0	0.0	0.0	0.0	0.0
Avg Life	4.5				

6-3b

Bond	Average Life No Z	W/Z
A	1.3	1.2
B	3.0	2.6

6-3c

See Figure 6-2.

6-3d

Bond	Average Life No Z	W/Z
A	2.55	1.83
B	7.29	4.60

6-3e

Bond	Average Life
Z	21.79

The negative principal cashflows increase average life.

6-4a

The maximum coupon on Bond B2 is 12%.

6-4b

Pro-Rata

Bond	Share	Coupon
B1	75%	8%
B2	25%	12%

	Principal		Interest	
Year	B1	B2	B1	B2
1	0.0	0.0	2.40	1.20
2	7.5	2.5	2.40	1.20
3	15.0	5.0	1.80	0.90
4	7.5	2.5	0.60	0.30
5	0.0	0.0		
6	0.0	0.0		
Total	30.0	10.0	7.20	3.60

6-4c

See Figure 6-3.

6-5a

The maximum cap on the floater is 12%.

6-5b

The maximum coupon on the inverse floater is 33%.

6-5c

The multiplier on the inverse floater is 3.

6-5d

Floater Formulas:

Floater = Min(LIBOR +1%, 12%)

Inverse Floater = Max(33% − 3 × Libor, 0)

6-5e

LIBOR	Floater	Inverse
3	4	24
6	7	15
9	10	6
12	12	0
15	12	0

6-6a, b, c

PAC and Support

Year	Principal	Bands 10%	Bands 30%	PAC Schedule	Scheduled Balance
1	20	10	30	10	40
2	20	10	30	10	30
3	20	10	30	10	20
4	20	10	10	10	10
5	20	60	0	0	0
6	0	0	0	0	0

Year	PAC Balance	PAC Principal	Support Balance	Support Principal
1	40	10	60	10
2	30	10	50	10
3	20	10	40	10
4	10	10	30	10
5	0	0	20	20
6	0	0	0	0

6-6d, e

		PAC Principal Cashflows			
Year	APP:	5%	10%	30%	40%
1		5	10	10	10
2		5	10	10	10
3		5	10	10	20
4		5	10	10	0
5		20	0	0	0
6		0	0	0	0
Avg life		3.75	2.50	2.50	2.25

		Support Principal Cashflows			
Year	APP:	5%	10%	30%	40%
1		0	0	20	30
2		0	0	20	30
3		0	0	20	0
4		0	0	0	0
5		60	60	0	0
6		0	0	0	0
Avg life		5.00	5.00	2.00	1.50

6-6f

	Average Life	
PSA	PAC	Support
75%	8.65	23.12
100%	7.56	20.46
175%	7.56	11.23
300%	7.56	3.35
400%	6.17	2.38

6-7a, b

Principal Only

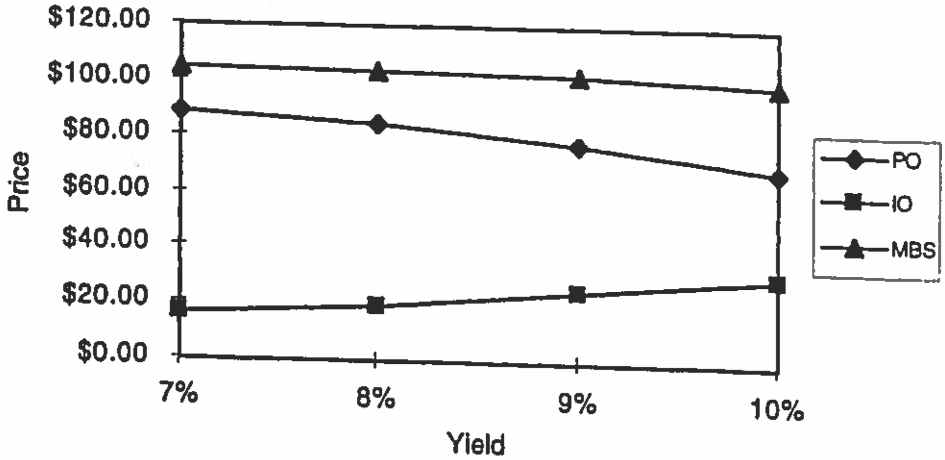
Year	Yield: APP:	7% 40%	8% 30%	9% 20%	10% 10%
1		40	30	20	10
2		40	30	20	10
3		20	30	20	10
4		0	10	20	10
5		0	0	20	60
6		0	0	0	0
	Price	$88.65	$84.66	$77.79	$68.95
	Effective Duration		6.4		

Interest Only

Year	Yield APP	7% 40%	8% 30%	9% 20%	10% 10%
1		10	10	10	10
2		6	7	8	9
3		2	4	6	8
4		0	1	4	7
5		0	0	2	6
6		0	0	0	0
	Price	$16.22	$19.17	$24.67	$31.05
	Effective Duration		-22.05		

	Price Comparison			
	7%	8%	9%	10%
PO	$88.65	$84.66	$77.79	$68.95
IO	$16.22	$19.17	$24.67	$31.05
MBS	$104.87	$103.83	$102.47	$100.00

Price Profile



6-8a

Bonds A, B, C, and D are PAC classes. PAC collars shift as prepayment speeds change. The "band" is the effective collar for a particular date, calculated given the actual historical prepayments. A PAC collar can be "broken" if prepayment speeds go above or below the guidelines specified for the PAC. In this situation, investors will have reduced protection against cashflow variability due to prepayments.

6-8b

Bonds A, B, C, and D also represent a sequential structure within the PAC grouping. Note how the average life increases from Bond A to Bond D.

6-8c

All of the other bonds represent the support bonds for the PACs. They will be subject to more average life variability as prepayment rates change than the PAC bonds.

6-8d

The easiest way to spot pro-rata bonds is to find bonds with the same average life. If their average lives are the same under a variety of prepayment speeds they are very likely pro-rata bonds. Thus F and S are pro-rata bonds. They represent a floater/inverse floater combination. Also SA, SB, E and FA are a pro-rata combination with a more complex coupon interaction.

6-8e

The original WAL was calculated at 300% PSA. "WAL@200" indicates the current WAL calculated using the Andrew Davidson & Co., Inc. Prepayment Model for the scenario where rates rise by 200 basis points. At higher rates, prepayments slow. WAL increases as prepayments decrease.

CHAPTER SEVEN

IO

IO

IO

Scenario Analysis

Susan continued to look over the investment proposal sent over from Bulls & Bears. The trade looked interesting, a recombination of an IO and PO off of FNMA 8s that looked 1/4 point cheaper than the collateral. "It's true," she said to one of her colleagues, "that the two securities aren't off the same trusts, but it looks like a good bet."

Susan spent some time trying to figure out what could go awry with the trade. Then she noticed that the WAC on the IO was .25% above the PO and that the same prepayment assumptions were being made in all the yield scenarios she was looking at. "What happens if the IO prepays differently than the PO in the yield scenarios?" she wondered. Using scenario analysis, she discovered that the combination underperformed collateral in a down 50 basis point scenario but only slightly outperformed when rates were rising 50 basis points. On balance, it looked like Bob from Bulls & Bears was up to his old tricks again.

■ INTRODUCTION

Previous sections have dealt with elements of static analysis and basics of MBS cashflows. We have been introduced to prepayments and factors affecting cashflows, and will now tie them together with an analytical tool called scenario analysis.

Table 7-1
Four-Step Process

Environment	Prepayments	Cashflows	Analysis
Holding Period	Static	Interim	Value
Interest Rates	Vector	Terminal	Risk
Re-investment	Model	Income	
Scenario Creation			

This chapter deals with fundamentals of:

- Total return calculations.
- Varying cashflows by scenarios.
- Relationship between scenarios and MBS analysis.

The factors under consideration follow the general framework established in the book, as shown in Table 7-1.

We will proceed through the various topics in this chapter, leading the reader through the development of the various techniques employed with scenario analysis. By the end of the chapter we hope to have developed an understanding of how to create a simple scenario analysis and how critical assumptions affect the analysis.

Environment

In order to understand how the method of scenario analysis can be applied to MBS, we will build off the basic cashflow examples developed earlier. Scenario analysis differs from yield analysis because we must make some explicit assumption regarding a holding period. While yield examines the value from holding a security until the last cashflow, scenario analysis examines performance over a specific time period. The length of the period will influence the weight of various assumptions and the interpretation of the results. In addition, there are a variety of ways to form a scenario resulting from the manner in which rates are shifted and how interim cashflows are re-invested.

Prepayments

The prepayment assumption must mirror the choice of interest rate scenario. We could either rely upon a prepayment model or apply static prepayment forecasts based on a specific scenario. By a static forecast we could assume, for example, that if interest rates remained constant, an MBS would prepay at 150% PSA and if rates were to drop, the MBS would prepay at 250% PSA. This static forecast would then be used for the entire life of the security.

Applying a prepayment model in the falling rate scenario would allow the prepayments to rise, but there would be additional fluctuations in prepayment rates due to seasonality, aging, and burnout. Prepayment models are considerably more useful when analyzing nonparallel yield curve shifts and when examining paths of interest rates.

Cashflows

Combining the change in interest rates with prepayment rates provides the information needed to project cashflows for the MBS. There are three general types of cashflows that will affect the total returns for securities: interim principal and interest, value of the remaining principal at the horizon, and income on the interim cashflows.

Analysis

Through examining the environment, prepayments, and cashflows, we build the various cashflow components of the scenario analysis. With the analysis methods we can put the pieces together and create some summary measures of performance and risk. These summary measures give an indication of both the absolute behavior of an MBS and provide a means to compare the relative performance of different securities.

In order to make matters somewhat more coherent, we introduce the concept of total return first. Having an understanding of the basic calculation will be important when examining the various factors that affect the measure.

■ COMPONENTS OF ENVIRONMENT

Total Return

Total rate of return (TRR) represents the basic measurement of value for scenario analysis. The TRR calculation compares the investor's final balance of cash against his starting balance. A growth rate is calculated to infer how the beginning balance could result in the ending balance.

This growth rate represents the overall cumulative gain and ignores the time of the holding period. The growth rate will generally be calculated as an annualized equivalent rate. Holding periods that are different than 12 months will require some normalization of the rate to place it on an annual basis.

Computation of Total Return

Assume we purchase a semi-annual coupon-bearing bond with a 5.9126% coupon at a price of par. At the end of six months, we receive a semi-annual coupon and we sell the bond at par. The total return computation is shown in Equation 7-1.

$$
\begin{aligned}
\text{6M TRR} &= \frac{\text{Coupon} + \text{Terminal Value}}{\text{Starting Value}} - 1 \\
&= \frac{\$2.9563 + \$100}{\$100} - 1 \\
&= 2.9563\%
\end{aligned}
$$

Equation 7-1
Total Return for a Six-Month Holding Period

Now, this is the total return assuming a six-month holding period. Normally we should put this into an annual equivalent. In our case, we would assume that the investor is able to put the money to work at

$\frac{A}{100} = \left(1 + \frac{S}{100}\right)^2 - 1$

the same rate for another six months. The annualized return is shown in Equation 7-2.

$\sqrt{(A+1)} * 100 = 1 + S$

Equation 7-2
Annualized Total Return

$$\text{Annual TRR} = \left(1 + \frac{\text{Six-month return}}{100}\right)^2 - 1$$

$$= \left(1 + \frac{2.9563}{100}\right)^2 - 1$$

$$= (1.0296)^2 - 1$$

$$= 0.06 \text{ or } 6\%$$

$S = \sqrt{(A+1)} * 100 - 1$

$\left(\sqrt{\frac{A}{100} + 1} - 1\right) * 100$

Exercise 7-1

Using the same security as above, assume that the security is sold at the end of six months for a price of $101. Compute total return over the six-month period, then convert the six-month holding period return to an annual equivalent.

$\frac{2.9563 + 101}{100} - 1 = .039563 = 3.96\%$

Annual $= (1.029663)^2 - 1$

$= 8.09\%$

Holding Period

Choice of holding period length will play a role in scenario analysis because it can skew the manner in which results are interpreted. The shorter the holding period, the greater the effect of final price on the total rate of return. As the holding period lengthens, more weight gets placed on the assumed rate of re-investment.

In order to keep total rates of return on a comparable basis, we normalize returns to be quoted on an annual basis. Recall from Equation 7-2 that for a six-month holding period we compounded the results to get the annual equivalent. This compounding effect may lead to distorted results when the holding period is very short. For example, a $1 price increase that occurred over one day would lead to an extremely large annual total return.

Usually, scenario analysis considers a 12-month holding period to minimize any distortions. In the interest of brevity, however, most of the examples and exercises in this chapter use a six-month holding period.

Holding Period Length

Using the same 5.9126% coupon bond as in Exercise 7-1, let's assume that the security is held one year and then sold for a price of $101. In this case, the total return would be as follows:

Total Return

$$= \frac{\text{Ending Cash}}{\text{Starting Cash}} - 1$$

$$= \frac{\text{Coupon Income}_{\text{Month 6}} + \text{Coupon Income}_{\text{Month 12}} + \text{Terminal Price}}{\text{Starting Price}} - 1$$

$$= \frac{2.9563 + 2.9563 + 101}{100} - 1$$

$$= \frac{106.913}{100} - 1$$

$$= 6.913\%$$

By moving the $1 gain in terminal value from month 6 to month 12, we lose the effect of compounding the price increase. As a result, the total return on a 6-month basis exceeds the 12-month return (from Exercise 7-1) by more than 1%. Note that in the example above, we have ignored the effect of re-investing the coupon income received in month 6. The re-investment effect will be considered next.

Reinvestment Rate

The main computation in the total return metric is the comparison of the final cash balance to the starting balance. When considering final cash balance, we must make some assumption regarding the re-investment of cash received during the holding period. Depending upon the length of the holding period and the level of prepayments, this re-investment rate can play a material role in the total return computation.

Choosing a conservative re-investment rate is a good policy when evaluating the range of potential investments. This approach will tend to keep an investor from relying on unrealistic assumptions when projecting returns. Normally we make an assumption that cash received from a security will be re-invested at short-term money market rates. These rates should vary along with the scenario. For example, if we were looking at a scenario in which rates fell 100 basis points, the expected re-investment rate would also decline by 100 basis points in the scenario.

The Effect of Re-investment Rates

To illustrate the effect of re-investment rates, let's examine an MBS over a six-month holding period. The security has a gross WAC of 9%, net coupon of 8.5%, and WAM of 355. We will assume a 15% CPR and that cash will be invested in a risk free asset earning 6%.

Cash received in the first month will be re-invested for five months. Consequently, to determine the value at the end of six months of $1 received in month 1 we would use the formula shown in Equation 7-3.

Equation 7-3
Re-investment Rate Calculation

$$\left(1+\frac{6\%}{12}\right)^5 = 1.02525$$

To show the overall effect of re-investment, examine Table 7-2:

Table 7-2
Effect of Re-investment

Month	Balance	Interest	Principal	Re-investment	Interest w/Re-investment	Principal w/Re-investment
0	100.00					
1	98.60	0.71	1.40	1.02525	0.73	1.44
2	97.22	0.70	1.38	1.02015	0.71	1.41
3	95.85	0.69	1.36	1.01508	0.70	1.38
4	94.51	0.68	1.34	1.01003	0.69	1.36
5	93.18	0.67	1.33	1.00500	0.67	1.33
6	91.87	0.66	1.31	1.00000	0.66	1.31
Total		4.11	8.12		4.16	8.23

The columns show the basic interest and principal. In the fifth column we include the horizon value of each $1 of interest and principal. We are implicitly assuming that cashflows occur at the end of the period. In the last two columns the interest and principal amounts received are multiplied by the horizon values to get the horizon amounts. In Table 7-3, we compute the total returns with and without re-investment.

Table 7-3

Total Returns with and without Re-investment

	No Re-investment	With Re-investment
Purchase Price	100.00	100.00
Horizon Price	102.00	102.00
Horizon Balance	91.87	91.87
Principal Terminal Value	93.71	93.71
Coupon Income	4.10	4.16
Principal Returned	8.13	8.23
Ending Cash	105.94	106.10
Total Return (annualized)	12.24%	12.57%

Reinvestment only adds $0.16 in additional income but this translates into 0.33% in total return.

Exercise 7-2

Using the data in Table 7-2 and 7-3, compute the total return assuming that we earn 8% from re-invested principal and interest.

Work area

Terminal	
Coupon	
Principal Returned	
Ending Cash	
Total Return (annualized)	

Exercise 7-3

Using the same security as Exercise 7-1, assume that the coupon payment received in month 6 is re-invested at an annual rate of 6% for six months. Continue to assume a horizon price of $101. What is the resulting total rate of return?

Scenarios

The scenario itself represents the essence of the scenario analysis method. Someone employing the method can adopt several approaches to examine the sensitivity of MBS to a variety of potential scenarios. Common methods to create scenarios include:

Parallel Shifts Changes in the term structure translate into parallel movements in yields. That is, rates shift by an equal number of basis points across the curve. These parallel movements can be associated with probabilities in order to weight the scenarios and calculate an expected return.

Nonparallel Shifts Allow for shifts in the shape of the yield curve. These nonparallel shifts can be used to judge effects such as flattenings, inversions, and steepenings of the yield curve.

Paths/Whipsaw Shifts Instead of trying to average returns over a variety of scenarios it may be more useful to examine the sensitivity to particular paths of rates. In a whipsaw scenario one could specify a future scenario where rates oscillated and then flattened out at a long-term level.

■ COMPONENTS OF CASHFLOWS

Interim Cashflows

Interim cashflows, represented by the principal and interest, will be governed by the characteristics of the security, such as the coupon and WAM, as well as the prepayments estimated for a particular scenario. We can derive interim cashflows for a scenario using the methods developed in previous chapters.

Assume an MBS with a gross coupon of 9%, net coupon of 8.5%, and a WAM of 355. Let's consider a scenario in which the prepayment rate remains constant at 15% CPR. Also, assume that the starting balance of the MBS is $1,000. The cashflows are shown in Table 7-4.

Table 7-4
Cashflows for 8.5% MBS at 15% CPR

Month	Balance	Amortized Principal	Prepaid Principal	Interest	Servicing	Investor Cashflow	CPR
0	1,000.00						
1	985.99	0.57	13.44	7.08	0.42	21.10	15%
2	972.17	0.57	13.26	6.98	0.41	20.81	15%
3	958.53	0.56	13.07	6.89	0.41	20.52	15%
4	945.09	0.56	12.89	6.79	0.40	20.23	15%
5	931.83	0.55	12.71	6.69	0.39	19.96	15%
6	918.75	0.55	12.53	6.60	0.39	19.68	15%

Exercise 7-4

Rising Prepayment Rates

Instead of having prepayment rates flat at 15% CPR, now assume prepayment rates rise 5% CPR per month starting in month 2 and continuing until month 6. Recreate the cashflows from Table 7-4.

Work area

Month	Balance	Amortized Principal	Prepaid Principal	Interest	Servicing	Investor Cashflow	CPR
0	1,000.00						
1							15%
2							20%
3							25%
4							30%
5							35%
6							40%

Terminal Value

At the end of a specific holding period, the investor has three basic types of funds: cash received from interim principal and interest, reinvestment income on the principal and interest, and the remaining value of the principal. This remaining value depends upon both the price and the remaining balance. Thus, an investor must be cognizant of the relationship between prepayment rates and terminal value.

Consider scenarios of falling rates. Higher prepayment rates will occur along with rising prices. However, the high prepayment rates experienced during the scenario lead to less remaining principal. As a result, the MBS investor does not benefit as fully from price appreciation due to falling interest rates as an investor in a nonamortizing security.

Another consideration is the method used to determine the price of a security at the horizon. Generally, the choice will be between a static method or an OAS-based method. In a static method some assumption must be made regarding the yield spread of an MBS relative to some benchmark. For example, pricing the MBS at some fixed number of basis points over an equivalent average life Treasury.

When using an OAS-based method the following procedure is usually followed:

1. Compute the OAS based on current market levels.
2. Project cashflows for the scenario.

3. Use the horizon yield curve to compute the price based upon the starting OAS.

OAS will be discussed in more detail in Chapter 9.

Exercise 7-5

Using the cashflows from Table 7-4 and assuming a starting balance of $1,000 compute the missing numbers in the following table. Use the first column, which assumes a terminal price of $95 as an example.

Work area

	Base Case	Case 1	Case 2
Starting Price	100.00	100.00	100.00
Terminal Price	95.00	100.00	110.00
Terminal Balance	918.75	918.75	918.75
Terminal Value	872.81		
Coupon Income	41.04	41.04	41.04
Principal Returned	81.25	81.25	81.25
Ending Cash	995.10		
Total Return (semi-annual)	–0.49%		
Total Return (annualized)	–0.98%		

Income

During the holding period, the investor benefits from the coupon income received. Coupon flows will be classified as the income component of a total rate of return. The income component will also be affected by prepayments—the greater the prepayment rate the less underlying collateral to provide coupon income. In addition, for floating rate MBS the income will be dependent on the reset levels of the coupon.

Coupon Income

Let's return to our basic cashflow example. Consider an 8.5% MBS with a gross coupon of 9% and a WAM of 355. Assuming a 15% CPR over the period, the coupon interest and servicing for the first six months have been calculated and displayed in Table 7-5.

The interest for the first period equals (8.5/1200) × 1,000 or 7.08. In each succeeding period we compute the interest on the outstanding balance for the beginning of the period. The servicing cashflow is calculated from the 50 basis point difference between the gross and net coupons.

Table 7-5
Cashflows for an 8.5% MBS at 15% CPR

Month	Balance	Interest	Servicing
0	1,000		
1	985.99	7.08	0.42
2	972.17	6.98	0.41
3	958.53	6.89	0.41
4	945.09	6.79	0.40
5	931.83	6.69	0.39
6	918.75	6.60	0.39

Exercise 7-6

Using the indicatives in the example above, compute the total interest received by the investor under cases of 5% and 20% CPR. You will want to re-use the cashflow calculator built for Exercise 7-4.

Work area

Month	15% CPR Interest	5% CPR Interest	20% CPR Interest
1	7.08		
2	6.98		
3	6.89		
4	6.79		
5	6.69		
6	6.60		
Total	41.04		

■ EFFECT OF CHANGING PREPAYMENT RATES ON TOTAL RETURN

In addition to the normal complexities of computing the total return, we must grapple with the effects of prepayment rates on the MBS. To illustrate the effects of prepayments and the relation to total return, let's consider our 8.5% MBS example. In Tables 7-6 and 7-7, we'll show the cashflows under two scenarios, 5% CPR and 15% CPR, and examine the influence on total return in Table 7-8.

Table 7-6
Cashflows with 5% CPR

Month	Balance	Interest	Principal
0	1,000.00		
1	995.17	7.08	4.83
2	990.36	7.05	4.81
3	985.56	7.02	4.79
4	980.79	6.98	4.78
5	976.03	6.95	4.76
6	971.29	6.91	4.74
		41.99	28.71

Table 7-7
Cashflows at 15% CPR

Month	Balance	Interest	Principal
0	1,000.00		
1	985.99	7.08	14.01
2	972.17	6.98	13.82
3	958.53	6.89	13.63
4	945.09	6.79	13.44
5	931.83	6.69	13.26
6	918.75	6.60	13.08
		41.04	81.25

Table 7-8
Total Return

	5% CPR	15% CPR
Terminal Balance	971.29	918.75
Terminal Price	100.00	100.00
Terminal Value	971.29	918.75
Coupon	41.99	41.04
Principal Returned	28.71	81.25
Ending Cash	1,041.99	1,041.04
Total Return (semi-annual)	4.20%	4.10%
Total Return (annualized)	8.57%	8.38%

When prepayment rates increase we receive less coupon income, usually leading to a lower total return. However, when we have situations where the coupon is lower than the reinvestment rate, the previous generalization may not hold true. In addition, the example makes a simplifying assumption that holds the horizon price constant in both scenarios. We would normally expect the price in the 15% CPR scenario to be higher, causing the total return to increase.

Exercise 7-7

Using the data from Table 7-2 change the prepayment rate to 40% CPR. Using the cashflow table below, fill in the various components of total return.

Month	Balance	Interest	Principal
0	100.00		
1	95.78	0.71	4.22
2	91.73	0.68	4.04
3	87.86	0.65	3.87
4	84.15	0.62	3.71
5	80.59	0.60	3.55
6	77.19	0.57	3.40

Work area

Horizon Price	105.00	106.00
Terminal Value		
Coupon		
Principal Returned		
Ending Cash		
Total Return (annualized)		

■ COMPARING MBS AND TREASURY TRR

We can now tie some concepts together through a comparison of MBS and Treasury total rates of return. This will help to show how the interaction of prepayment rates and terminal pricing assumptions plays a large role in affecting the relative returns between securities.

For the comparison, six-month total returns were calculated for a FNMA 8% 30-year MBS and the five-year on-the-run Treasury. Parallel shifts of the yield curve from +300 to –300 basis points were considered. The FNMA 8% had an initial price of 102-12. In addition to the total returns, the terminal prices of the MBS and its average life have been calculated. Total returns were calculated using the TRA function on the Bloomberg.

Table 7-9
Total Return for Different Scenarios

Scenario	TRR MBS	TRR 5-Year Treasury	MBS WAL	MBS Terminal Price
-300	12.57	29.97	1.64	105.16
-200	9.70	21.58	1.64	103.59
-100	8.21	13.57	2.16	102.73
0	7.47	5.92	6.51	102.24
100	-2.32	-1.39	9.83	97.10
200	-12.83	-8.38	10.46	91.60
300	-22.68	-15.06	10.87	86.44

In the unchanged yield curve case, the MBS has a significant total-return advantage over the Treasury. However, as the yield curve shifts we see significant divergence between the MBS and the Treasury. When rates fall, the MBS has a lower total return and when rates rise the performance of the MBS is also worse than the Treasury. This result is related to the negative convexity of the MBS, which will be explored in greater detail in the following section.

To understand the behavior of the MBS we must consider the role of prepayments and total return. As interest rates drop, accelerating prepayment rates keep the price of the MBS from rising very much. On the other side, as rates rise the extension of the average life causes the price to decline more than expected.

The underperformance of the MBS is also related to the manner in which terminal prices are determined. The total return example prices the MBS relative to the yield of the interpolated average-life Treasury. As we move out on the yield curve the benchmark yield rises. As interest rates fall investors are likely to widen the expected yield spread between the MBS and the Treasury benchmark.

■ RISK MEASURES

While an investment will normally be evaluated on a total return basis, some consideration must be made regarding the interim price risk of a security. The road is littered with MBS investment crackups in which the investment manager made an investment in a high yielding security and was subsequently wiped out before realizing the expected yields. These situations occur because of extreme price sensitivity to changes in interest rates and other environmental factors.

The common risk measures used to examine fixed-income securities are duration and convexity as briefly described in Chapters 3 and 4. These measures are important because they can be used to determine the approximate price change of a security for a given change in yield (which we term Δyield). This equation, called the Taylor series expansion, is as follows:

$$\text{Percentage Price Change} = \text{Duration} \times \Delta\text{Yield} + 0.5 \times \text{Convexity} \times \Delta\text{Yield}^2 + \text{Higher Order Terms}$$

Equation 7-4
The Taylor Series Expansion for a Percentage Price Change

The equation above assumes that we are using the option-adjusted effective modified duration and effective convexity. For most applications of the equation, we ignore the higher order terms beyond convexity.

Duration

We define duration as the percentage change in price for a basis point change in yield. Duration comes in many flavors—the one commonly used for MBS is the option-adjusted effective modified duration. The option-adjusted component implies that we consider the effects of changing prepayment rates and pricing assumptions as interest rates move. To compute the effective duration, the formula previously shown in Chapter 3, Equation 3-16, is commonly employed.

$$\text{Effective Modified Duration} = \frac{-100}{\text{Price}_{\text{Base}}} \times \frac{\text{Price}_{+\Delta\text{Yield Scenario}} - \text{Price}_{-\Delta\text{Yield Scenario}}}{2 \times \Delta\text{Yield}}$$

Equation 7-5
Effective Modified Duration

To employ the formula, we need to know the current price for a security, denoted by $\text{Price}_{\text{Base}}$. Then we consider some change in interest rates, ΔYield. This change in yield needs to be large enough to induce some change in the expected prepayment rates. Usually shifts of 25 or 50 basis points are used.

The numerator in the equation contains the prices in the up and down shift scenarios. In many cases an OAS model will be used to project the prices. Using the base case price, an OAS will be calculated. The yield curve will then be shifted up and down, and the new value will be calculated using the OAS from the base case.

In the absence of an OAS model it would be acceptable to change the prepayment model and to modify the yield spread. Some investors use a simpler, yet effective method called the empirical duration. They derive the shifted prices by examining current market prices. For example, an 8% MBS would trade with a similar price as an 8.5% MBS in a down 50 basis point shift and like a 7.5% in an up 50 basis point shift.

Computing the Effective Duration

An example of the effective duration calculation is as follows:

$$\text{Price}_{\text{Base}} = 99.906$$

$$\Delta\text{Yield} = 50 \text{ basis points}$$

$$\text{Price}_{-\Delta\text{Yield}} = 102.127$$

$$\text{Price}_{+\Delta\text{Yield}} = 97.393$$

$$\text{Effective Duration} = \frac{-100}{99.906} \times \frac{97.393 - 102.127}{2 \times .50}$$

$$= 4.738\%$$

To interpret the effective duration we would say that, assuming convexity to be 0, if interest rates declined 100 basis points we would expect the price of the security to rise 4.738%. Strictly speaking, duration should be quoted as a negative number that reflects the inverse relationship between price and yield (IO securities excluded). The convention is to quote duration as a positive number and the terms in the duration equation have been arranged to give a positive result.

Exercise 7-8

Compute the effective duration for the securities in the following table.

Work area

	Bond 1	Bond 2	Bond 3
–50bp	55.83	105.19	23.03
Base	45.00	104.22	26.75
+50bp	36.38	103.03	29.92
Effective Duration			

Convexity

For most fixed-income securities the relationship between price and yield is not exactly linear, as shown in Chapter 4, Figure 4-8. The nonlinear aspect is termed convexity and most noncallable bonds have "positive" convexity. A security with "positive" convexity will experience prices rising at a greater rate than they fall for a corresponding change in interest rates. However, callable securities, such as MBS, have "negative" convexity. With a "negatively" convex security, the prices will rise less than they fall for a corresponding change in interest rates.

If we return to Equation 7-4, which approximates the price change for a change in yield, we see that the convexity is an additive term. When convexity is positive, the change in yield will result in higher prices when yields fall. In the case of a rising yield scenario, the effect of positive convexity will be to keep prices from falling further than would have been expected from the projected effect of duration. That is, positive convexity adds some buffer from a rising yield scenario (remember that we square the change in yield).

For securities with negative convexity we would see the opposite effect. When yields fall the convexity component results in a drag on price performance. However, when yields rise, prices fall at an accelerated rate.

Effective convexity is generally calculated as follows:

$$\text{Effective Convexity} = \frac{100}{\text{Price}_{\text{Base}}} \times \frac{\text{Price}_{+\Delta\text{Yield Scenario}} + \text{Price}_{-\Delta\text{Yield Scenario}} - 2 \times \text{Price}_{\text{Base}}}{\Delta\text{Yield}^2}$$

Equation 7-6
Effective Convexity

The multiplication by 100 is performed for scaling purposes.

Computing Effective Convexity

An example of the effective convexity calculation is as follows:

$\text{Price}_{\text{Base}} = 99.906$

$\Delta\text{Yield} = 50$ basis points

$\text{Price}_{-\Delta\text{Yield}} = 102.127$

$\text{Price}_{+\Delta\text{Yield}} = 97.393$

$$\text{Effective Convexity} = \frac{100}{99.906} \times \frac{97.393 + 102.127 - 2 \times 99.906}{0.5^2} = -1.169$$

Exercise 7-9

Using the prices from the previous exercise, compute the convexity of the three securities:

	Bond 1	Bond 2	Bond 3
–50bp	55.83	105.19	23.03
Base	45.00	104.22	26.75
+50bp	36.38	103.03	29.92
Convexity			

Estimating the Price Change

Now that we know the estimated duration and convexity, we can approximate the price change for a given parallel shift in the yield curve using Equation 7-4. Using our bond from the previous examples, let's project the percentage change in price for a 25 basis point decline in yield.

$$\text{Percentage Price Change} = \text{Duration} \times \Delta\text{Yield} + 0.5 \times \text{Convexity} \times \Delta\text{Yield}^2$$
$$= -4.738 \times (-0.25) + 0.5 \times (-1.169) \times (-0.25)^2$$
$$= 1.185 - 0.037$$
$$= 1.148$$

For a 25 basis point decline in rates, we would expect prices to increase by 1.148%. This would lead to the price rising from \$99.906 to \$101.053.

Exercise 7-10

Using the calculated durations and convexities compute the percentage price changes and the resulting prices for 25 and 100 basis point shifts in the yield curve for the three bonds in Exercises 7-8 and 7-9.

Work area

Price Change	Bond 1	Bond 2	Bond 3
25bp			
100bp			

Prices	Bond 1	Bond 2	Bond 3
25bp			
100bp			

REVIEW QUESTIONS

Suppose an investor were contemplating a purchase of an inverse floating rate bond. What would be the usefulness of examining a non-parallel shift in the yield curve?

Now assume that the stability of a PAC bond were being examined. How could a whipsaw scenario be used to track the stability of the security?

Why would one want to use a prepayment model to project prepayment rates in cases of changes in the shape of the yield curve?

ANSWERS TO EXERCISES

7-1

Semi-annual return = 3.96%

Annual equivalent return = 8.07%

7-2

Terminal	93.71
Coupon	4.17
Principal Returned	8.27
Ending Cash	106.15
Total Return (annualized)	12.68%

7-3

Total Rate of Return = 7%

7-4

Month	Balance	Amortized Principal	Prepaid Principal	Interest	Servicing	Investor Cashflow	CPR
0	1,000.00						
1	985.99	0.57	13.44	7.08	0.42	21.10	15%
2	967.27	0.57	18.15	6.98	0.41	25.70	20%
3	943.81	0.56	22.90	6.85	0.40	30.31	25%
4	915.63	0.55	27.62	6.69	0.39	34.86	30%
5	882.83	0.54	32.27	6.49	0.38	39.29	35%
6	845.54	0.52	36.77	6.25	0.37	43.55	40%

7-5

	Base Case	Case 1	Case 2
Starting Price	100.00	100.00	100.00
Terminal Price	95.00	100.00	110.00
Terminal Balance	918.75	918.75	918.75
Terminal Value	872.81	918.75	1,010.62
Coupon Income	41.04	41.04	41.04
Principal Returned	81.25	81.25	81.25
Ending Cash	995.10	1,041.04	1,132.91
Total Return (semi-annual)	-0.49%	4.10%	13.29%
Total Return (annualized)	-0.98%	8.38%	28.35%

7-6

Month	15% CPR Interest	5% CPR Interest	20% CPR Interest
1	7.08	7.08	7.08
2	6.98	7.05	6.95
3	6.89	7.02	6.82
4	6.79	6.98	6.69
5	6.69	6.95	6.56
6	6.60	6.91	6.44
Total	41.04	41.99	40.53

7-7

Horizon Price	105.00	106.00
Terminal Value	81.05	81.82
Coupon	3.83	3.83
Principal Returned	22.81	22.81
Ending Cash	107.6853	108.4572
Total Return (annualized)	15.96%	17.63%

7-8

	Bond 1	Bond 2	Bond 3
–50bp	55.83	105.19	23.03
Base	45.00	104.22	26.75
+50bp	36.38	103.03	29.92
Effective Duration	43.22	2.07	–25.76

7-9

	Bond 1	Bond 2	Bond 3
–50bp	55.83	105.19	23.03
Base	45.00	104.22	26.75
+50bp	36.38	103.03	29.92
Convexity	19.64	–0.84	–8.18

7-10

Price Change	Bond 1	Bond 2	Bond 3
25bp	11.42	0.49	–6.70
100bp	53.04	1.65	–29.85

Prices	Bond 1	Bond 2	Bond 3
25bp	50.14	104.73	24.96
100bp	68.87	105.94	18.77